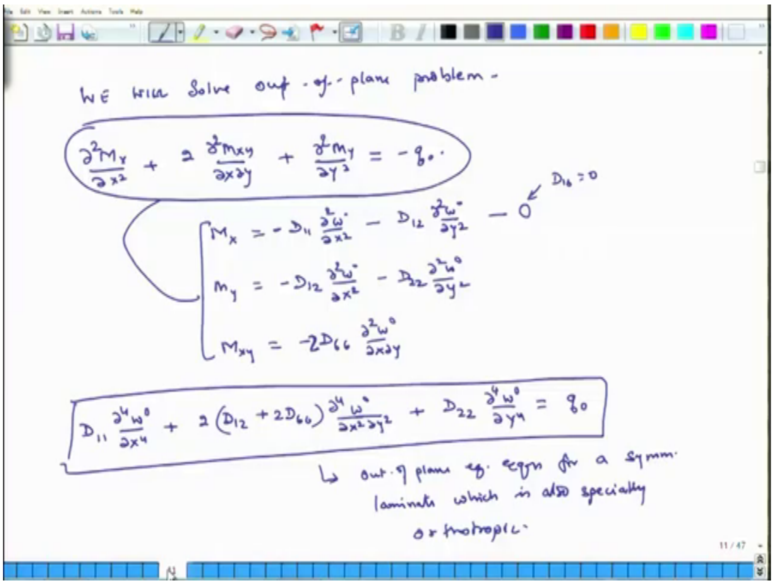
Advanced Composites
Prof. Nachiketa Tiwari
Department of Mechanical Engineering
Indian Institute of Technology, Kanpur

Lecture - 47
Example Based On a Finite Rectangular Plate (Part-II)

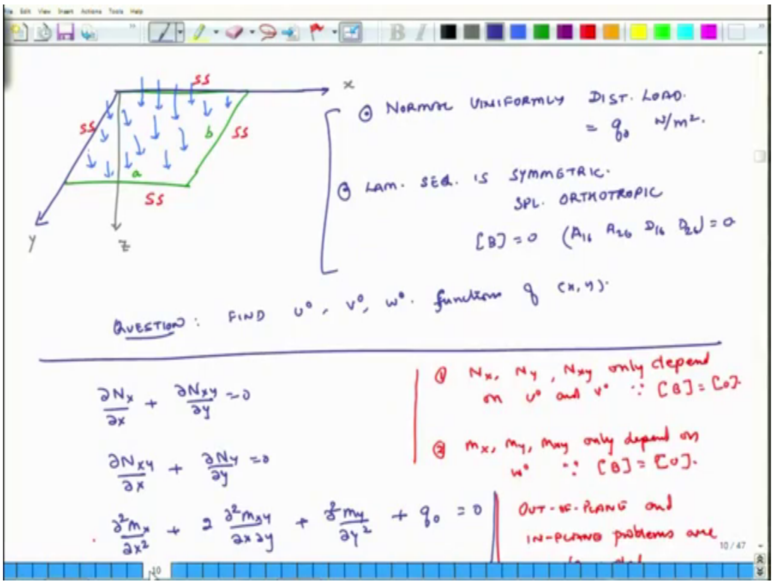
Hello. Welcome to Advanced Composites. Today is the 5th day of the ongoing class in the 8th week of the course.

(Refer Slide Time: 00:28)



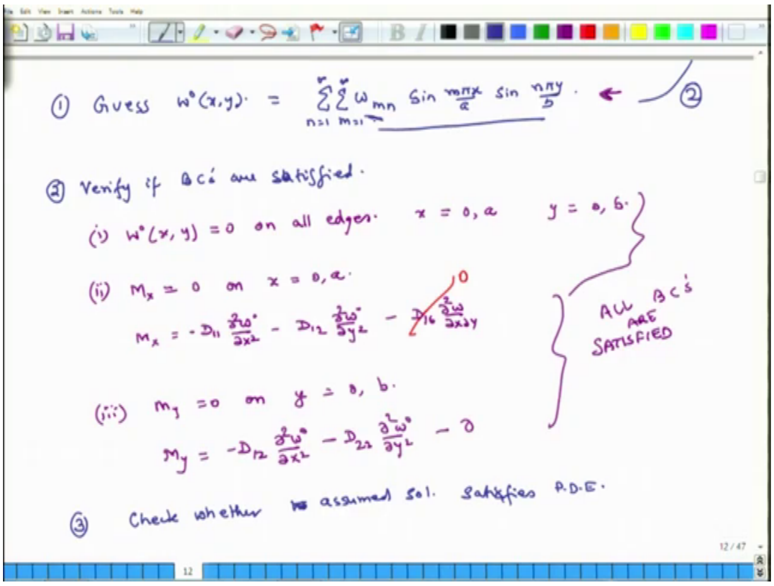
And yesterday we developed the governing differential equation for a plate which is symmetrically laminated and also it is specially orthotropic in nature, because it is having such a lamination sequence, it is out of plane and in plane solutions gets decoupled. And the governing differential equation assumes the form for as shown here. So, this is the governing differential equation for variable w which is out of plane deflection.

(Refer Slide Time: 00:59)



So now what we will try to do is, we will try to solve this equation in the context of the problem described, that is the plate is having simple supports on all the 4 edges. And also it is uniformly loaded in the transverse direction.

(Refer Slide Time: 01:13)



So, the first step is we guess w naught x y, guess w naught x y. So, our guess is that so, we as guesses series solutions. So, this is equal to w m n which is a constant times sin m pi x over a sin n pi y over b. The reason we are guessing it such a ways, so, w m n is a constant that, if nothing else this function will satisfy the condition on w; which is that w

0 at both the edges on all the 4 edges. And this is a summation on both the terms m so, m can be from 1 to m and n can be from 1 to n ok. And this capital mn or we can make it an infinite.

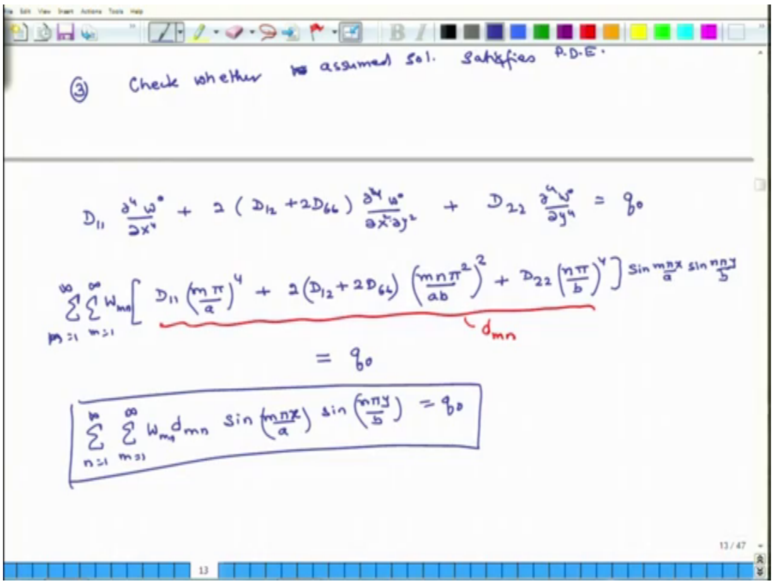
It could be anything, it could be an infinite series. So, second verify if BC's are satisfied. So, the first BC is w naught x y equals 0 on all edges ok. So, what are the edges x is equal to 0 or a. So, when we look at the problem definition, x is equal to 0 on this edge, and x is equal to a on this edge. Similarly, y is equal to 0 on this edge and y is equal to b on this edge. So, these are the 4 edges, y is equal to 0 and b. So, when we plug in x is equal to 0, in this equation w becomes 0. And when we plug in y is equal to 0. So, this solution the boundary condition on x is equal to 0 edge is satisfied. Similarly, when we plug in y is equal to 0 in this equation, it gets satisfied. So, boundary condition on y is also satisfied.

Similarly, x is equal to a then we vote it till here, then it is equal to m pi sine of m pi and sine of m pi is again 0. So, boundary condition is satisfied on both the x edges and similarly on both the y edges. So, at least from stand point of boundary condition on w, this function satisfies are requirement. The second is M x is equal to 0 on x is equal to 0 and a. Now what is M x? M x is equal to D 1 1 del 2 w over del x square minus minus D 1 2 del 2 w over del y square minus D 1 6 del 2 w over del x del y, but D 1 6 is 0, because of lamination sequence. So, it is D 1 1 second derivative of w in x plus D 1 2 second derivative w n y.

Now, let us see whether this boundary these 2 boundary conditions are satisfied. So, when I differentiate this function of w with respect to x twice I again get the sine term. And that sine term will still be 0 on x is equal to 0 and x is equal to a. So, this boundary condition is also satisfied. And the third boundary condition is M y is equal to 0 on x, on y is equal to 0 and b. So, what is the definition of M y? M y is equal to minus D 1 2 del 2 w naught over del x square minus D 2 2 del 2 w naught over del y square and then my D 2 6 term which is 0. And again when we plug in the value of w from this form, we again see that it satisfies. So, these 2 boundary conditions are also satisfied. Actually all the 3 boundary conditions are satisfied, all BC's are satisfied

So, are assumed function w it satisfies all the BC's. So, the next thing is that it should also satisfy the governing differential equation. So, we check that. So, then the thing is check whether assumed solution satisfies PDE, ok. So, when we plug in so, what we have to do is, we are plugging in this equation. So, this is equation 1 and this is equation 2. So, we are plugging in this series solution for w in equation 1. So, what do we get? And we so, when we plug it in, what we get is; so, I will write this equation once again.

(Refer Slide Time: 07:37)



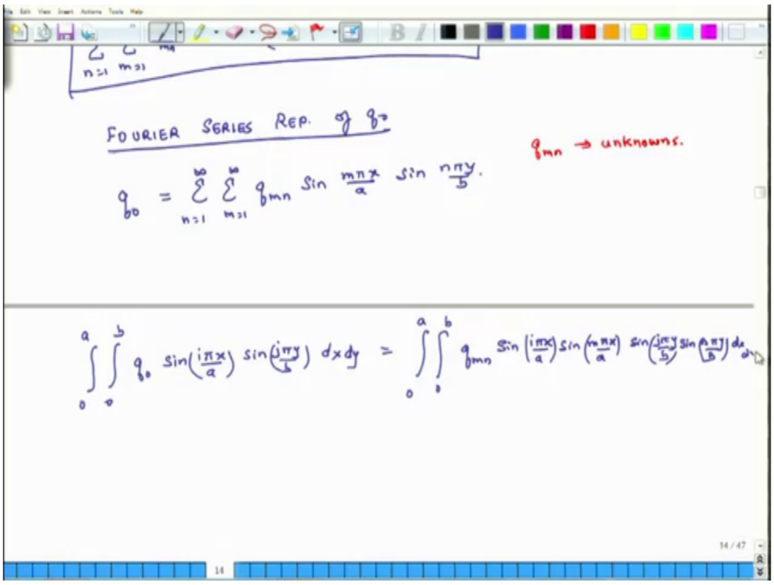
So, once I plug in the solution I get a series term; m is equal to 1 to infinity, n is equal to 1 to infinity. In the brackets we get D 1 1 m pi over a to the power of 4. See when I plug in and I differentiate this term, this with respect to x 4 times, I will get m pi over a to the power of 4. So, I get this plus 2 D 1 2 plus 2 D 6 6 times m n pi square over a b.

Student: (Refer Time: 09:12).

Plus, D 2 2 n pi over b to the power of 4 multiplied by sine m pi x over a, sine n pi y over b. And this equals q naught, and q naught is the constant; q naught is the constant. So, what I will do is, this long expression I will call it d m n, just to make our life, I do not want to write these long things. So, I will just rewrite this equation very quickly again; d m n sine m pi x over a sin n pi y over b is equal to q naught.

Now if we can show that in some special condition, this left side and right side are equal then our assumed solution is correct. If we cannot show that these sides are equal, then we cannot claim; oh there is one thing I forgot to mention. There was a w m n which I omitted. So, this entire thing is also multiplied by w m n. If w m n is not known, and if we can select some special values of w m n; such that this entire thing equals q naught, then we have solved the problem. So, let us call this equation 3.

(Refer Slide Time: 11:21)



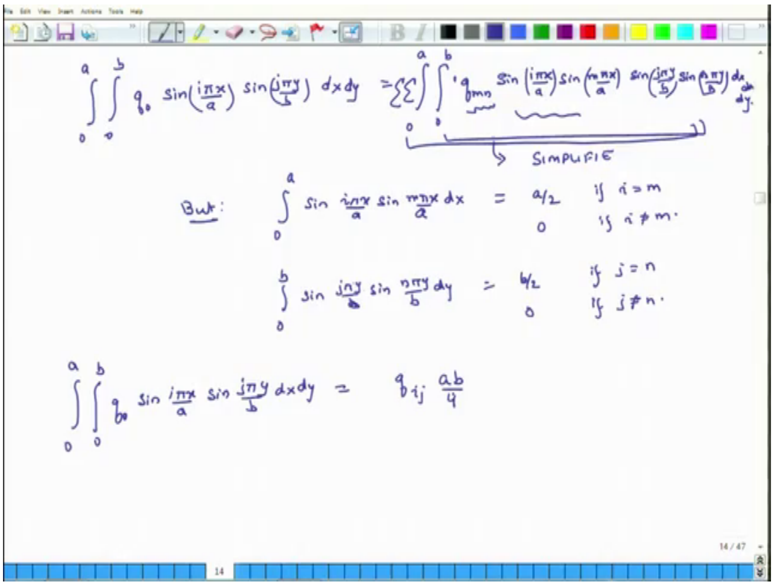
Now, q naught is a constant, q naught is a constant. And we know that we can express constants as Fourier series expansions. We can express constants as Fourier series expansions. So, what will be what we will do is, we will express this q naught in terms of a Fourier series expansion ok, but most of the times when we deal with Fourier series we talk of only one dimension. So, we say that constant c 1 is equal to sine of 2 pi x over a plus sine of 3 pi x over a and so on so far. So, here both dimensions are involved. So, this will be a 2 dimensional Fourier series.

So, what we will do is, we will do Fourier series representation of q naught. So, we will assume q naught which is a constant is equal to double sum of q m n sine m pi x over a sine n pi y over b. And m is equal to 1 to infinity n is equal to 1 to infinity. So, the larger the number of terms we take the more accurate this q will become. And then these are unknowns so, we have to calculate these unknowns, you have to calculate these unknowns ok.

So, how do we calculate them? So, this is again standard Fourier procedure, just expanded to 2 dimensions. We multiply both sides of this equation by sine i pi x over a and sine j pi by over b, and we integrate this both sides over the domain of the problem. So, so we do what we are doing is we have integrated, we are integrating after multiplying both sides by sine i pi x over a sine j pi y over b, D x D y equals 0 to b 0 to a q m n sine i pi x over a sine m pi x over a sine j pi over b sine n pi y over b and integrate it with respect to D x and D y.

So, this is D x times D y ok. So, I cannot write the everything on that one line. So, it is D x times D y. So, this is how we compute these unknown constants q m n ok. So, the left side or the right side; so, we what we do is we use the results so, we know.

(Refer Slide Time: 15:20)

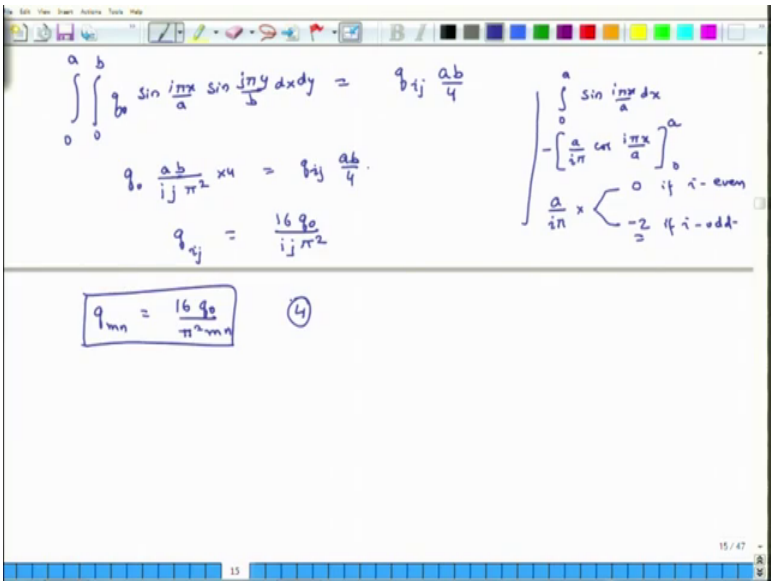


But so, we know, that 0 to a for instance, sine of i pi x over a sine of m pi x over a, D x see when we. So, this is equal to a over 2, if I equals m and it is equal to 0, if i is not equal to m. This is the standard trigonometric identity. And similarly we know that for the y integral. So, this is equal to b over 2 if j equals n and 0 if j is not equal to n.

So, essentially what this means is, that this entire integral, I can express it as and of course, this is this has to be summed up for all values of q m n. Because if summation you forgot to put this, because we said that q naught is summation of this. So, this summation we vomited ok. So, what this means is that, this entire integral, this entire integral gets simplified if I use this result. So, what it becomes is; so, this becomes so, I will rewrite this integral 0 b 0 a q naught sine i pi x over a sine j pi y over b D x D y equals. And i n so, the integral of sine i pi and sine m pi that will be a over to only when i is equal to pi. Student: I is equal to (Refer Time: 17:49)

I is equal to m right, i is equal to m. So, essentially if I use this relation, I end up with q i j times a b over 4, ok. Q i j times a b over 4, and what do I get on the left side? On the left side so, in so this is equal to so, the left side becomes q not.

(Refer Slide Time: 18:23)

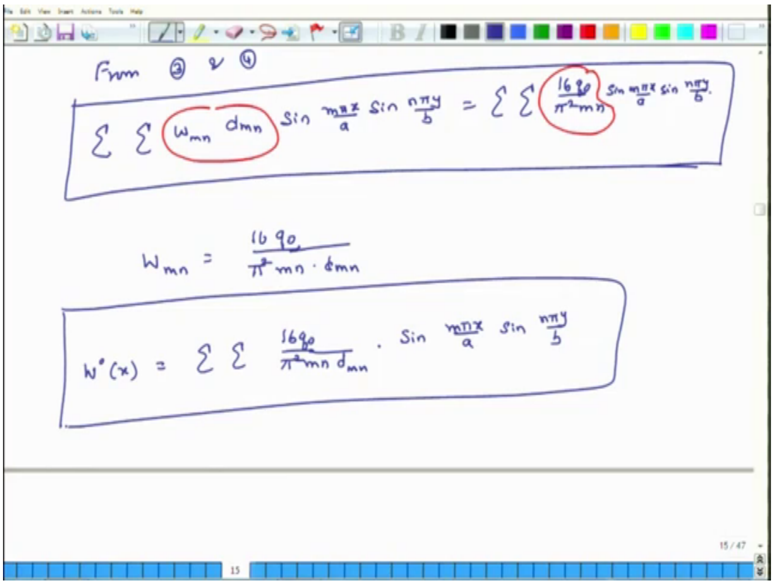


So, what is integral of 0 to a let us say sine i pi x over a times D x. So, it is equal to 0 to a, it will be equal to a over i pi, and there is a negative sign. Cosine i pi x over a evaluated I am sorry, there is a negative sign; evaluate between limits 0 and a. If x is odd then this becomes so, this. So, this becomes I over I pi times 2 possible values.

If x is even if x is even if no, I am not said if is even, then this becomes 0. And if i is odd, then it becomes minus 2. Cosine of so, we will consider suppose i is 1, then it will be cosine of pi, times a by a a cancels out right. So, cosine of pi is minus 1, and cosine of 0 is minus 1. Cosine of 0 is 1 so, minus 1 minus 1, right minus 1 minus 1. So, it becomes minus 2, right.

So, if so, this is if i is even, and this is if i is odd. So, we get basically q, and similarly we have similar integral for sine j pi y over b also. So, essentially we get q naught a by i times j times pi square b into 4 is equal to q i j, I am sorry q i j times a b over 4. Because minus 2 gets multiplied by minus 2 I still get positive 4, ok. So, the value of q i j is equal to 16 q naught over i times j times pi square, or I can also say that q m n is equal to 16 q naught over pi square m n, ok. So, this is equation 4. And I plug this equation 4 in equation 3, I plug this equation 4 in equation 3.

(Refer Slide Time: 21:51)



So, I get so, if I combine from 3 and 4 what do we get? This is the thing we get, ok. Now this left side and right side will be equal, it will be equal only if this term is equal to this term. Because other terms are same so if that is the case then we can say that w m n is equal to 16 q naught over pi square m n times d m n.

So, the solution for w so, now we have seen that this assumed solutions satisfies all the boundary conditions, and all the and the differential equation also. So, the solution for w is double summation 16 q naught over pi square m n d m n times sine m pi x over a sine n pi by over b. So, this is the solution for the out of plane problem for a simply supported plate; which is a laminated composite plate which is simply supported on all the 4 sides.

And this plate is also not only symmetric in nature, but it is also specially orthotropic. So, for that kind of a plate, this is the solution for out of plane displacement that is w. And the geometry of the plate that is it is dimensions a and b, they are embedded in 2 parts, they are embedded in here, they play a role, and they also have a role in definition of d m n, ok.

So, this is the solution and what we will do tomorrow is will continue with this discussion on this simply supported plate and all the 4 sides. And then once we finish the discussion on this problem then we move on to some other problems ok.

So, thank you and have a great day. Bye.