Advanced Composites
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Lecture – 64
Buckling of an Infinitely Long Composite Plate

Hello, welcome to advanced composites, today is the fourth day of the ongoing eleventh week of this course, yesterday we had started developing the solution for buckling of an infinitely long plate, which was having a width of b meters and we had developed the following equation.

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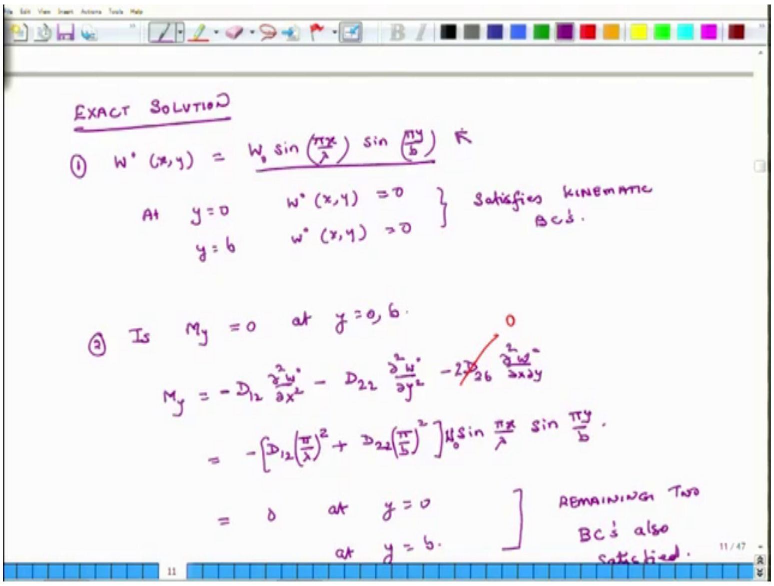

What we had also done was that, we had assume that displacement function for w between displacement out of plane in the out of plane direction and we have seen that if we use this kind of a function than this function satisfies all the boundary conditions associated with the infinitely long simply supported plate.

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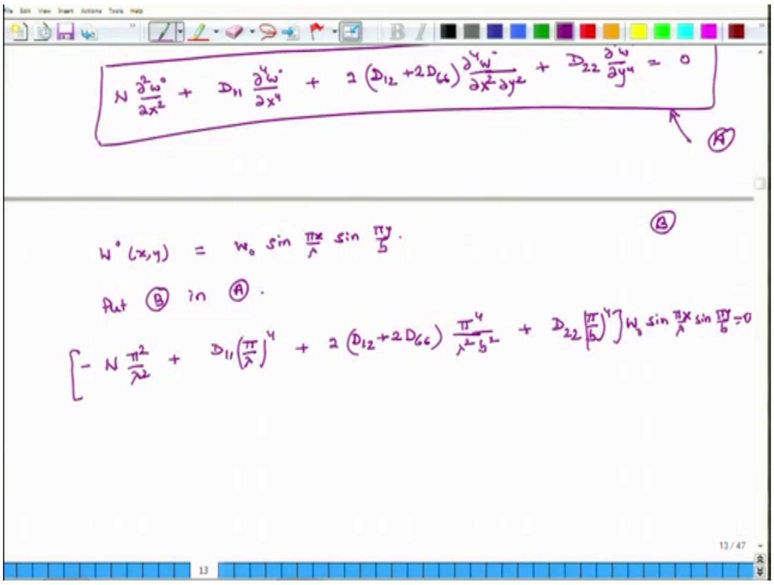

And then the next step was to somehow ensure and explore, whether this function also satisfies the boundary condition. So, what we will do is that in this modified differential equation, we will plug in the value or the function for w naught and see what it implies? So now, what we do is w naught x y is equal to w naught sin pi x over lambda sin pi y over b and lambda is a parameter and will interrupt what that lambda is b is the physical dimension of the plate its b meters wide. So, if I plug in what I so, I so, this is this equation A, this is B. So, we put B in A. So, what we get we get minus N and this is differentiated twice in x the first term.

So, it becomes pi square over lambda square plus D 1 1 pi over lambda to the power 4 plus 2 D 1 2 plus 2 D 6 6 pi to the power 4 over lambda square b square plus D 2 2 pi over b to the power 4 w naught sin pi x over lambda sin pi y over b and this equals 0.

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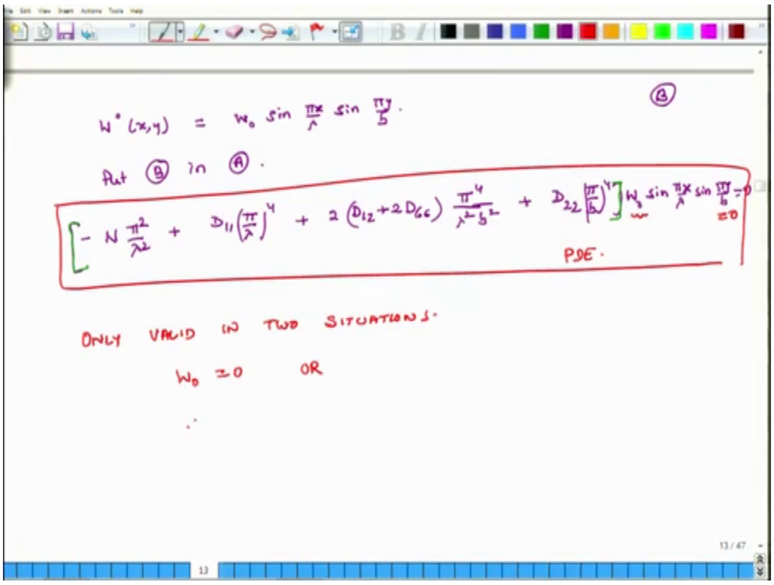

So, this is the equation and this equation we are getting. So, this is equal to 0, this equation we are getting from the PDE from the partial differential equation. So, this equation should be valid at all times, you should be valid at all times for values of x and for all values of y. Now, this is true only in 2 conditions. So, what I will do? I will put this thing in green brackets. So, only this is valid in 2 situations, what are the 2 situations? One is w naught is equal to 0 ok.

If w naught is equal to 0, which means that I have a perfectly flat plate and it is in an ideal situation, it does not experience any ideal disturbances it is homogenous, it is perfectly symmetric all the conditions which, we have described way back in the beginning of this week and then if I keep on pressing it using externally compression load then, it will never buckle. If it is perfectly flat and it does not get any disturbances and if it is perfectly homogenous and it is the lamination sequence is perfectly symmetric and so on and so forth.

And the loading is also perfectly at the mid plane, it is not slightly of centre. So, that is this thing because, this w naught is here. So, w naught is 0 then it will never buckle or.

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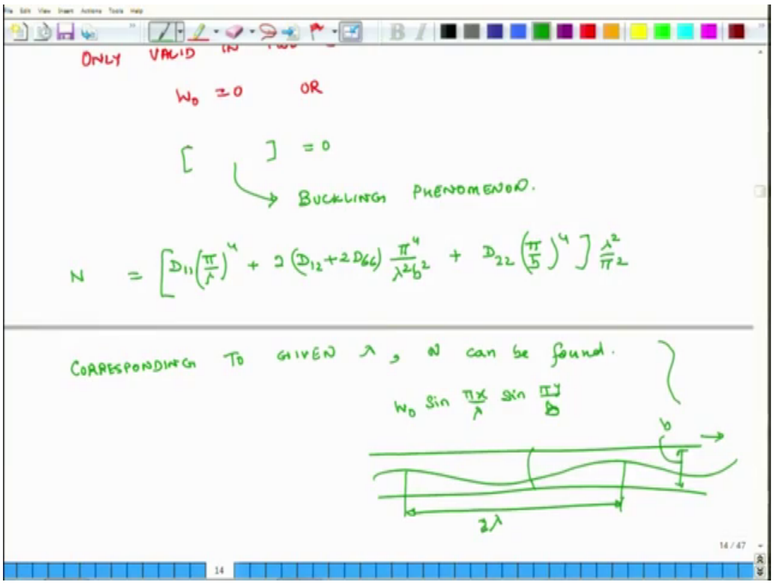

So, this is one condition or this term in green brackets, this becomes 0 and that condition corresponds to the buckling phenomena. So, either w is 0 or W can be non 0, but the term in the green brackets that has to be 0.

So, this term in the green brackets relates to buckling phenomena ok. So, when is it 0. So, let us write down. So, if this thing in green and then the green brackets, this is equal to 0 then it implies that N pi square over lambda square is equal to D 1 1 pi by 4 oh I am sorry, pi over lambda to the power 4 plus 2 D 1 2 plus 2 D 6 6 pi 4 over lambda square b square plus D 2 2 pi over b to the power 4 ok. So, this condition is 0, if this condition is satisfied then the term in the parenthesis is 0 or what I can do is, I can remove this thing and I can multiply this entire thing by lambda square by pi square and this pi ok.

So, that is my critical. So, corresponding to given lambda N can be found, what does that mean? That if so, what is what is w sin pi x over lambda sin pi y over b what does it mean? That this tube, this plate is there, which is infinitely long and the deflection pattern in this direction in the x direction is like this and. So, this is lambda, this is lambda, this is the wavelength of the buckling pattern in the x direction. Similarly, the wavelength in the y direction is b or actually, it is half wave length because, what is the full wave length? Full wave length will be like this.

So, this is this is b. So, we will half wavelength in the y direction is b and in the x direction, it is actually, this is 2 lambda ok. So, that is there. So, corresponding to any lambda any lambda, I can calculate in N. So, what; that means, is that the plate you keep on loading it and after sometime it will buckle and as I keep on increasing the load as I keep on increasing the load wavelength will keep on changing and this change will keep on happening continuously, it will not happen at discrete values, it will keep on happening continuously.

So, as I keep on increasing the load, this lambda will become as I increase N, what will happen to lambda? Lambda will become smaller. So, essentially what; that means, is that if I keep on pressing it, this wavelength keep on keeps on between smaller and smaller wavelength between keeps on between smaller and smaller. So, that is what it mean and this change will happen, continuously ok. This is what it means, another way to look at is that, if I want to know that associated with a lambda, there is a value of N, another question is that suppose, I imposed on it, some value of N then corresponding to that value of N, what will be the value of lambda?

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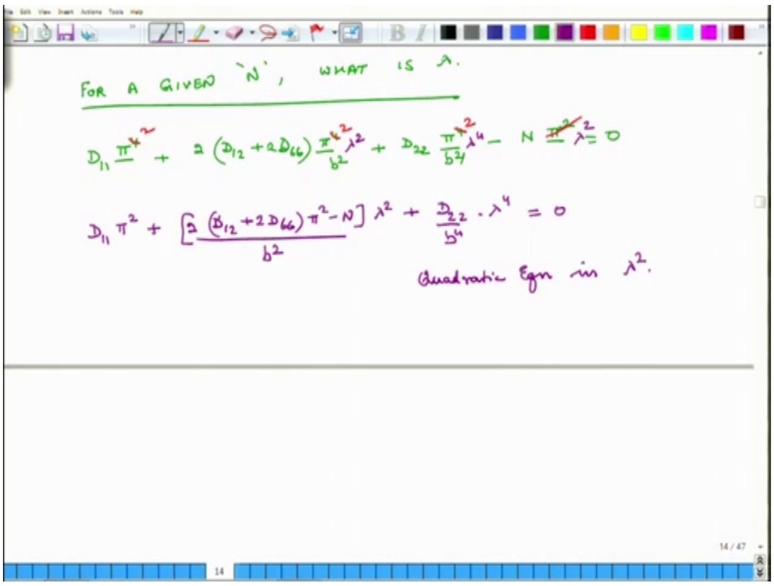

So, this is the other question. That for a given, what is lambda? So, that also we can solve this equation and we can find it out, because this is a equation, which is having terms in lambda 4 and lambda square, lambda to the power of 0. So, this like a quadratic equation lambda square and I can solve this quadratic equation and I can compute it. So, that is what we will do right now. So what I do is, I multiply this equation by lambda to the power 4. So, I get D 1 1 pi 4 over lambda to the power of 4. So, what I am doing is, I am reorganizing this entire equation, I am reorganizing this entire equation.

So, this is plus 2 D 1 2 plus 2 D 6 6 pi to the power 4 over lambda square b square plus D 2 2 pi to the power 4 over b square minus N pi square over lambda square is equal to 0, this is b to the power 4 ok. So, I delete pi square, this becomes pi square this becomes pi square and this becomes pi square and the other thing, I do is I multiply this entire equation by lambda square. So, I get no lambda to the power 4.

So, if I multiply the entire equation by lambda to the power 4, this goes away and this goes away and this goes away and instead, what I have is lambda to the power 2 here, lambda to the power 4 here and lambda to the power 2 here ok. So, I reorganized D 1 1 pi square plus 2 D 1 2 plus 2 D 6 6 pi square minus N times lambda square plus D 2 2 by b to the power 4 times lambda to the power 4 is equal to 0.

Student: what about b 2 (Refer Time: 13:26).

And there is a b square here also, there is a b square here also. So, this is again a quadratic equation in what it is in lambda square. So, from that we can compute value of lambda square ok.

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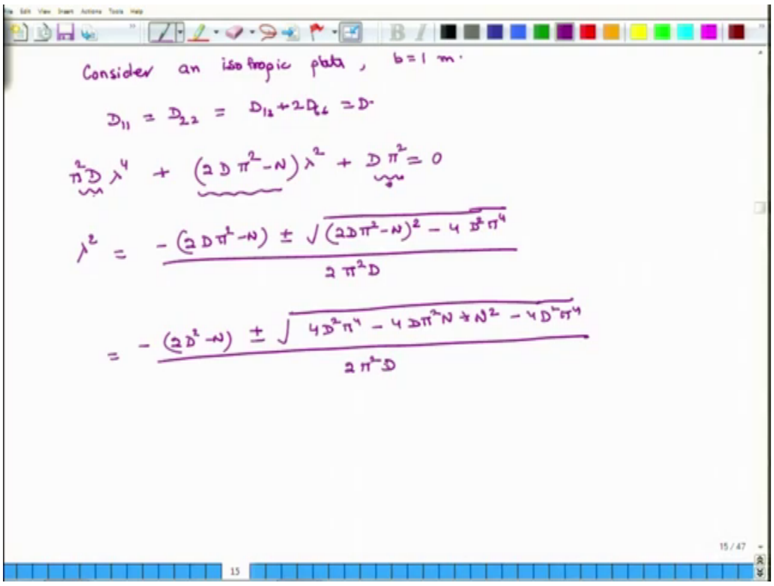

So now, we will consider the case consider an isotropic plate and which is 1 meters in width so, 1 meter wide. So, for an isotropic material we have shown earlier that D 1 1 is equal to D 2 2 is equal to D 1 2 plus 2 D 6 6 is equal to D ok. So, this equation becomes D times lambda to the power 4 plus 2 D or this pi square also here times pi square minus N times lambda to the power 2 plus D is equal to 0 right.

So, there should be pi square here also, there should be pi square here also and there should be a pi square here also. So, if I solve this, what do I get? Lambda square is equal to. So, this is a quadratic equation and I can solve for it. So, it is what is it minus b plus minus b square minus 4 a c divided by 2 a so, minus b. So, this is b. So, minus 2 D pi square minus N plus minus D square minus 4 a c. So, this is this is squared 2 D pi square minus N, whole thing square minus this times this. So, 4 D square pi to the power of 4 divided by 2 a.

So, it is 2 pi square D ok. So, we will continue this. So, this is equal to minus 2 D square minus N plus minus, what is this? So, this is 4 D square pi 4 minus 4 D pi square N minus N square or plus N square minus 4 D square pi 4 divided by 2 pi square D.

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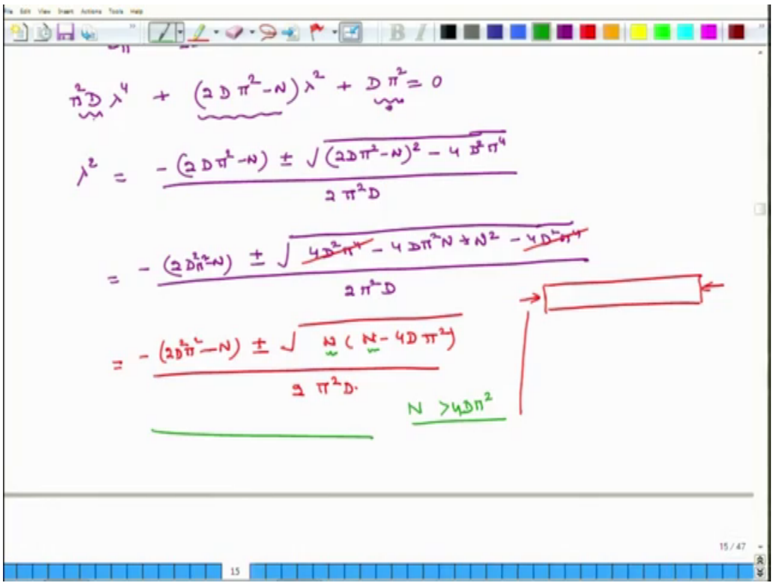

So, this term and this term they cancel out. So, this is equal to minus 2 D square minus N plus minus N into N minus and there is actually I am sorry, I missed the pi square here. So, so, this is N minus 4 D square right divided by 2 pi square D.

Student: (Refer Time: 17:48).

Yeah pi square.

Student: we starting also the 2 D square pi square minus N.

1 second. So, this is minus 2 D square pi square minus N and this is N minus 4 D pi square, 4 D pi square ok. So, what does this tell us? If N is very small so, what is N? This is the plate, infinitely long and I am applying N and N is compressive ok. Now if N becomes tensile, if N becomes tensile, this number is becomes negative and this number becomes negative right. So, anyway so, point what I am trying to is N is very small, if N is very small then the term under this thing will become negative. Suppose N is 0 then the term under the square root sin will be negative and what; that means, is this entire thing, it will become, it will have an imaginary route ok. This will become only positive non imaginary route, only when N exceeds D pi square 4 D pi square only N.

So, the plate will start buckling only once N exceeds a certain number because, otherwise the wavelength, which is lambda, it will be imaginary because, lambda square will be imaginary. So, lambda will also be imaginary. So, what this tells us is that initially, the plate will not buckle, but as I keep on increasing my external load, the plate after certain point of time, it will start buckling and once it starts buckling, it will develop it is wavelength and we can calculate the wavelength from this relation and we will get 2 values for each value of N. So, we have to pick up the smaller value. So, that is there.

And before buckling, what is the solution? Before buckling, the solution is w 0 equals 0, which we have seen here. So, before buckling, this is the solution w naught equal 0, the plate is flat and after the plate buckles, the wavelength corresponding to the N can be computed from this relationship. So, this is what I wanted to show today. Tomorrow we will continue this discussion. So, what we have discussed in so far is in context of infinitely long plates, tomorrow we will see, how we can use the same theory and same understanding in context of finite plates, plates which are rectangular, but of finite dimensions. So, that is pretty much it for today and I look forward to seeing you tomorrow.

Thank you.