Advanced Composites

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Lecture – 41

Thermal Effects in Composite Laminates (Part-I)

Hello welcome to Advanced Composites. In the current seventh week of this course today is the fifth day. And what we start plan to start today is a new theme of discussion and that relates to Thermal Effects in Composite Laminates. The reason we are going to discuss these effects because these effects are extremely important; in context of composite laminates because most of times when composite laminates are fabricated, they are fabricated at elevated temperatures which could be more than 100 degrees or 100 and 50 degrees centigrade.

So, it is at that temperatures that several phases of the laminate fibers and matrix they are processed and they are heated till that temperature. And then at that temperature the reaction happens and everything solidifies and then the composite is brought down to room temperatures. So, as the material is brought down to room temperatures in built stresses may develop because of temperature effects even though there may not be any external forces applied on the system.

So, how do we compute these stresses and strains in composites? It is an is extremely important question. And what we planned to do today tomorrow and may be 1 day in the next week is to discuss how these effects can be accounted for and calculated. So, that is what is the agenda for next few days.

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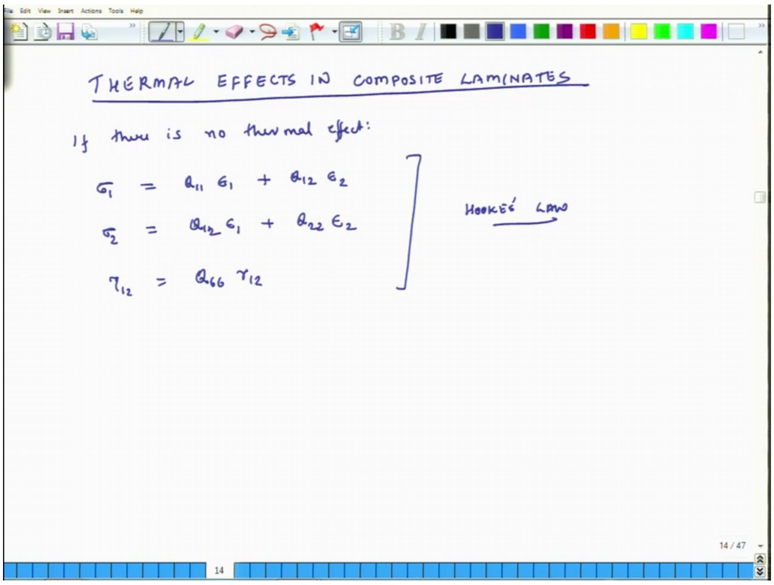

So, what we are going to discuss is thermal effects, thermal effects in composites. Now, if there is no thermal effect then we already know that stress strain relations between for a single layer and that is sigma we are talking about an axis system which is aligned to the material axis of the layer. So, we are talking about the stresses that are at an individual layer. So, this is equal to Q 1 1 times epsilon 1 plus Q 1 2 times epsilon 2. And there is no Q 1 6 because this is the material axis system is aligned to the loading axis.

Similarly, sigma 2 is equal to Q 1 1 excuse me Q 1 2 times epsilon 1 plus Q 2 2 times epsilon 2 and tau x y no sorry tau 1 2 equals Q 6 6 gamma 1 2. So, these expressions essentially come out from the original Hooke's law. But when temperature effects are there then this Hooke's law has to be modified.

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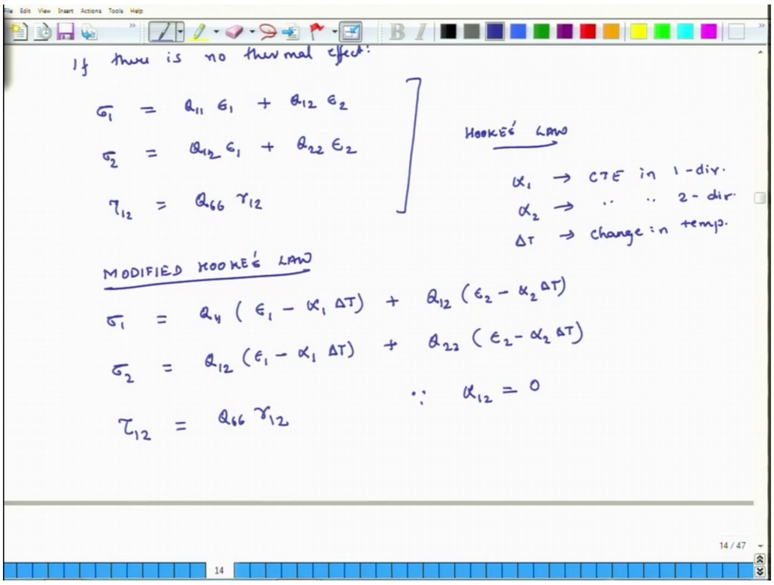

And once it is modified these are the relations for modified Hooke's law sigma 1 equals Q 1 1 and instead of epsilon 1, I have epsilon 1 minus alpha 1 times delta T plus Q 1 2 times epsilon 2 minus alpha 2 times delta T. So, this is a modified Hooke's law and what are alpha? Alpha 1 is coefficient or thermal expansion of the material in 1 direction. Alpha 2 is coefficient of thermal expansion in 2 direction ok. And delta T is change in temperature. It could be positive or negative.

So, similarly sigma 2 using modified Hooke's law is Q 1 2 epsilon 1 minus alpha 1 delta T plus Q 2 2 times epsilon 2 minus alpha 2 delta T. And tau 1 2 is equal to Q 6 6 times gamma 1 2. Because alpha 1 2 bar is (Refer Time: 06:00) is alpha 1 2 bar ok.

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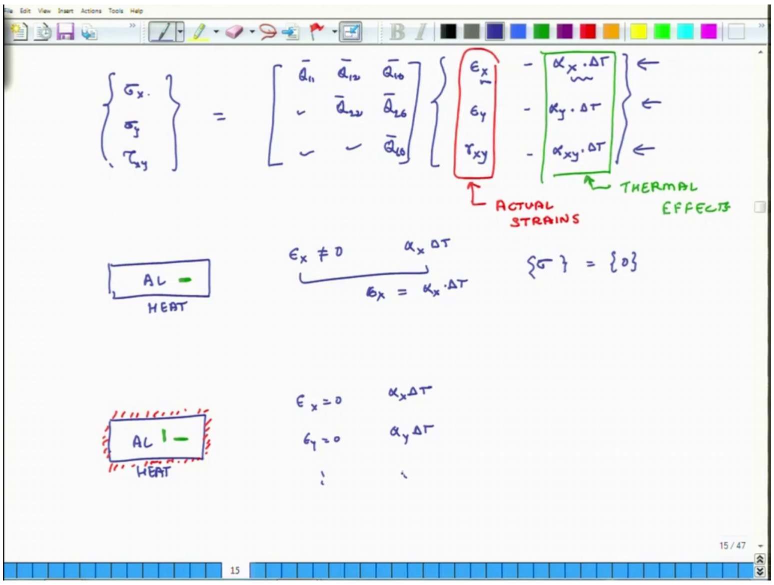

So, this is the modified Hooke's law. Now if I want to move from 1 2 axis system to an x y axis system. Then what do I do? So, sigma x sigma y tau x y is equal to so, this is in the 1 2 axis system. So, we will write down the relations for x y coordinate system also. So, for x y coordinate systems we have a Q bar matrix. So, Q 1 1 bar Q 1 2 bar Q 1 6 bar Q 2 2 bar Q 2 6 bar Q 6 6 bar and all these symmetric matrix. And we know how to compute it is values, but then it is not just simplified to epsilon x epsilon y and gamma x y vector, but it also accounts for thermal strains.

So, this is epsilon x epsilon y gamma x y minus alpha x times delta T minus alpha y times delta T and minus alpha x y times delta T. So, it should be remembered that in x y coordinate system there is an alpha x y which is non zero, but in the 1 2 coordinate system alpha 1 2 is 0. Now what is the difference between epsilon x and this thing. So, these strains are the actual strains in the system. These are actual strains.

So, if you what do you mean by actual strain? If you put a strain gauge on it we will measure it. And whatever is the measured value these are those strains actual strain physically observable strains. These strains they are thermal effects. These are thermal effects. So, we will illustrate by an example. Suppose we have an aluminum bar and I heat it and the bar is free to expand, the bar is free to expand. Then what will happen when it expands epsilon x will be not 0 right. It is expanding so, if I put a strain gauge here. If I put a strain gauge here it will actually measure the strain. So, epsilon x what do I say I said these are the actual strains.

So, epsilon x will not be 0. And alpha x and delta T will also be not 0. And it will be just that it will expand by the same amount which will be equal to alpha x times delta T right. So, in this case epsilon x will be equal to alpha x times delta T. It will expand by that amount and then the unit per unit expansion is called the strain. So, this epsilon x minus alpha x times delta T when you add these up it will come to be 0. The sum of this and this will come out to be 0.

So, what will be the stress when a aluminum bar is free to expand? It is not experiencing any stresses. So, our model should be such that it should predict 0 stress. So, all these three vectors in that case components of the vectors they will be 0. And when they are 0 it will mean that sigma x will be Q 1 1 times 0 plus Q 1 2 times 0 plus Q 1 6 times 0. So, external stress will come out to be 0 understood. And that is what is the reality so, it works.

Now, consider the case other case when we have an aluminum bar are and again we are heating it. And we are again we are putting some strain gauges to measure the expansions and xy and all these things. But one thing what we are doing is we are constraining. So, it cannot expand and we are heating it. So, what will happen the actual strain and the bar will be 0, actual strain in the bar will be 0 and alpha x times delta T will not be 0.

Similarly, epsilon y will be 0 and alpha y times delta T will not be 0 and so on and so forth. So, one diet happens the sum of this epsilon x and alpha x delta T will not be 0, but it will be negative quantity and when it is a negative quantity and it gets multiplied by Q 1 1 Q 1 1 2. And all these you will find that sigma x sigma y and tau xy they may not be necessarily 0 then may be compressive entities ok.

So, this is how we compute thermal stresses. So, in the strain vector we have not just epsilon x epsilon y and gamma xy which is the actual strain, but we subtract from that thermal effects. And then we multiply that modified strain vector with the Q bar matrix to get stresses at each ply level. This is very important to understand. So, the next question is that alpha 1 and alpha 2 these are material properties. And they are so, we can measure them we can measure them. But, if we move from 1 2 axis system to x y axis system, how do we calculate alpha x alpha y and alpha x y because, unless we know these I cannot compute all these numbers in the green block.

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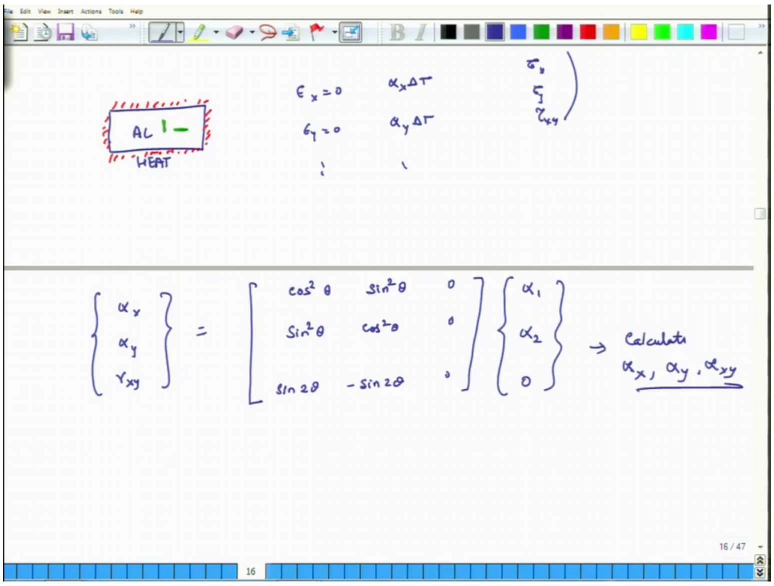

So, that is the next thing I will write down. So, these are basically strain transformation equations. So, alpha x alpha y and gamma xy is equal to cosine square theta sin square theta sin square theta cosine square theta sin 2 theta minus sin 2 theta. And actually the third column it really does not matter and the reason it does not matter is because alpha 1 is not 0 alpha 2is not 0 alpha 1 2 is always 0 ok. So, using these relations you can calculate alpha x alpha y alpha xy. So, this is the modified Hooke’s law. Now if we have this modified Hooke’s law then how do.

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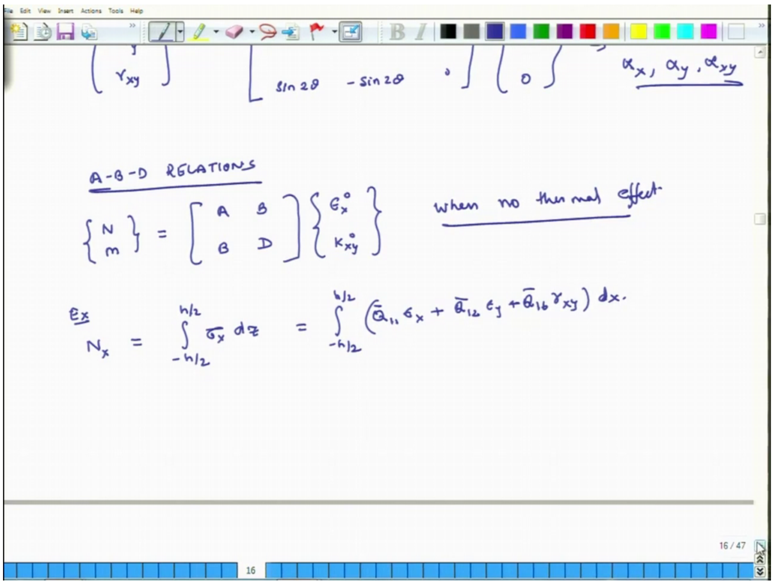

So, next thing is A B D relations. How do they change, how do they change ok. So, our original relations are N M is equal to A B B and D. epsilon x naught and k xy naught right these are the original relations. So, these relations are valid when no thermal effects, no thermal effects. Now let us look at what happens when thermal effects are present.

So, we will just do an example what is a N x N x we have defined was integral from minus h by 2 to h by 2 sigma x dz. This is how we had defined x right it is the fourth resultant that is force per unit length. And this was this is sigma x is what Q 1 1 bar epsilon x ha plus Q 1 2 bar epsilon y plus Q 1 6 bar gamma xy dx ok.

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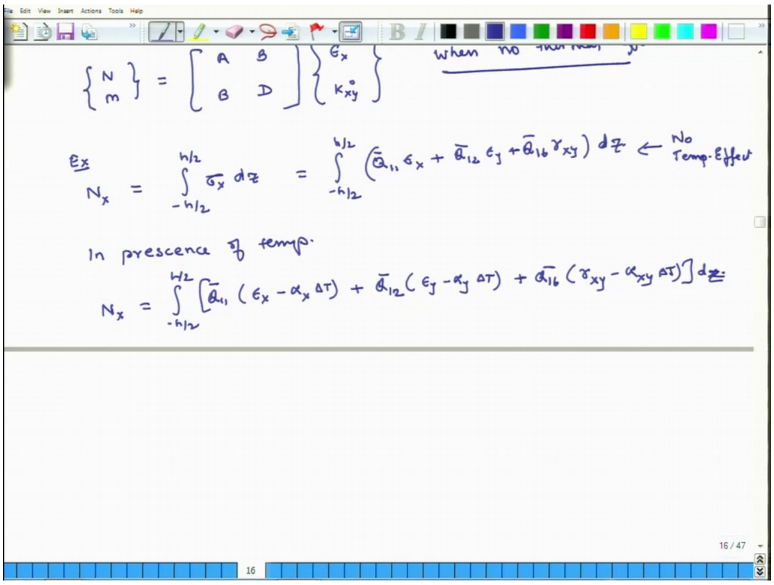

So, this is the, this was the relation when no temperature effect. But now we have a modified Hooke's law right. So, in presence of temperature in presence of temperature what is N x if I have to compute N x then it will be minus h by 2 to h by 2 Q 1 1 bar times epsilon x minus alpha x delta T right plus Q 1 2 bar times epsilon y minus alpha y times delta T plus Q 1 6 bar times gamma x y minus alpha x y times delta T. And this entire thing has to be integrated dz. So, there should be here also it should be d z.

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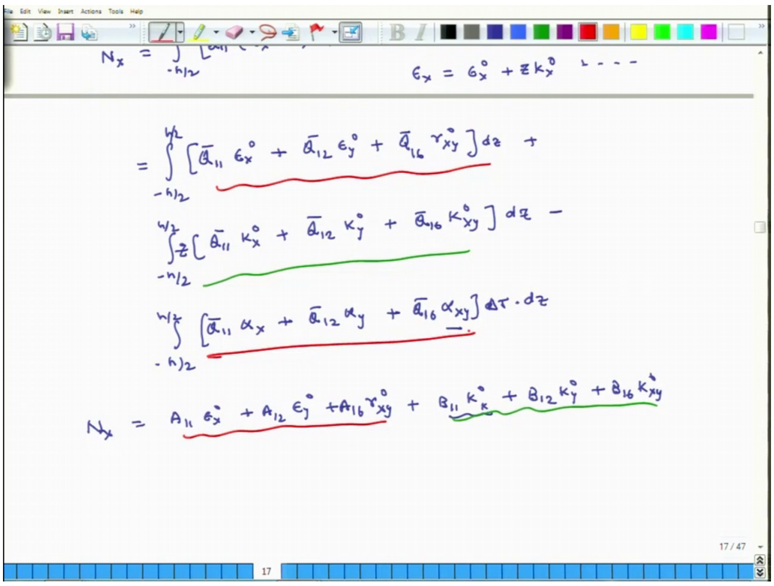

And minus h by 2 to h by 2; if I integrate it now I can break it up in to a temperature related component and a non temperature related components. So, Q 1 1 bar and we also know that what is epsilon x, this is equal to epsilon x naught plus z k x naught this is right. So, this is and so on and so forth. So, I do that substitution. So, I get epsilon x naught plus Q 1 2 epsilon y naught plus Q 1 2 bar gamma x y naught.

So, these are the mid plane strain components d z plus minus h by 2 h by 2 I get Q 1 1 bar. And then there is a z here k x naught plus Q 1 2 k y naught plus Q 1 6 bar. I am sorry this should be Q 1 6 1 6 and this is also Q 1 6 Q 1 6 bar k xy bar d z plus the temperature component, that is the temperature component. So, when the temperature component I have negative. So, I will put a minus sign here minus sign h by 2 to h by 2 minus. And I have Q 1 1 bar alpha x delta T. Actually I will take delta T outside plus Q 1 2 bar times alpha y plus Q 1 6 bar times alpha x y times delta T times d z.

Student: (Refer Time: 20:55).

Ha.

Student: Alpha x y term.

Alpha x y term is here.

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[FL] Because when we do the strain transformations you are seeing this see this. So, alpha x y is there and alpha x y and alpha x and alpha y you can compute from these relations. Alpha x y is not necessarily 0. So, now when I do all this essentially what I end up getting is N x is equal to A 1 1 epsilon x naught plus A 1 2 epsilon y naught plus A 1 6 gamma x y naught.

So, these terms are coming from the first integration. Then from the second integral I get the B terms B 1 1 k x y naught. I am sorry k x naught plus B 1 2 k y naught plus B 1 6 k x y naught. So, this I am getting from the second integral from here. And the first component set of components are coming from the integral. And then from the third integral I can add up all these things, all these things ok.

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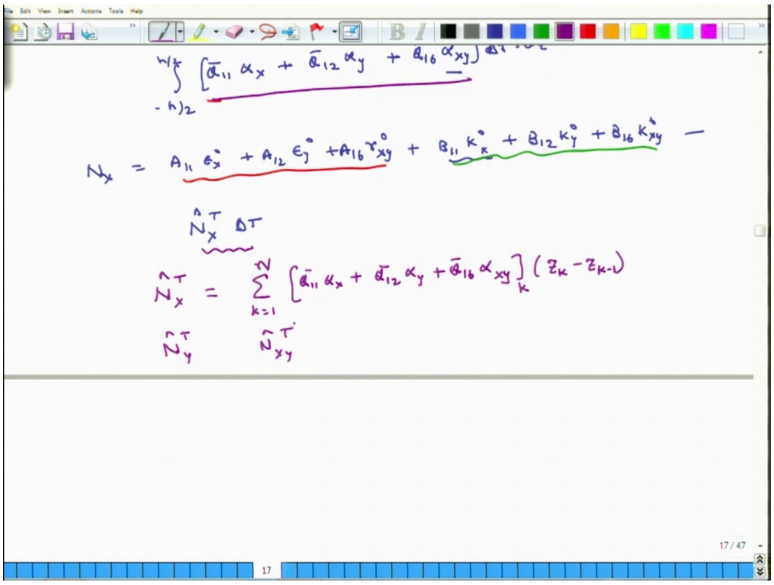

So, I get a third thing from the third one so, I get minus and I get N x T hat times delta T. And this I am getting from the third integral N x hat ok. So, what is N x hat? It is equal to so, if I am integrating it is and because this is our laminated composite I have to replace this integral by a summation sign. So, this is equal to k is equal to 1 to n Q 1 1 alpha x bar plus Q 1 2 alpha y bar plus Q 1 6 alpha x y.

And this is we compute for the k th layer and then we add up all these things and I am sorry so, this has to be multiplied by z k minus z k minus 1, basically the thickness of the layer. So, this is N x hat ok. So, likewise I can compute other things and y hat and N xy hat. So, I get so, these are the three relations I get for N. And then I get similarly three other relations for moment resultants.

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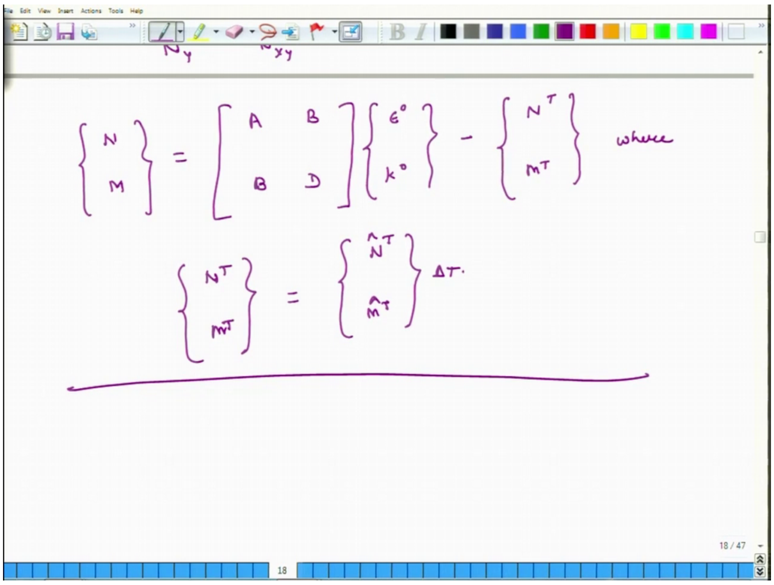

So, overall my set of equations become N M equals A B B D times epsilon naught and k naught curvature matrix minus N T M T, where N T and MT equals N T hat M T hat times delta T ok. Because, there is a delta T term here if you remember So, the integral of everything in the bracket over the thickness is N x hat N y hat and so on and so forth.

But, then we have to multiply it by delta T term. So, this is what I wanted to cover today. Tomorrow we will extend this discussion and we will also see what happens to the governing differential equations and the boundary conditions and then we will start solving an actual problem and see where it leads us. So, that is all for today and I look forward to seeing you tomorrow.

Thank you.