Advanced Composites

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Lecture - 57
Role of D_{16} and D_{26} Terms On Laminated Plate Response (Part-III)

Hello, Welcome to advanced composites. Today is the third day of the 10th week of this course. Over last two days we had been developing the solution for a plate which was rectangular simply supported symmetric in nature, but its D 1 6 and D 2 6 terms were not zero.

And what we have found is that if the plate is not symmetric, and if the plate is symmetric, but its D 1 6 and D 2 6 terms are not zero, then in the first thing what happens is that in the differential equation we get these additional terms which are cosine terms if the assumed solution for the deflection pattern is sinusoidal. The differential equation also generates cosine terms which are multiplied by D 1 6 and D 2 6. And all that gets reflected in the solution. So, with that understanding now what we will do is we will start exploring how does the presence of D 1 6 and D 2 6 influence the solution and other stuff related to the plate.

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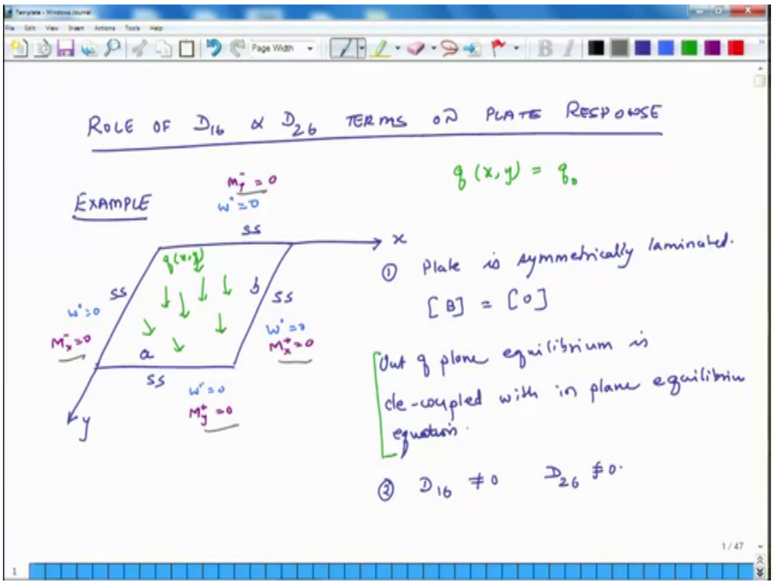

Now in this plate we had said that this plate is simply supported on all the four edges, which means that on all the four edges moments respective moments are 0. So, on the edge x is equal to 0, and x is equal to a, M x is 0 and on edges y is equal to 0, and y is equal to b and y is 0. Let us see what our solution gives us. So, we have figured out how to calculate W 1 1, W 1 2, W 2 1, and W 2 2.

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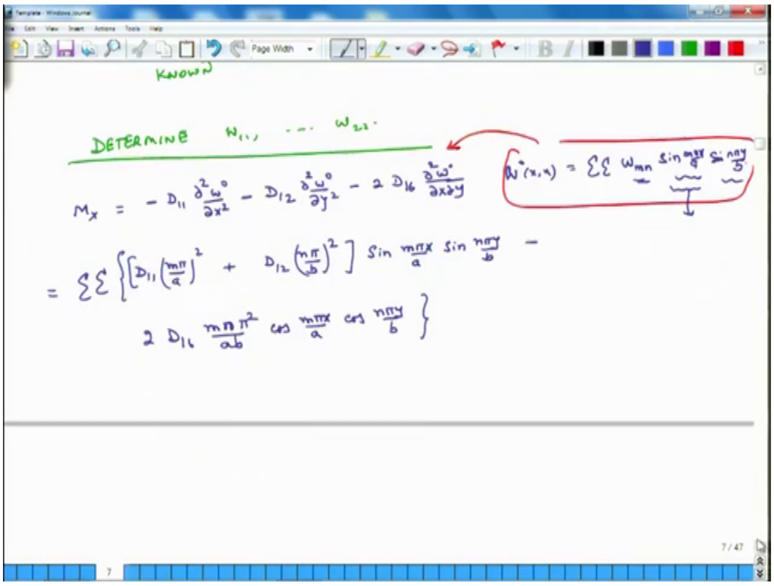

Now, if we want to compute M x we know that M x is equal to minus D 1 1 del 2 w naught over del x square minus D 1 2 del 2 w naught over del y square minus 2 D 1 6 del 2 w naught over del x del y. And here W naught x y is equal to sum of w m n sin m pi x over a sin n pi y over b. And we can find these coefficients w m n exactly by using the process which we had described earlier. We have developed the process for a two term solution, but we can extend the same process for solutions which have three terms or four terms or whatever. And as we increase the number of terms our solution is going to get more and more accurate.

So, if I plug in this relation back into the expression for w M x what do I get, so I get M x is equal to so this sin term is getting differentiated twice. So, I get D 1 1 m pi over a whole square plus D 1 2 n pi over b whole square. And this thing is multiplied by sin m pi x over a sin n pi y over b. And then now we have a D 1 6 term. So, I have minus so the negative sign here, this negative sign and this negative sign they disappear because this thing is being differentiated once or twice, so I get sin then I get cosine. And when I cosine differentiate cosine, it becomes negative right and same thing happens here. So, actually I should still have a negative sign here right.

So, I get one negative sign because I am differentiating this twice, yes. So, I am sorry, so this is this becomes positive. Now, when I differentiate this w with respect to x and y, there is no reversal of sin because sin just gets differentiated into cosine. So, this negative sign remains and I get 2 D 1 6 m n m n pi square over a b cosine m pi x over a cosine n pi y over b. And this entire thing I have to sum up over the index m n n, this is what I get.

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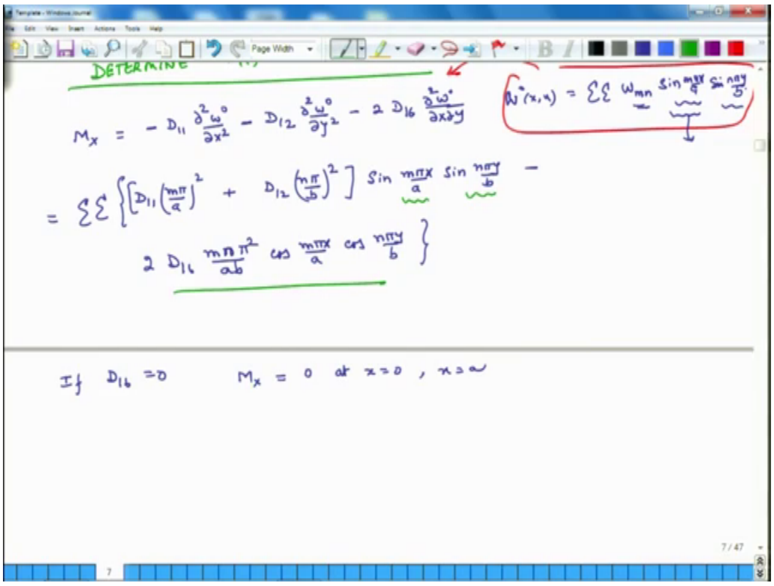

Now, let us look at the value. So, what is the value of, so if D 1 6 was 0. In case D 1 6 was 0 that is each ply was orthotropic in nature if D 1 6 was 0, then M x would be 0 at x is equal to 0, and also at x is equal to a. Because if D 1 6 was 0, then this term does not exist and sin terms vanish at x is equal to 0 and at x is equal to a. So, M x condition is satisfied. Similarly, if we want to compute M y, we will find that M y if D 2 6 was 0, it will remain 0 on y is equal to 0 and y is equal to b edges.

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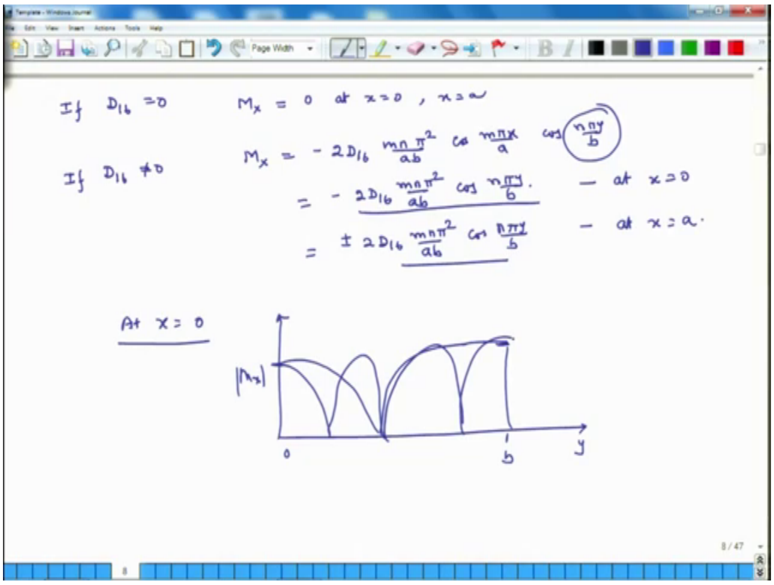

But if D 1 6 is not 0, D 1 6 is not 0, then what happens then M x is equal to minus 2 D 1 6 m n pi square over a b cosine m pi x over a cosine n pi y over b. And what is its value it at x is equal to 0, it will be minus 2 D 1 6 m n pi square over a b. And if x is equal to 0, then its value is cosine n pi y over b because (Refer Time: 08:00).

So, this is at x is equal to 0. And this is equal to minus 2 D 1 6 m n pi square over a b cosine m pi y over b. And this will be plus minus at x is equal to a. Because if at x is equal to a, what is the value of cosine M x over a, it will be cosine m pi. So, suppose m is 1, then it will be negative 1. So, this negative 1 will get multiplied by minus and it will become a plus; if x is.

Student: (Refer Time: 08:50) you just find out.

M pi over.

Student: point (Refer Time: 08:57).

And at if m is equal to 2, then it will be cosine 2 pi which will be 1. So, this will oscillate anyway. So, at x is equal to 0, what is the shape of M x. So, this is 0, so this is x, suppose this is x is equal to 0 edge actually, suppose I am doing y on this axis and this is 0 to b. So, it will be, and here if I am plotting M x, and this is I am plotting at x is equal to 0, so it will be this sort of thing. So, it will be something like this. I am just plotting the magnitude and it depends on the value of n.

So, if n is equal to 1, then it will be pi y over b cosine pi over b. So, it depends so, it can be it can do this, but anyway, the point is that it will start from some maximum value and it will end at the same maximum value it can do this also and so on and so forth.

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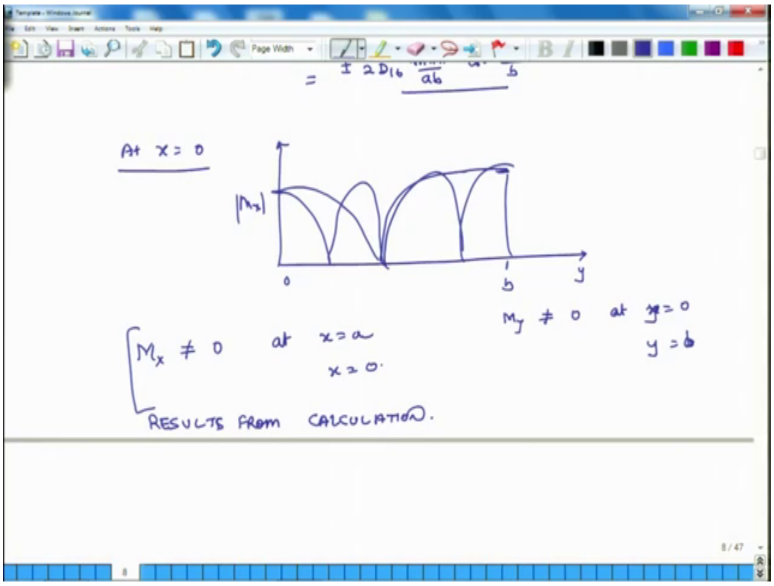

So, what we see is that M x is not equal to 0, at x is equal to a, and at x is equal to 0. Similarly, we see there we can show that M y is not equal to 0 at y is equal to 0 and y is equal to b. So, this is what computation tells us. So, this is the result from computation, results from calculation. So, calculation tells us that M x and y are not 0 at the simply supported edges, but these edges are simply supported so, it has to be 0 right. So, we get the solution or the calculation of M x we get an error.

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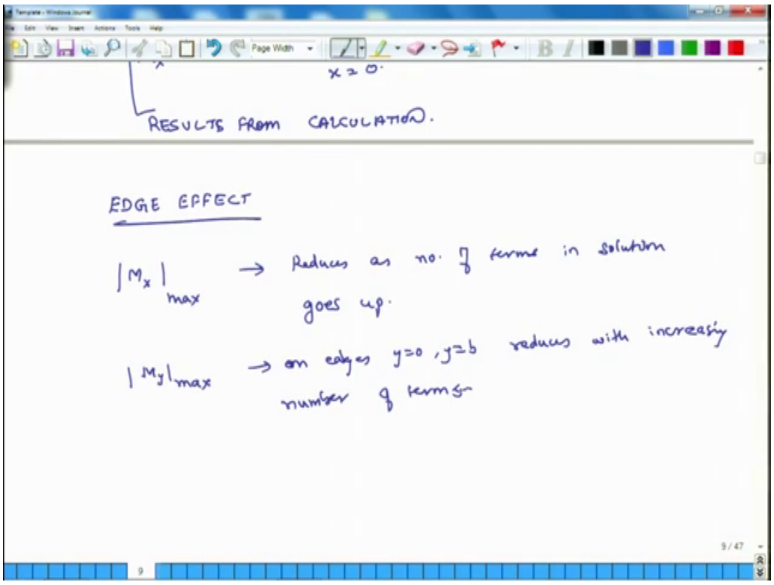

So, this difference between actual value and the computational value, there is a difference and this thing is known as the edge effect. This is known as edge effect and the magnitude of M x, so suppose and on the edge it will vary from point to point. So, suppose its maximum value is something M x max, this magnitude this reduces as number of terms in solution goes down or goes up. Same thing is true for M y also. The value of M y max on edges y is equal to 0, y is equal to b, it reduces with increasing number of terms. So, this is that.

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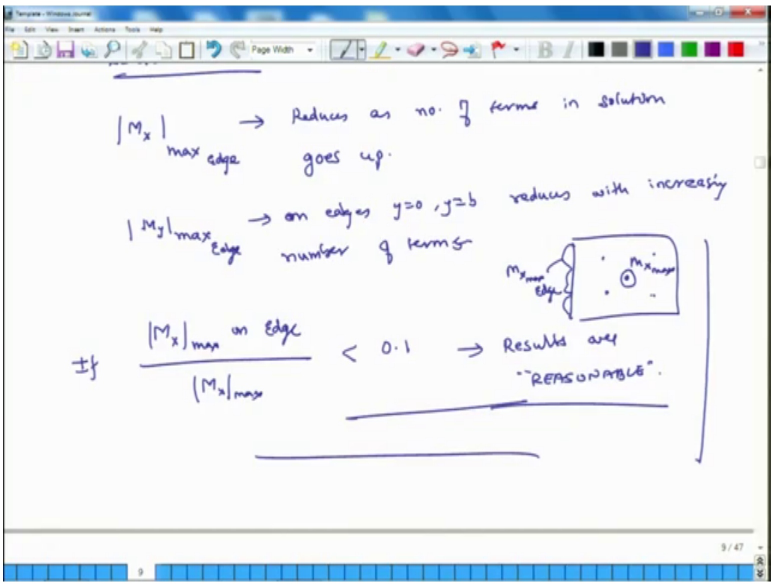

And typically we say that this is in a reasonable border if M x max on edge is a divided by M x max. So, actually here I will say edge and this is also I will say edge. So, M x will have a maximum value on edge; and M x will have an even higher value somewhere in the middle of the plate. On edge it will be small, but it will be nonzero, and but in the plate if you compute somewhere here the bending moment will be maximum. So, M x will be maximum here, so that is what I call M x max and M x is maximum on the edge and that maxima I call as M x max edge.

So, if this ratio is typically if it is less than 1 if this ratio, so we keep on increasing the number solution till this ratio becomes less than 10 percent then we say, then we can say that results are reasonable. So, this problem appears in analytical formulations like we did it using the special Galerkin method or even if you try to do the same thing in finite element analysis. We again come across this edge effect, and it comes in again and again.

So, this is a very important feature and because of this we tend to get some artificially high stresses also on the edge which may not necessarily be there in the real plate because in the real plate they mean there are no moments on the on the edges. So, this is one feature or one artifact which comes up because of D 1 6 and D 2 6.

So, another thing is related to the position of the values of M x for at different locations and how those vary as we introduce the terms D 1 6 and D 2 6 so that is another thing we will look tomorrow. And we will continue this discussion on D 1 6 and D 2 6 tomorrow as well.

So, thank you, and I look forward to seeing you tomorrow. Bye.