

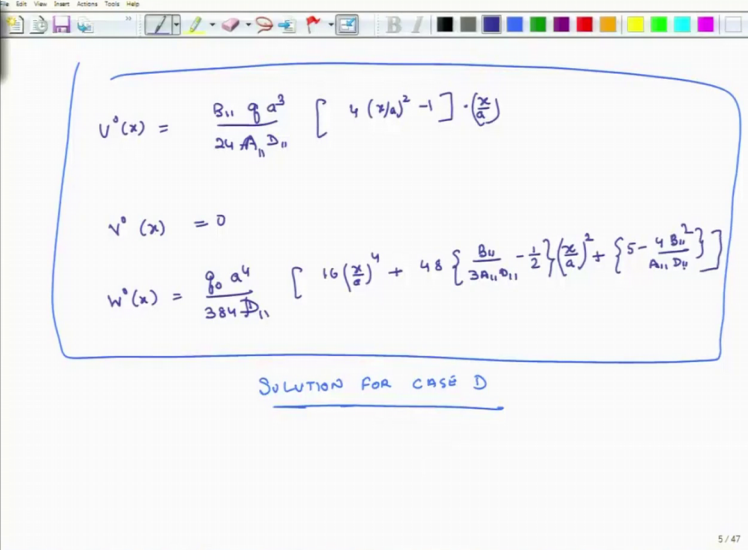
Advanced Composites
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Lecture - 38

Nature of Displacement $u_0(x)$ and how it gets Influence by Important Parameters of Lamination Sequence

Hello welcome to Advanced Composites today is the second day of the ongoing week, which is the 7th week of the course. Yesterday we finished the solution for the last case which we have been discussing over last 2 3 days that is the case D and we came up across we come up came upon the solution.

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The image shows a handwritten slide titled "SOLUTION FOR CASE D" containing three equations:

$$U^0(x) = \frac{B_{11} q a^3}{24 A_{11} D_{11}} \left[4 \left(\frac{x}{a} \right)^2 - 1 \right] \cdot \left(\frac{x}{a} \right)$$

$$V^0(x) = 0$$

$$W^0(x) = \frac{q_0 a^4}{384 D_{11}} \left[16 \left(\frac{x}{a} \right)^4 + 48 \left\{ \frac{B_{11}}{3 A_{11} D_{11}} - \frac{1}{2} \right\} \left(\frac{x}{a} \right)^2 + \left\{ 5 - \frac{4 B_{11}^2}{A_{11} D_{11}} \right\} \right]$$

SOLUTION FOR CASE D

As expressed in these equations for U nought, V nought and W nought. What I like to discuss for next 5, 7 minutes is the nature of W nought excuse me the nature of U nought and how it gets influenced by important parameters related to the lamination sequence. So, let us look at this expression once again, U nought x .

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FOR CASE D

$$U''(x) = \frac{B_{11}}{q a^3} \left[4 \left(\frac{x}{a} \right)^2 - 1 \right] \left(\frac{x}{a} \right)$$

Sign of $U''(x) \rightarrow x$ and B_{11}

$B_{11} > 0$

Diagram showing the sign of $U''(x)$ for $B_{11} > 0$:

- For $x < 0$, $U'' > 0$
- For $x = 0$, $U'' = 0$
- For $x > 0$, $U'' < 0$

So, this is for case C I am sorry for case D. So, $U''(x)$ equals $B_{11} q a^3$ divided by $24 A_{11} D_{11}$ into $4 x^2$ over a^2 minus 1 times x over a ok. Now let us look at all these parameters q is positive a is positive A_{11} is always positive and so, is D_{11} . So, this entire thing will always be positive number ok.

Then let us look at this term in the so, this is always positive number, then this term in the bracket the maximum value of x will be $a/2$ I am sorry it will be $a/2$ because, our plate the total length of the plate is a and our coordinate system starts from the midpoint. So, the maximum value of x will be $a/2$, which means that the maximum value of this entire thing in the brackets will be 0, when x is equal to $a/2$ then this will be 0, otherwise this thing in the parenthesis will always be negative.

So, this is a negative number ok and then this parameter it can be positive or negative this can be positive; if x is positive then this is positive otherwise it is negative. And the same thing is true for B_{11} this can be positive or it can be negative and we will see that later why in ones what situation B_{11} is positive and when is it negative.

So, the sign of U it depends on x and B_{11} right, if it depends on the on x and B_{11} depends. So, if I switch the sign from x to minus x it switches similarly if I change B_{11} from positive to negative it gets switched ok. So, we will consider two cases if B_{11} is more than 0, B_{11} is more than 0 and let us say this is my plate and this is the midpoint at x is equal to 0.

So, what is the value of this thing, at x is equal to 0 the displacement will be 0 because x is 0 right and as I move away from the plate as I move away from the plate. So, at x is equal to 0 U will be 0 because of this term $[FL]$ because x is 0 here. So, this 0 gets multiplied by this entire thing ok or you can if you want to simplify this entire term you can consider it as $4x^3$ over a^3 minus x over a . So, when x is 0 then it is 0 and as I move away from the so, x is 0 here U is 0 what about at the ends at x is equal to a over 2 this is again 0 and x is equal to minus a over 2 it is again 0 ok. and in between it becomes large and then again it starts decaying. So, this is how U is changing over the length of the plate.

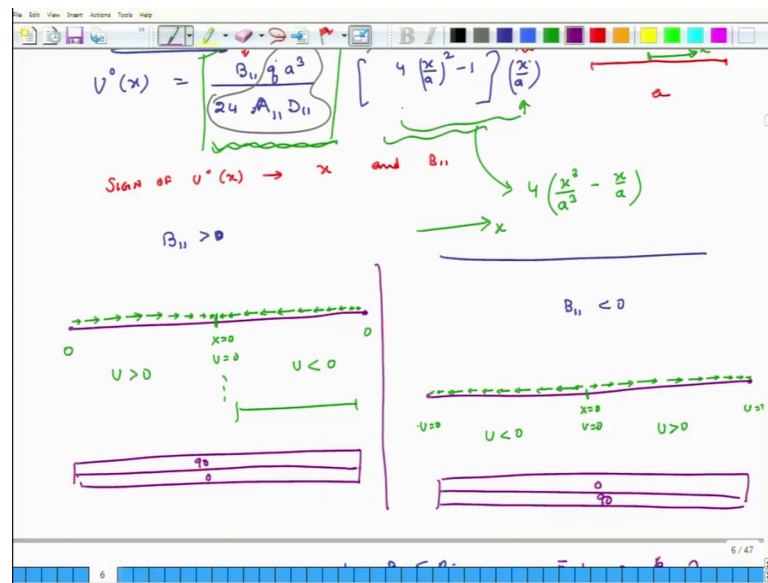
Now when B_{11} is more than 0, then this entire thing is positive this is negative ok. So, this entire thing is positive, when B_{11} is larger than 0 when B_{11} is positive then this entire expression is positive ok. So, then the sign of U depends on the sign of x . So, in this zone in this zone so, sorry so, we will go back. So, then the sign of this thing depends on x , now this is always a positive number negative number this thing and if x is negative then U will be positive ok.

So, in this region U will be more than 0 in this zone and in this zone U will be less than 0 ok and how is it going to vary at ends at 0 and a over 2 it will be 0. So, what will happen it will be initially 0, then it will become slightly larger then more large then it may become it will become somewhat maximum then it starts becoming small like this ok.

And how does U change in this , but in all the direction of all the arrows is in the positive x direction, this is our positive x direction, that is how we had developed the coordinate system this is x this is origin. So, our so, in the first half of the plate U is positive and it initially is 0 it starts growing and then it starts becoming smaller small till at the midpoint it becomes 0 and what happens when be on the other side just the opposite of that happens.

So, it is very small then it becomes larger it becomes even more and then it becomes maybe at a maximum point and then it starts becoming smaller like this ok. So, this is when B_{11} is more than 0 and when B_{11} is less than 0 for the other case just the opposite will happen ok.

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So, if this is the length of the plate at x is equal to 0, x is 0 here, U will be 0, U also U is 0 here at the boundaries because, that is how we have prescribed the boundary conditions, but because B_{11} is negative, this term in green it becomes a negative entity this is always negative this term is always negative. So, negative time negative becomes positive and what; that means, is that whenever x is positive U will be positive.

So, here U will be more than 0 and here U will be less than 0 ok. Now what does that mean physically so, we will draw that so, here U is positive, but very small, then it starts becoming larger and then it becomes small like this and on the other side it is negative like this ok. So, the question is in what case B is positive and what case B is negative this is something we have to think about.

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WHEN is $B_{11} > 0$ and $B_{11} < 0$.

Laminate sequence: $[0_2 / 90_2]$

Midplane at $z=0$. Layers at $z = -2$ and $z = +2$.

Definition of B_{11} :

$$B_{11} = \sum_{k=1}^N \bar{Q}_{11} (z_k^2 - z_{k-1}^2) \times \frac{1}{2}$$

Given values:

- $\bar{Q}_{11}|_{0^\circ} = 20 \times 10^6$
- $\bar{Q}_{11}|_{90^\circ} = 10^6$
- $N = 2$

Calculation:

$$B_{11} = \frac{1}{2} \left[20 \times \{ 0^2 - (-2)^2 \} + 1 \times \{ 2^2 - 0^2 \} \right] \times 10^6$$

$$= \frac{10^6}{2} [-80 + 4] = -\frac{76 \times 10^6}{2} = -38 \times 10^6$$

So, what is the so, when is B_{11} positive and when is B_{11} negative, it depends on the lamination sequence so, consider a lamination sequence 0 2 90 2. So, the lamination sequence is 0 2 90 2 which means top two layers are 0 degrees and the bottom two layers are 90 degrees and in this case this is the mid plane so, here z is equal to 0. And let us say each layer is 1 millimeter thick so, here z is equal to minus 2 and here z is equal to plus 2 ok.

So, what is the definition of B_{11} ? B_{11} is equal to Q_{11} times z_k square minus z_{k-1} square into 1 over 2 and this is actually \bar{Q}_{11} and this I summate from k is equal to 1 to n ok. Now effectively here it has two layers even though we can call it a four layers, but the two layers of 0 we can combine them as a single layer and two layers of 90 we can combine them as a single layer.

So, in this case N is equal to 2 and \bar{Q}_{11} , let us say for the 0 degree layer will be much higher than \bar{Q}_{11} for 90 degree layer right because the laminate is much more stiff in the one direction and it is not that stiff in 90 degrees direction ok. So, we will put some values on this. So, let us say this is 20 so, we will say that \bar{Q}_{11} for 0 degrees is 20 into 10 to the power of 6 and \bar{Q}_{11} for 90 degree layer is 10 to the power of 6 it is that much different ok. So, for this laminate it will be B_{11} equals half which I take out and so, this is 20 into 0 minus. So, 0 square which is z_k so, I am just for calculating for the 0 degree layer.

So, 0 square minus 2 square plus 1 into 2 square minus 0 square ok. So, this works n times 10 to the power of 6. So, this comes out to be 10 to the power of 6 over 2 and this is 20 times minus 4 so, minus 80 plus 4 is equal to minus 76 into 10 to the power of 6 by 2. So, you can say it 38 into 10 to the power of 6 ok so, this is for 0 degree 0 90s laminates.

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Handwritten notes on a whiteboard showing the calculation of the B_{11} stiffness coefficient for a laminate with layers 0 and 90 degrees.

When is $B_{11} > 0$ and $B_{11} < 0$.

For the 0/90 laminate, the layers are defined as:

- Top layer (0): $z = -2$ to $z = 0$
- Bottom layer (90): $z = 0$ to $z = 2$

The general formula for B_{11} is:

$$B_{11} = \sum_{k=1}^N \bar{Q}_{11} (z_k^2 - z_{k-1}^2) \times \frac{1}{2}$$

For the 0/90 laminate, the calculation is:

$$B_{11} = \frac{1}{2} \left[20 \times \{0^2 - (-2)^2\} + 1 \times \{2^2 - 0^2\} \right] \times 10^6$$

$$= \frac{10^6}{2} [-80 + 4] = -\frac{76 \times 10^6}{2} = -38 \times 10^6$$

For the 90/0 laminate, the calculation is:

$$B_{11} = \frac{1}{2} \left[1 \times \{0^2 - (-2)^2\} + 20 \times \{2^2 - 0^2\} \right] \times 10^6$$

$$= \frac{10^6}{2} [-80 + 80] = 0$$

Now, we will consider another lamination sequence so, here it is 90 0 so, this is 90 2 and this is 0 2. So, in this case B_{11} equals half so, once again we will assign z is equal to 0 here z is equal to minus 2 and z is equal to plus 2.

So, B_{11} equals half times for the 90 degree layer what is it is 1 into 0 square minus 2 square plus 20 times 2 square minus 0 square ok. So, if you do the math this comes out to be plus 38 times 10 to the power of 6. So, what that means, is that the direction of U will be like this so, here B_{11} is more than 0, which means if the lamination sequence if the plate is like this, B_{11} more than 0 means that the top layer is 90 and the bottom layer is 0.

Then the deflection of U will be in this pattern. And in this case B_{11} is made less than 0, which means that here I have just taken the same plate, but I have just switched it and made it upside down. So, here the top layer is 0 and the bottom layer is 90 and when that happens and when I load it from using this vertical load Q then the deflection of U just switches because of the role of B_{11} ok.

So, this is again important to understand that this lamination sequence is not only lamination sequence is important, also how we place the plate relative to the load that is also important whether we are putting load on the 90 degree ply or on the 0 degree ply and so, on and so, forth. So, this is something very important to understand so, this is what I wanted to cover today.

Tomorrow [FL] [FL] tomorrow we will start a different problem. So, till so, far we have discussed all these 4 cases ABC and D 2 of these cases were the symmetric laminates 2 are not symmetric laminates, but in all the 4 cases the load on the plate was transverse and its intensity was Q Newton Newtons per meter. Now what we will do is we will have a new problem where we will not have a transverse load, but we will have a load in the x direction and see how plates behave. So, that is what we plan to do tomorrow and till then have a great day.

Thank you.