Advanced Composites
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Lecture - 53
Beam (two term solution)

Hello. Welcome to advanced composites. Today is the fifth day of the ongoing week which is the ninth week of this course. Last 3 days we have been discussing the principle of virtual work and how it can be used as a powerful method to solve problems related to deflections in composite plates; where the analytical solution may not necessarily be either available or it may be non-obvious.

Today we will continue this discussion, and we will solve the problem in context of a beam, but here we will not have just one term solution, but a multiple term solution. And we will see how we can have multiple that is a series solution for deflections and we will illustrated through the example of a beam which is isotropic, and pinned at both ends not simple both ends

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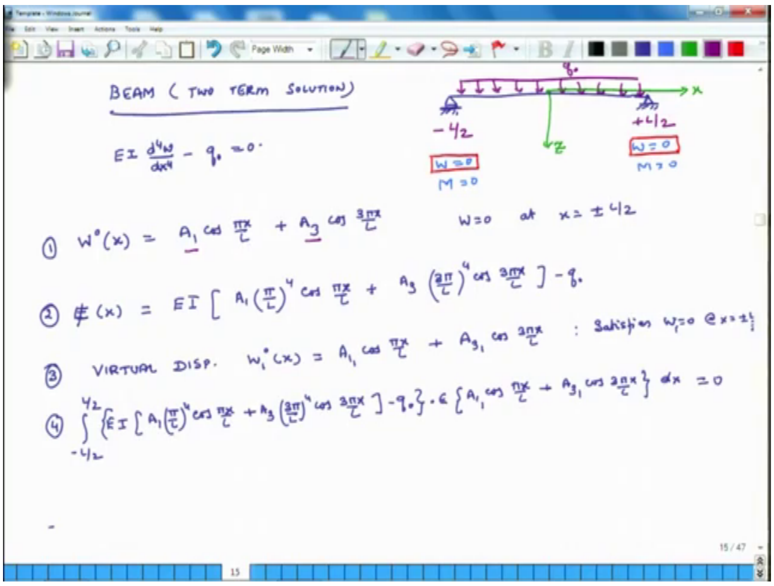

So, we will generate the solution for a beam for 2 term solution. So, if we know how to do 2 we can do 3 4 term or whatever. So, we will generate a 2 term solution, and once again the beam is simply supported on both sides. So, beam axis is at the centre. So, this is my x axis, this is my z axis and I am interested in finding out the deflections for the beam. The beam is again experiencing uniformly distributed normal load; which is q Newton per meters (Refer Time: 02:20)

The length of the beam is L. So, this is minus L over 2 and this is plus L over 2. What are the boundary conditions? The boundary condition is that w is equal to 0 at this end and also the moment is 0 at this end and w is 0 at this end, and the moment is 0 at this end, because it is simply supported on both the points. And to remind you, if we use virtual work, then we will only worry about the kinematic boundary conditions; that is boundary conditions which relate to displacement and slopes, not related it does not relate to a kinematic boundary condition, do not relate to forces and moments. So, we will worry only about w is equal to 0 at both ends.

Before we do that ok; so, what is the governing differential equation? The differential equation for the system is EI d 4 w over d x 4 minus q naught is equal to 0. This is the governing differential equation; we have seen this earlier. And we will say that the displacement is having 2 terms A 1 cosine pi x over L plus another term A 3 cosine 3 pi x over L. So, this we have chosen this function, this function W is equal to 0 at x is equal to minus both plus and minus L over 2. So, this function satisfies both the kinematic boundary conditions. So, we are fine with this function ok. So, this was a step one.

The next step is that we have 2 find the error, error force; error and here it is only a function of x is if I plug this w back into the governing differential equation. So, this is equal to EI A 1 pi over L 4 cosine pi x over L plus A 3 3 pi over L to the power of 4 cosine 3 pi x over L minus q naught. This is the error oh, I am sorry this q naught has to be outside the bracket. So, this is the error or the resistive.

Third we have to write the statement for virtual work is equal to 0, and for that we have to first select a function for virtual displacement. So, virtual displacement now here is the trick, in the function for w there are 2 terms. So, we and what are unknowns? The unknowns are A 1 and A 3. So, when we choose the error the function for displacement, there also we choose a 2 term virtual displacement field. That will make things easier.

So, virtual displacement is W 1 x, and again we use the Galerkin method special Galerkin method. So, we use similar functions. So, this is equal to A 11; excuse me A 1 1 cosine pi x over L plus A 1 2. I am sorry, A 3 1 so, this extra subscript implies that it is virtual displacement, cosine 3 pi x over L. And we have to make sure that this virtual displacement fields satisfies all the kinematic boundary conditions. Because this function is same as the actual displacement field, it satisfies all the kinematic boundary conditions. So, satisfies W is equal to 0 at x is equal to plus minus L over excuse me over 2 so, that is there.

So, the last step is now we say that we have to compute the virtual work and equate it to 0. So, what is virtual work? It is error force times virtual displacement integrated over the domain.

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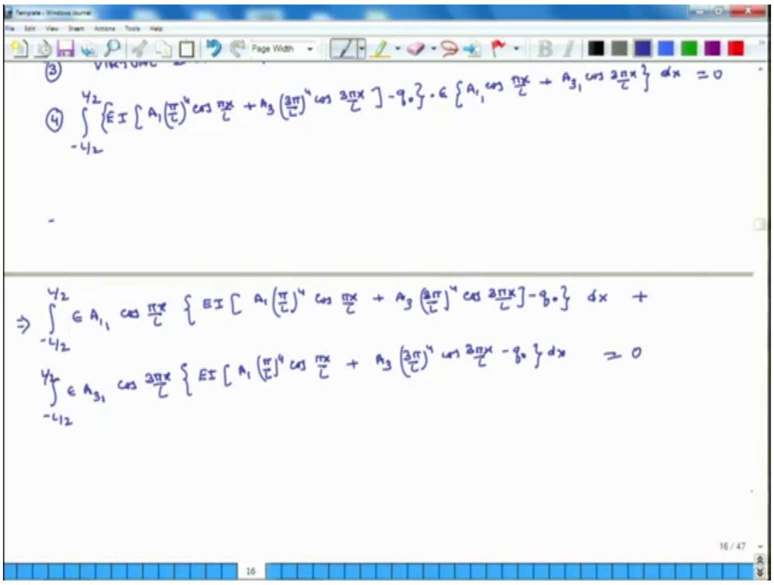

So, minus L over 2 to L over 2 EI A 1 pi over L to the power of 4 cosine pi x over L plus A 3 3 pi over L 4 cosine 3 pi x over L minus q naught. This is the error times parameter of a smallness which is epsilon times virtual displacement field A 1 cosine pi x over L plus A 3 1 cosine 3 pi x over L dx. And this is the overall virtual works. So, it has to become 0 as per the principle of virtual work.

Now, what we do is, we have a multiple of virtual work with respect to error force, and we reorganise these things. So, so what I get is minus L over 2 to L over 2. And first I multiply everything inside, and I collect all the parameters which are multiplied by A 1 1 , ok. So, epsilon A 1 1 cosine pi x over L times EI A 1 pi over L to the power of 4 cosine pi x over L plus A 3 3 pi over L to the power of 4 cosine 3 pi x over L minus q naught. So, d x so, this is one integral plus minus L by 2 to L by 2, epsilon A 3 1 cosine 3 pi x over L EI A 1 pi over L to the power of 4 cosine pi x over L, plus A 3 3 pi over L 4 cosine 3 pi x over L minus q naught dx is equal to 0. So, I just reorganise whatever was there in the integral into 2 separate integrals that is all I have done.

Now, what I will do is, I will move this integral sign. So, I will take the epsilon A 1 1 out.

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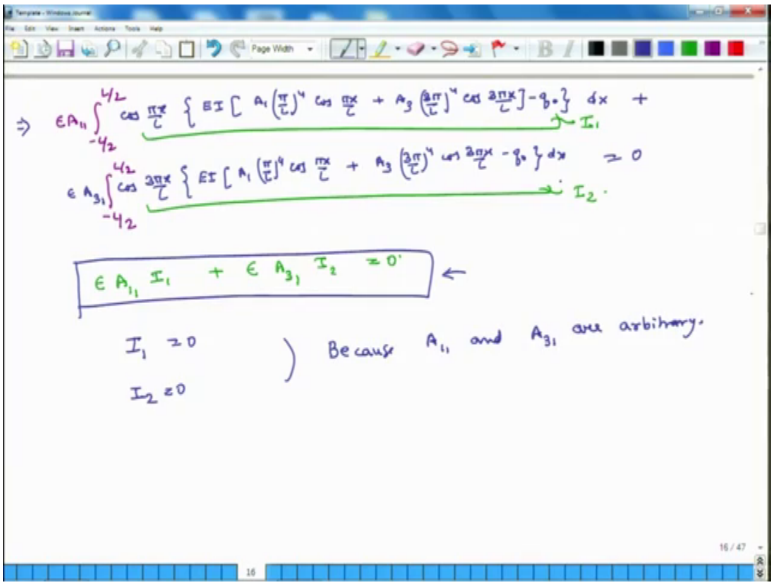

Like this, and I will do the same thing to the second integral. Because A 3 1 and A 1 1 they are constants. And I call this entire integral I one and I call this entire integral I 2. So, essentially what I get is epsilon A 1 1 I 1 plus epsilon A 3 1 I 2 is equal to 0.

Now, this relation this A 1 1 and A 3 1 they are arbitrary numbers, they can be anything they can be anything. So, for this relation to be true, the first term has to be individually 0 and the second term has to be individually 0. Because A 1 1 and A 3 1 are arbitrary numbers, they can be anything and they are not 0.

So, what that means is that, I one should be 0 and I 2 should be 0. Because A 1 1 and A 3 1 are arbitrary, they can be anything, ok. But this equation has to be true for all values of A 1 1 and A 3 1. So, it can be true only if I one is individually 0 and I 2 is individually 0.

Now, if you look at I one you get basically you get an equation in A 1 and A 3. And if you look at I 2 you get another equation in A 1 and A 3, ok. So, you get 2 equations with 2 unknowns. So, basically you will get when you solve this I 1 and I 2 and you reorganise, you will get a 2 by 2 equations system and this is how it will become, ok.

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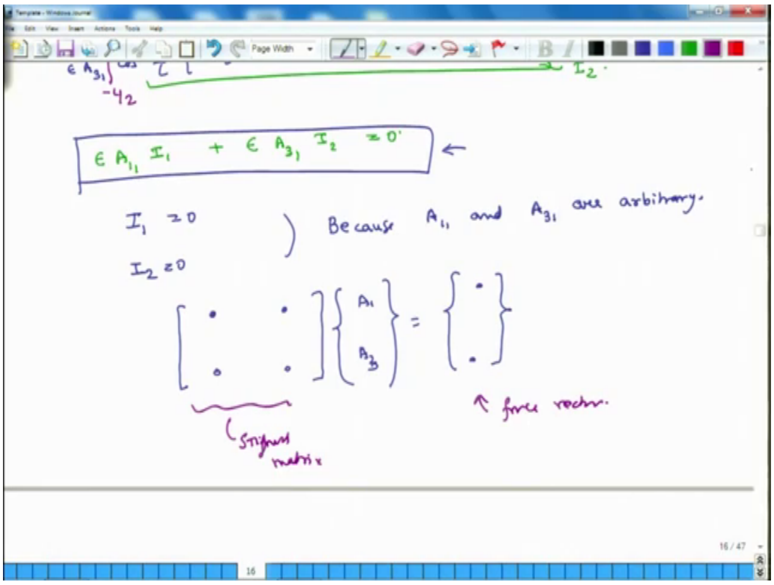

So, you will have a 2 by 2 matrix multiplied by vector A 1 A 3, and on the right side left right side you will have another matrix. So, this is called the stiffness matrix, and this is called the force vector. And A 1 and A 3 are amplitudes of the displacement component, right.

What is the first component? First component is A 1 the cosine pi x over L, second component is A 3 cosine 3 pi x over L. So, you solve for these and you get A 1 and A 3, ok. So, all it requires a just basic algebra integral calculus, and these are simple function. So, it is not difficult to integrate them at all. So, once you do that you get the values are A 1 and A 3.

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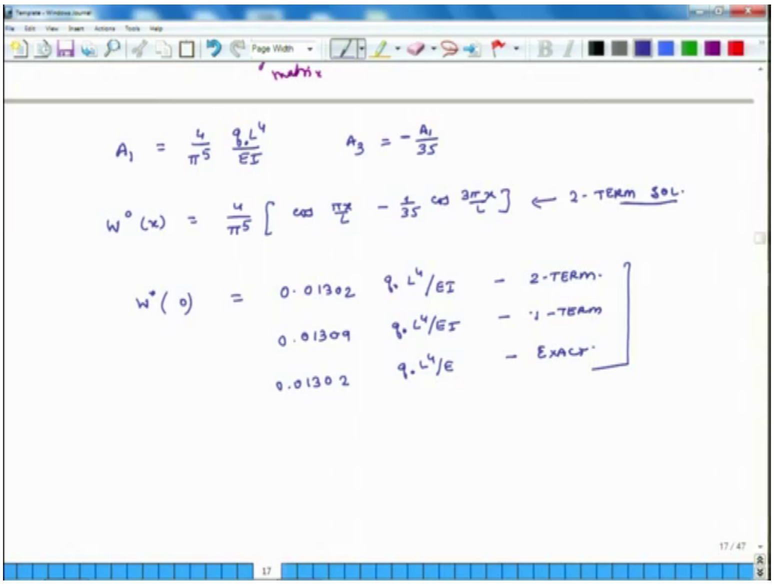

And the results are a 1 is equal to 4 over, excuse me, pi to the power of 5 q L 4 divided by EI, and A 3 is equal to a 1 by 35 negative of that result what we find out. So, w naught x is equal to 4 over pi to the power of 5 cosine pi x over L minus 1 over 35 cosine 3 pi x over L. So, this is the 2 term solution.

Similarly, if we wanted to do A 3 term solution, what will we do? We will go back to our expression for Ww, we will have 3 terms maybe I just extend this. So, I will say A 1 cosine pi x over L plus A 3 cosine p pi x over L plus A 5 cosine pi x pi over x over L. So, I will have 3 terms.

Next I will compute the error force, then I will have my virtual displacement. And in virtual displacement again I will have 3 terms, involving A 1 1 A 3 1 and A 5 1, and then I equate the virtual work to get 0. And then here I have a 2 equations involving A 1 and A 3, there I will get 3 equations. And I solve for A 1 A 3 and A 5 and that is so I do it. So, this is the thing and just for comparison purposes, w naught at 0 which is the origin is equal to 0.01302 q naught L 4 by EI, this is the 2 term solution. And it is 0.01309 q naught L 4 by EI this is the one term solution and it is 0.01302 it is q naught L 4 over EI this is an exact solution.

So, we see that in this case, just by having 2 terms the midpoint the deflection of the midpoint of the beam is pretty close to the actual value at least up to 4 places of decimal just by having 2 terms. So, this is the overall approach, and we can use a very similar approach for plate also. So, that is what I plan to do today we will continue this discussion tomorrow as well.

Thank you.