

Advanced Composites
Prof. Nachiketa Tiwari
Department of Mechanical Engineering
Indian Institute of Technology, Kanpur

Lecture - 36
Particular Solution for Semi-Infinite Plate (Case D)

Hello, welcome to Advanced Composites. Today is the last day of this week which is the 6th week of this course and what we have discussed throughout this week is solutions for semi-infinite plates. In 2 cases, we have solved the situation when the plate is symmetric and now we are working on plates which are having unsymmetric lamination sequences.

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Handwritten mathematical derivations for Case C and D on a whiteboard:

$$w^0(x) = \frac{1}{D_{11}} \left[\frac{q_0 x^4}{24} - c_2 \frac{x^2}{2} + \left(\frac{B_{11}}{A_{11}} \cdot c_1 - c_4 \right) \frac{x^2}{2} + \left(\frac{B_{11}}{A_{11}} c_5 - c_7 \right) x + c_8 \right]$$

$$u^0(x) = \frac{1}{A_{11}} \left[\frac{B_{11}}{A_{11}} \frac{q_0 x^3}{6} - \frac{B_{11}}{A_{11}} \frac{c_2 x^2}{2} + \left(c_1 - \frac{B_{11}}{A_{11}} c_4 \right) x + \left(c_5 - \frac{B_{11}}{A_{11}} c_7 \right) \right]$$

$$v^0(x) = (c_2 x + c_6) \times \frac{1}{A_{66}}$$

$D_{11} = D_{11} - \frac{B_{11}^2}{A_{11}}$ (Reduced Bending Stiffness)
 $A_{11} = A_{11} - \frac{B_{11}^2}{D_{11}}$
 $M_x = -\frac{q_0 x^2}{2} + c_3 x + c_4$
 $N_x = c_1$
 $M_{xy} = -D_{12} \frac{d^2 w^0}{dx^2}$
 $N_{xy} = c_2$
 $M_{xy} = 0 \therefore B_{12} = 0$

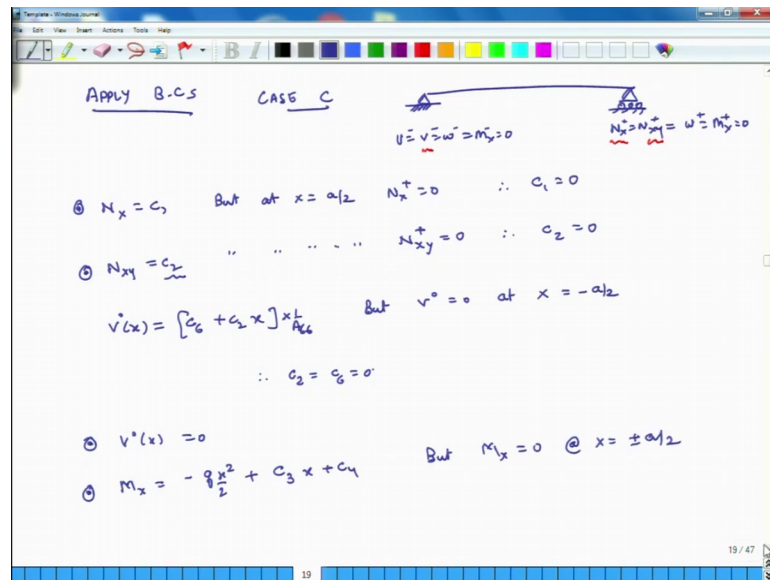
CASE C & D

And, for cases C and D we have developed the solutions for U V and W as well as for M x M y and M xy and D 1 2. And, we have found that instead of just a simple D 1 1 we get a reduced bending stiffness term which is script D 1 1. Similarly, in the denominator in the expression for U x we have a script A 1 1 and in both these cases what we see is that the presence of bending this coupling matrix coupling stiffness matrix; it tends to reduce the bending stiffness as well as the in plane stiffness of the plate. So, that is one thing.

The other thing is that we have till so far not calculated the integration constants C 1 through C 8 and the value of these constants will be determined by implementing the boundary conditions. So, that is what we plan to do now. So, we have all these solutions for U V W also for M x M y and M xy and the next thing we are going to do is we are

going to implement the solutions for these integration constants. We are going to calculate these integration constants. One thing I omitted writing in terms of the solution in the last class was that I did not explicitly write down the values of N's. So, N_x is equal to C_1 N_{xy} is equal to C_2 so, that is still there.

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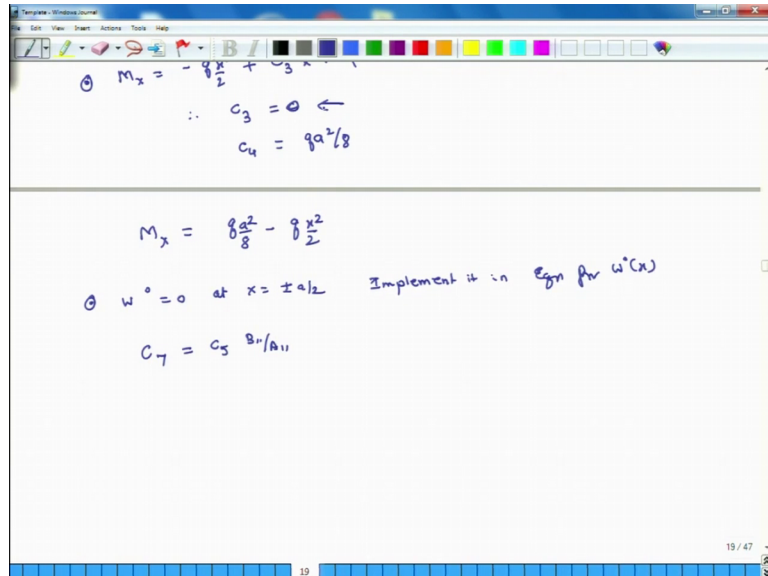


So, now we apply the boundary conditions and the first will be for case C and in case C the B C S are $U = V = W = M_x = 0$ at the at x is equal to minus a over 2 . And, here it is N_x is equal to N_{xy} is equal to W is equal to M_x is equal to 0 at x is equal to plus a over 2 . So, N_x is equal to C_1 , but at x is equal to a over 2 N_x plus is equal to 0 so, therefore C_1 equals 0 . So, we have implemented the first boundary condition which is this one ok. Next we apply the second boundary condition which is on N_{xy} . So, we know that N_{xy} is equal to C_2 , but at x is equal to a over 2 N_{xy} plus is equal to 0 therefore, C_2 equals 0 . So, now we have implemented 2 boundary conditions.

The third is we will see what is V . So, V_x is equal to $C_6 + C_2 x$ times 1 over A_{66} ok, but we know that V_{naught} is equal to 0 at x is equal to plus minus a over 2 . So, if that is the case so, therefore C_2 is equal to C_6 is equal to 0 . So, V_{naught} is 0 ; oh I am sorry V_{naught} is 0 not at x is equal to plus minus a over 2 , it is just at V_{naught} 0 at x is equal to minus a over 2 and C_2 is already 0 we have calculated it. So, C_6 also comes out to be 0 ok. So, we have implemented now 3 boundary conditions out of 8 ok. The next boundary condition we will apply will be on M . So, we know that M_x is equal to

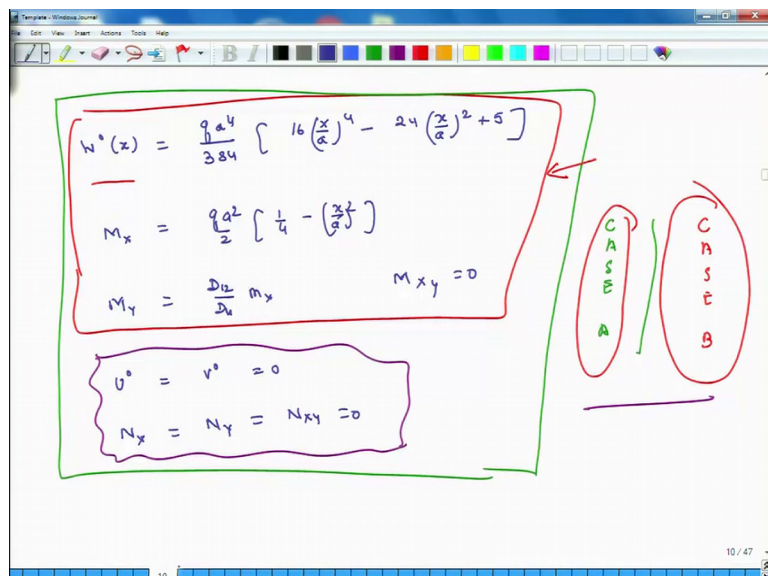
minus $q x^2$ over 2 plus $C_3 x$ plus C_4 ok, but we know that M_x is equal to 0 at x equal to plus minus a over 2.

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So, we do all these we apply both these boundary conditions and we find that therefore, C_3 is equal to 0 and C_4 is equal to $q a^2$ over 8. So, because of that M_x is equal to $q a^2$ over 8 minus $q x^2$ over 2 same relation or I can have the same expression which is this M_x is equal to $q a^2$ over 2 times $1 - 4x^2/a^2$. So, this is my relation for M_x .

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So, now we have applied 6 boundary conditions ok. So, we have calculated the value of C_1 , C_2 , C_6 , C_3 , C_4 and what we have not yet implemented our conditions on U and W . So, those are the conditions which I have not yet been implemented. So, the expression for W is this long thing and let us implement the so, we know that W , W is equal to 0 at x is equal to plus minus a by 2.

So, if we implement this condition implement it in equation for W naught x , what do we get? We get 2 conditions C_7 is equal to C_5 times B_{11} by A_{11} , this is 1 condition we get because, then we implement this condition in this equation for W this term has to be 0 ok, this term has to be 0. So, when we do that we get this relation and we also get the other thing and C_3 is already 0. So, that term also goes away.

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The image shows a digital whiteboard with the following handwritten content:

$$M_x = \frac{8a^2}{8} - \frac{8x^2}{2}$$

① $w'' = 0$ at $x = \pm a/2$ Implement it in Eqn for $w(x)$

$$C_7 = C_5 \frac{B_{11}}{A_{11}}$$

$$C_8 = \frac{5qa^4}{384}$$

$$w''(x) = \frac{qa^4}{384} \left[16\left(\frac{x}{a}\right)^4 - 24\left(\frac{x}{a}\right)^2 + 5 \right]$$

So, so the other condition we get is C_8 is equal to $5qa^4$ divided by 384. So, if we put all these things together what we get is W naught x equals a to the power of 4 times of course, q divided by 384 subscript D_{11} into $16x$ by a^4 minus $24x$ by a^2 plus 5. So, we have now applied 7 boundary conditions. So, the only boundary condition, which is left is now on U ok. So, now we apply the 8th boundary condition which is on U and we apply on the equation for U naught ok.

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$w'' = 0$ at $x = \pm a/2$ Implement it in eqn for w''

$$C_7 = C_5 \frac{B_1 A_{11}}{D_{11}}$$

$$C_8 = \frac{5 q a^4}{384 D_{11}}$$

$$w''(x) = \frac{q a^4}{384 D_{11}} \left[16 \left(\frac{x}{a}\right)^4 - 24 \left(\frac{x}{a}\right)^2 + 5 \right]$$

Applying B.C on $v''(x) = 0$ at $x = -a/2$.

$$v''(x) = \frac{B_1 q a^3}{24 A_{11} D_{11}} \left[4 \left(\frac{x}{a}\right)^3 - 3 \left(\frac{x}{a}\right) + 1 \right]$$

So, applying the B.C on U naught x is equal to 0 at x is equal to minus a over 2, what do we get? We get ultimately this is the equation we get if you do the math carefully. So, this is equal to $B_{11} q a^3$ divided by $24 A_{11} D_{11}$ into $4 x$ by a cube minus $3 x$ by a plus 1 ok. So, we will compile the results.

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FOR CASE C

$$v''(x) = \frac{B_1 q a^3}{24 A_{11} D_{11}} \left[4 \left(\frac{x}{a}\right)^3 - 3 \left(\frac{x}{a}\right) + 1 \right]$$

$$v''(x) = 0$$

$$w''(x) = \frac{q a^4}{384 D_{11}} \left[16 \left(\frac{x}{a}\right)^4 - 24 \left(\frac{x}{a}\right)^2 + 5 \right]$$

So, U naught x so, these are the results for case C. So, what are the equations? The equations are U naught x is equal to $B_{11} q a^3$ divided by $24 A_{11} D_{11}$ times D_{11} into $4 x$ over a cube minus $3 x$ over a plus 1. And, V naught V naught is equal to 0 and W naught is equal to $q a^4$ divided by $384 D_{11}$ $16 x$ over a minus $24 x$ over a plus 5, oh this is 2 the power of 4 and this is to the power of 2 ok.

So, these are the relations we get for W and for U and V and what we will do is we will continue this discussion next week also because; this is what we have solved for case C . Similarly, what we will also do is we will solve for case D and see whether these solutions are same or different. So, that concludes our discussion for today and I look forward to seeing you next week at the same time that is on Monday.

Thank you very much. Have a great weekend bye.