Advanced Composites
Prof. Nachiketa Tiwari
Department of Mechanical Engineering
Indian Institute of Technology, Kanpur

Lecture – 55
Role of D_{16} and D_{26} Terms On Laminated Plate Response (Part –I)

Hello. Welcome to Advanced Composites. Today is the start of the 10 week of this particular course. And over this week will continue our discussion of laminated composite response. And specifically we will touch 2 important topics. Till so far we have solved different types of problems for composite plates.

Semi-infinite beams, rectangular plates, square plates, but in every single case which we have done, we have assumed that the terms D 1 6 and D 2 6 are 0. And then for these types of plates we have solved the problem. Now, today what we are going to do is we are going to explore the role of D 1 6 and D 2 6 as to how it influences the behavior of the plates in terms of different parameters.

So, that is one thing we will discuss in detail and will try to get developed some quantitative as well as qualitative understanding. The other thing we will do is, that we have till so far developed solutions for plates which were in static equilibrium. So, the other thing we will do over this week is we will also extend our equilibrium equations to the situation where things are in dynamic equilibrium.

And when such an equilibrium how to plates behave, and is specifically we will learn how to compute the natural frequencies of these composite plates; so, that is the overall agenda for this week. And that is what we plan to do over this week. So, as I said the first thing we are going to explore is figure out how does how do these terms D 1 6 and D 2 6 terms they influence the response of the plate. That is what we want to discuss.

Now, we have seen earlier several weeks back, that these terms D 1 6 and D 2 6, they couple the bending and twisting response of the plates. So, if a plate is being subjected to pure bending forces bending moments, then the plate in presence of D 1 6 and D 2 6 it not only bends, but it also tries to twist, because the terms D 1 6 and D 2 6 they couple the bending moments M x and M y to the twist curvature k x y on one side, and the twist curvature M x y to bending curvature k x and k y. So, that is one thing I would like to add. But now what we will do is, we will begin into this issue a little deeper and see the mathematics underlying the role of D 1 6 and D 2 6 terms.

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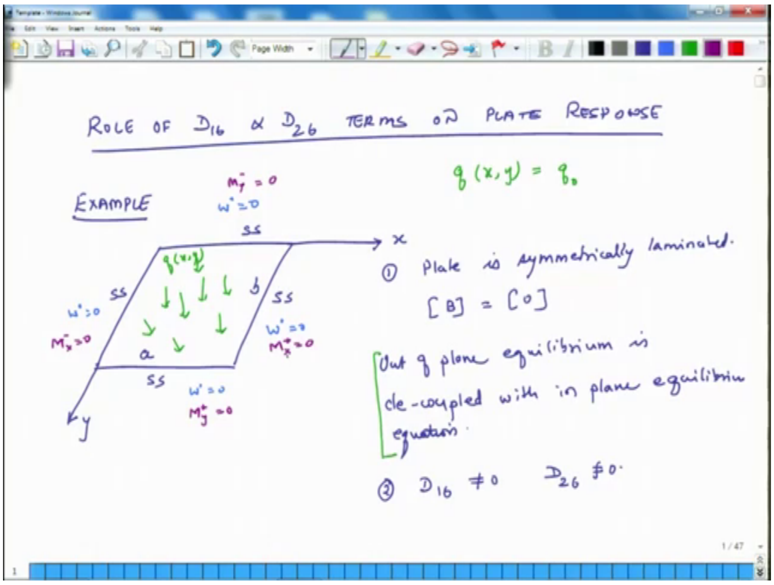

So, our theme is going to be role of D 1 6 and D 2 6 terms on plate response. So, what we will do is, we will consider a simply supported plate; so, we will learn through example. And the example is of a plate which is simply supported. So, this is my x, this is y direction and the plate is simply supported on all the 4 sides. We assume that the length of the plate is a and the width of the plate is b, and we make the following assumptions. One, we assume that plate is symmetric, plate is symmetrically laminated.

So, because it is symmetric eliminated, the bending extension coupling matrix is 0. And because, of this we can say that out of plane equilibrium equation here is decoupled with in plane equilibrium equation. Also one thing, I would like to add is that this simply supported plate is see a vertical it is transversely looted in the vertical direction by distributed load call q x y. And let us assume for purpose is a simplicity that q x y is constant.

So, we say that q x y is constant. So, it is uniformly distributed load and it is q naught. So, because the plate a symmetrically located, we can say that the out of plane equilibrium equation is decoupled with the in plane equilibrium equation, and we have discussed this several times. And we also assume that the plate lamination layers are not orthotropically oriented. So, because of this D 1 6 is not equal to 0 and D 2 6 is not equal to 0. So, then we what do we do? So, if I have to find the response of the plate I have to find the out of plane equilibrium.

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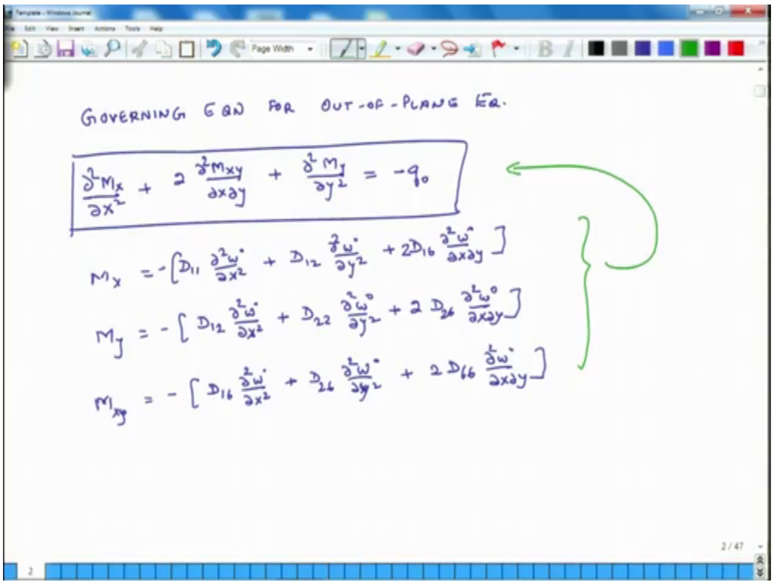

And the governing equation for out of plane equilibrium for out of plane; so, what is the governing equation? It is del 2 M x over del x square plus 2 del to M x y over del x del y plus second derivative of M y with respect to y. And that equals minus q naught so, it should be q x y, but here q x y is same is constant and it is q naught.

So, this is the governing equation. And we would like to solve for this governing equation. So, the first thing we will do for solving it is we will express M x M y and M x y in terms of w. So, we know that M x from the A B D relations is equal to D 1 1 del 2 w naught over del x square, and this is everything is negative. So, it minus D 1 1 so, plus D 1 2 del 2 w naught over D y square plus D 1 6 del 2 w naught over del x del y.

And actually there is a 2 here, because the twisting curvature is twice of second derivative of w with respect to x and y. Similarly, M y equals minus D 1 2 del 2 w naught over dell x square, plus D 2 2 del 2 w naught over del y square plus 2 D 2 6 del 2 w naught over del x del y. And M x y equals minus D 1 6 curvature in x direction plus D 2 6 del 2 w over del del y square plus 2 D 6 6 del 2 w naught over del x del y. So, we take these definitions, and we plug it into the governing differential equation. So, once we do that the overall equation we get is a long one.

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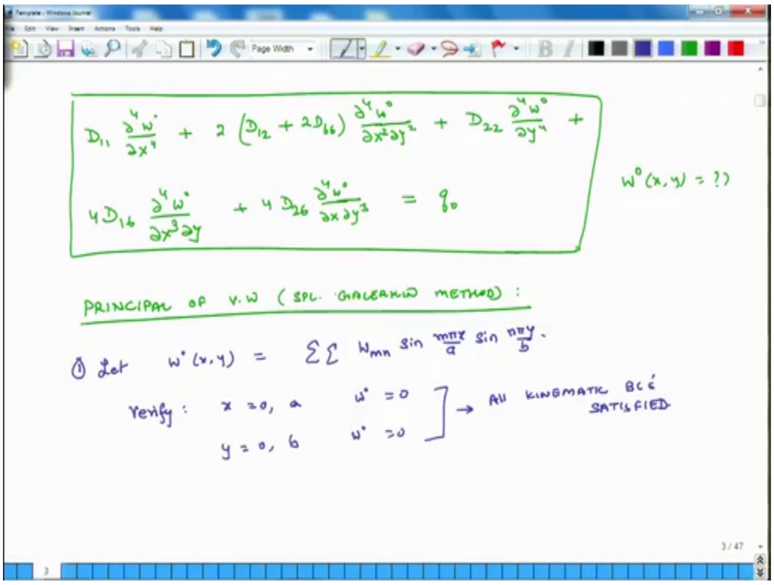

So, what we get is D 1 1 del 4 w naught over del x 4, plus 2 D 1 2 plus 2 D 6 6 del 4 w naught plus over del x square del y square plus D 2 2 del 4 w naught over del y 4, plus D 1 6 and in D 1 6 we get. So, this one D 1 6 term in M x and another D on 6 in M x y; so, I get 2 D 1 6 here, and this term gets differentiated twice in x.

So, I get D w D 3 D 4 w D x cube D y. And from here this is being differentiated again with respect to x and y. So, I get the same thing here. So, I get basically 4 D 1 6 del 4 w del x cube del y. So, I get 4 D 1 6 del 4 w del x cube del y, and likewise I also get 4 D 2 6 del 4 w del x del y cube, and then I get a D 6 6 term. So, that we have already taken care of D 6 6. So, that you are fine with. So, I get these 2 extra D 6 6 terms, and all these things add up to be equal to q naught.

So, our aim is to find the solution for w. So, given this w this is what we want to find from this equation. Now what we will do is we will use the principle of virtual work, virtual work and in that context we will actually use the special the Galerkin method to solve this equation. So, when we do that, when we are trying to use special Galerkin method what is it? First we have to see: what are the boundary conditions.

So, boundary condition here is w on this edge is equal 0 w 0 on this edge, w 0 on this edge and w 0 on all the 4 edges. This is one boundary condition. So, this is the boundary condition associated with kinematic constraint which is w. And the other boundary condition is on this edge m x is equal to 0 m x is equal to 0. So, this is plus, this is minus on this edge it is m y plus equal 0 and here m y minus equal 0. So, our choice of the trial functions for a special Galerkin method; requires the only thing it requires is that we should be able to satisfy only the kinematic boundary condition. We do not need to satisfy the condition related to moments being equal to 0 or all the 4 edges, ok.

So, we choose so, we say let w naught which is a function of x and y b a sum of w m n sin m pi x over a sin n pi y over b. And we see we check whether this function satisfies all the conditions related to w. So, we see that so, we verify that at x is equal to 0 and a w is equal to 0. And at the other 2 edges y is equal to 0 and b w is equal to 0. So, as long as we are satisfying all the kinematic boundary conditions which are that is with those which are related w we are. So, all kinematic boundary conditions satisfied, all kinematic boundary conditions satisfied.

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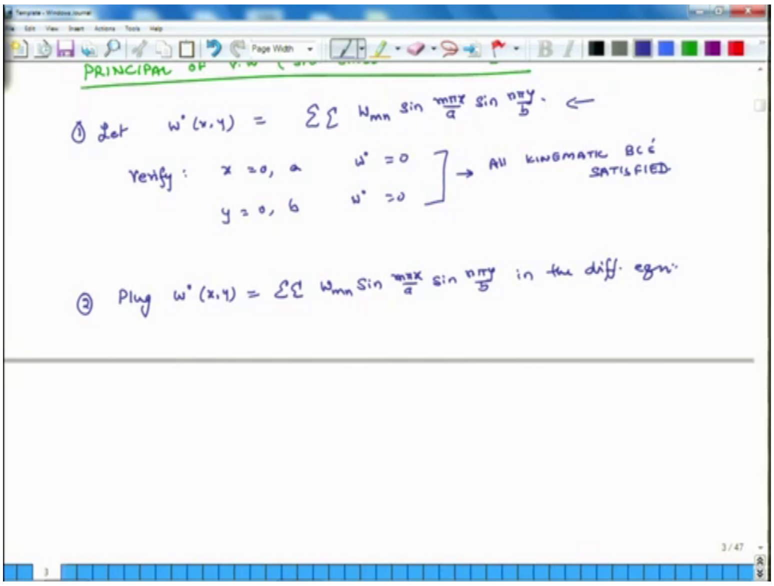

So, the second thing is we plug this w m n because we are using this special Galerkin method, and we plug it into the governing differential equation. And for making our life a little simpler actually first we will just plug it in and then we will make some simplifications. So, the second thing is this we plug this expression for w back into the governing differential equation. So, plug w not x y is equal to summation of w m n sin m pi x over a sin n pi y over b in the differential equation.

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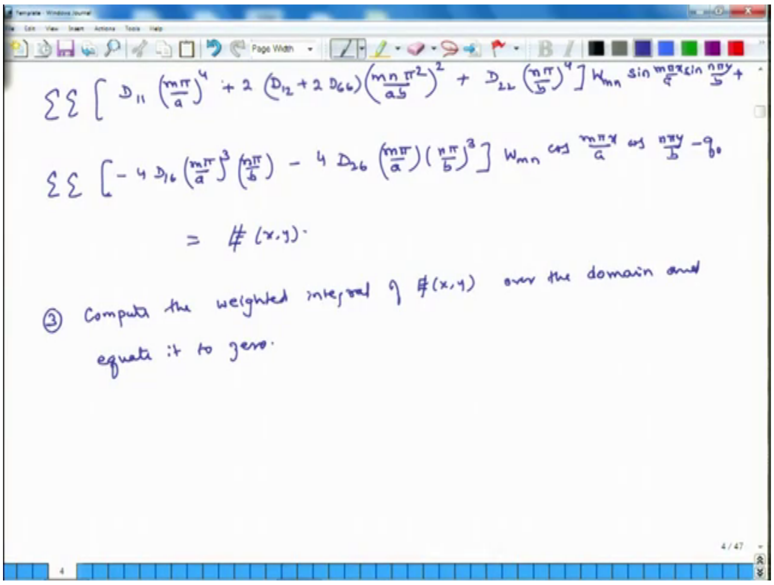

So, we do that and what do we get. So, if I plug everything in and I do all my math this is what I get I get a long expression. And this is D 1 1 m pi over a to the power of 4 plus 2 D 1 2 2 plus D 6 6 m n pi square over a b whole square plus D 2 2 n pi over b to the power of 4. And this entire thing is multiplied by w m n sin m pi x over a, sin n phi y over b plus. And then there are terms involve in D 1 6.

And in D 1 6 we have third derivative of w with respect to x and the D 2 6 involves the derivative of w with respect to yeah. So, the point is that here we will get cosine terms, ok. So, we get another summation and we have minus 4 D 1 6 m pi over a cube n phi over b minus 4 D 2 6 m pi over a n pi over b cube w m n cosine m phi x over a cosine n phi y over b. So, this is there.

And then minus q naught equals 0. Now, once we plug in clearly, all this summation does not necessarily add up to q naught and if it does not add up to q naught, it means that there is an error in this equation. So, this should be ideally equal to 0, but in reality it is not equal to 0. So, we call this so, this entire thing is equal to error.

And this error changes with respect to x and y, error with respect to x and y; so, that is the second step that we compute the error function. We compute the error in when we plug this assume displacement function back in the differential equation I get an error, and then because it is a special Galerkin method. So, we multiply this error by a trial function, and because it is a special Galerkin method this trial function is of the same shape as the original displacement function. And then we integrate the whole thing over the domain of the system. So, essentially what we do is third compute the weighted integral, weighted integral because here of this error over the domain and equate it to 0, ok. So, what do we do?

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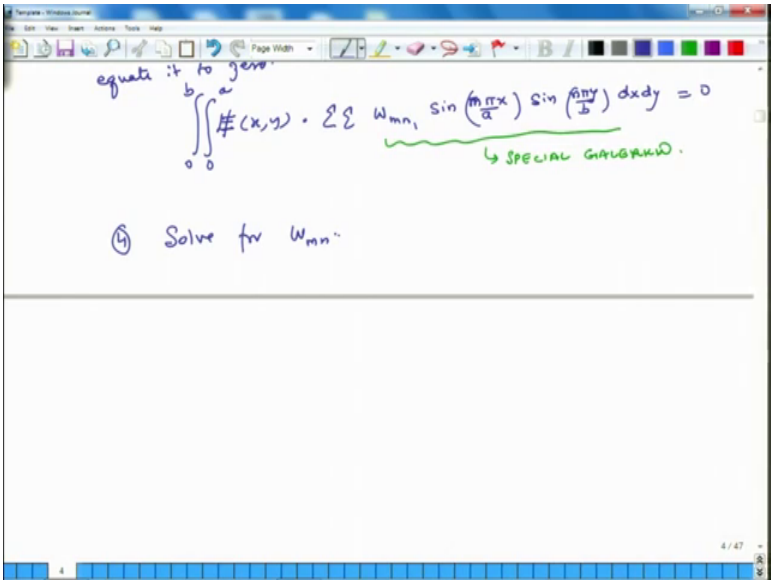

Mathematically what it means is, that I take this error and I know what this a long function is. I multiply it by a trial function; so, let that trial function be w m n 1 this is the virtual displacement and it is having the same shape as the original except that it is amplitude is w m n 1 not w m n m pi x over a sin n pi y over b. And I integrate it over the domain.

So, I am integrating it from 0 to a and 0 to b. So, if there are n m times n terms in the error component, then they are same number of terms are there in the weighted function this w m n 1. So, the total number of terms in this entire integral will be m times n whole square; which is m square times n square. And this entire thing is equated to 0.

So, this is we this choice is because we are using a special Galerkin. If we were using general Galerkin and then it could have been any functions. But in a special Galerkin we use the same function, ok. So, we say that the error is not exactly 0 at every single point, but the weighted integral of this error over the entire domain is 0.

So, if so in this way we get an approximate solution; in similar approaches also using finite element analysis. So, this is the third step. Now what we do here is, we go to the 4th step and then we actually solve for. So, the 4th step is solve for w m n. And we will actually illustrated by in the in the next step.

So, solve for w m n. So, how do we do? So, the next step is that we will actually solve for this, and what we will do is we will actually show how to solve for it using a 2 terms series for w m n. So, it will have 4 terms for w m n sin pi x over a times sin pi y over b plus 2 pi x over a times hence so on and so forth. So, we will then in that case we will actually solve for different coefficients. And that is something we will do in the next class.

So, till then I will close this discussion and will meet once again tomorrow, ok.

Thanks.