Advanced Composites
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Lecture – 26
Force Equilibrium in Z-direction

Hello, welcome to Advanced Composites. Today is the 2nd day of the ongoing week, and yesterday we just finished developing two equilibrium equations. The first one was related to force equilibrium in the X-direction and the next one was force equilibrium in the Y-direction. Today we will extend that discussion, because ultimately we have to satisfy 6 different equilibrium conditions. So, the third equilibrium condition is that sum of forces in the Z-direction should be equal to 0.

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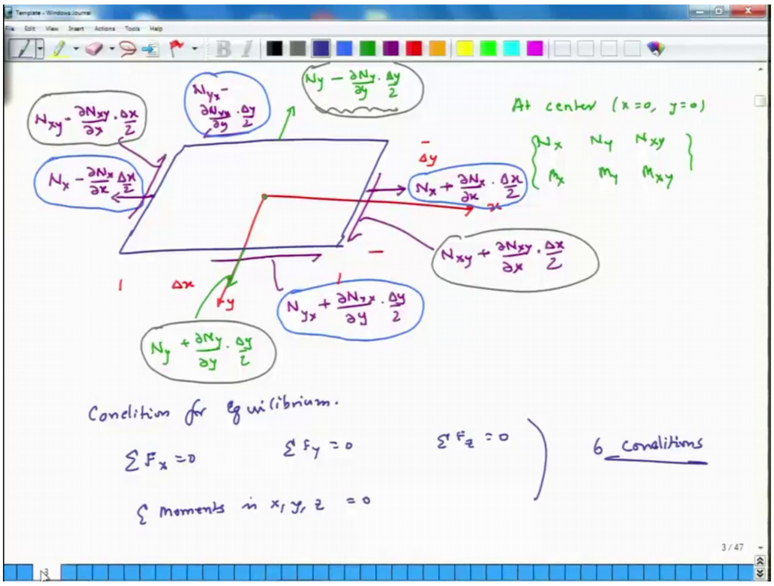

Now, yesterday when we started discussing and we were talking about force equilibrium X and Y-directions, we had only listed forces in X and Y-directions because those are the only forces we wanted. We could have put other forces also in Z-direction also, but it would have made this picture much more complex. So, to avoid complexity and this picture meets our needs, we did not put any forces in Z-direction.

So, today we will start seeing what kind of forces are there on the plate.

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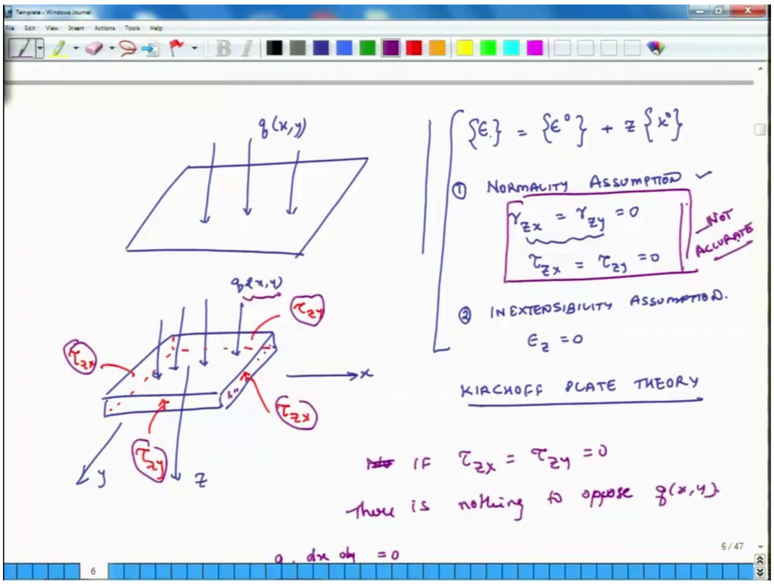

So, if you look at the plate and typically for instance consider this; this is a piece of wood. It could be considered as a composite and if I put some load on it, there is a vertical force which is acting on this plate, ok. There is a vertical force which is acting on this plate. And in a lot of cases, plates are required to handle vertical loads, because for instance this roof or whatever this table, these are plates and they experience external loads in Z-direction. So, there is load in Z-direction and this load could be uniform or it could be a point load or it could be a distributed load. So, we will say that there is a load distributed in the plate in the vertical direction and it is q newtons per square meter, ok. This is the vertical load.

So, this is one load, but for equilibrium we have to see all the loads which are there on the plate, ok. So, the other thing is that when we were developing our equations to connect midplane strains and when we were developing these equations epsilon is equal to epsilon naught minus or actually it is z times midplane curvatures, we had made two important assumptions. What were those assumptions?

The first assumption was that if I bend the plate, there was a normality assumption, right. So, whatever if I am bending a plate and suppose this edge is normal to this line, and this line and when I bend the plate, then this edge remains normal after it is bent, ok. So, we had a normality assumption and a consequence of this normality assumption was that we said that gamma z x is equal to gamma z y equals 0, ok. So, this was the normality assumption, and because of this we also said that because shear stress z x, shear stress shear strain z x and shear strain z y is 0 because at each layer the fibers are running parallel to the plane.

We also said that tau z x is equal to tau, z y is equal to 0. So, this is tau z x and is equal to tau, z y is equal to 0, and the second one was that in extensibility assumption and what does that mean that if I am bending the plate. And I am stretching it, the dimension of the plate in Z-direction, it is not changing. So, we had said that epsilon z z or epsilon z is 0, ok. Epsilon z is equal to 0.

Now, these assumptions belong to a theory of plates called Kirchhoff Plate Theory. So, the first assumption is related to normality and the second one is related to inextensibility. Now, consider this if there is a plate and suppose it is seeing some external transverse load q x y and I am actually just drawing the three-dimensional because it has some thickness. I am not drawing, just the midline, but also the thickness. What is tau?

So, this is my x axis, this is my y axis and this is z axis based on the sign convention. What is tau z x? Tau z x will act in this direction. This is tau z x. What is tau z y? This will be tau z y and likewise we will also have shear stresses tau z x on the other side and tau said why on so we will have tau z y on this plane, and tau z x on this plane, right based on our sign convention, but based on our assumptions, all these stresses are 0. All these stresses are 0 because we have assumed gamma z x is equal to 0; gamma z y is equal to 0.

Now, if these stresses are 0, then there is nothing to resist. If tau z x is equal to tau z y is equal to 0, there is nothing to oppose q xy, right because there is no other external force on the system. There is no other external force. I am just cutting a piece of this plate and on this surface the only force which can resist has to be created because of tau z x. If tau z x is 0 at our z y 0, nothing will oppose this q x y, ok.

So, what does that mean? Then, the condition for equilibrium will be q times dx times dy is equal to 0 which means that q should be 0, q should be 0, but we know that plates when we put loads on them, we are actually applying lower qs and q is not 0 in reality because most of these plates, they are subjected to external transverse loads and they are able to bear those loads all that. This discussion means that our assumptions related to gamma z x and gamma z y. They are not necessarily true and they are not accurate because they contradict in reality.

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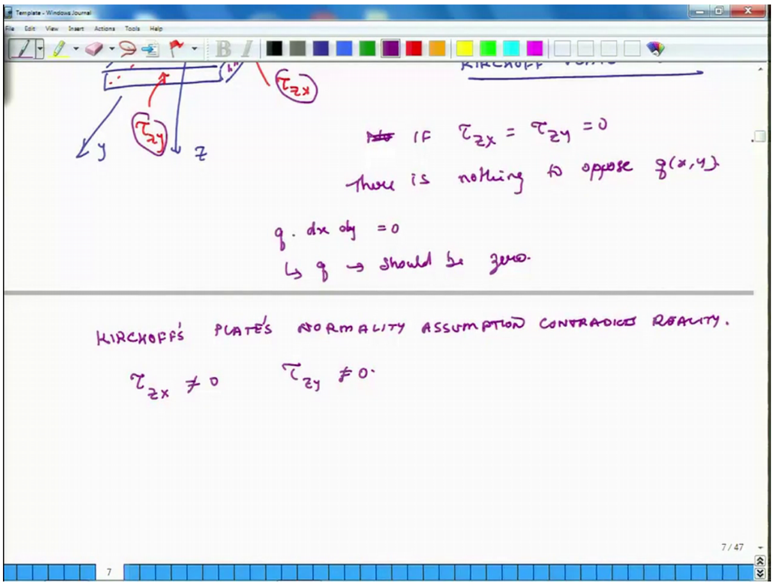

So, Kirchhoff plates normality assumption it is not, it contradicts in reality. So, there is something important to understand. So, we cannot say and because of this we cannot say that tau zx is 0. It cannot be 0 in general and similarly tau z y it cannot be 0 in reality. So, there is a defect in Kirchhoff’s plate theory, and we have identified this defect and now, we will correct this defect. So, we say that well these shears are not necessarily 0, and I integrate this shear along the thickness of the plate.

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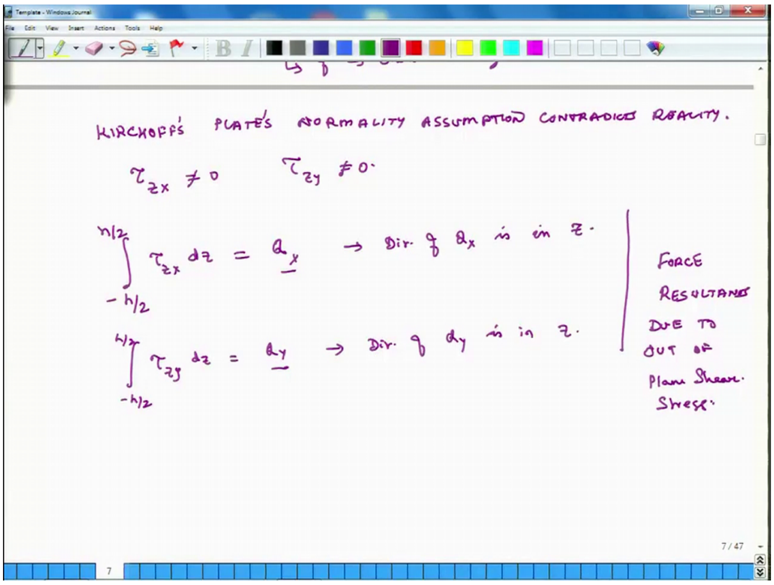

So, if the plate is thick plate, thickness is minus s by 2. 2 s by 2, then I get just as I get a force resultant in x and force resultant in Y-direction. Similarly, I get a force resultant in Z-direction and because it is an integral of tau zx, I call it Q x. What is the direction of Q x? Direction of Q x is in Z-direction, because tau z x is acting in the vertical direction, and we call it Q x because it is acting on the x plane, it is acting on this surface.

So, that is why we call it x, but its direction is in the Z-direction, understood. Similarly if tau z y is not 0, then if I integrate it, then I call it Q i and the direction of Q i is in Z-direction also, ok. So, note these are not forces, Q x is not force, Q y is not force. These are force resultants and they are due to out of plane shear, out of plane shear stress. They do out of plane shear stress.

So, now that we have developed the theory for Q x and Q y, we will again draw the mid plane of the plate with all the vertical forces or force resultants acting on it, ok. So, let us look at this plate.

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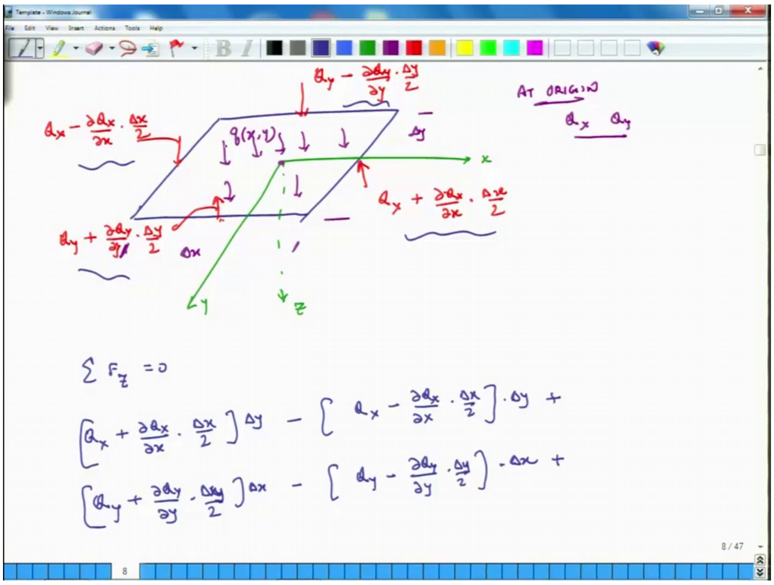

So, again we will draw a mid plane. So, this is the mid plane, my x axis is here. This is y axis, this is z axis, and this plate is seeing a vertical force which is distributed in nature. So, this is Q xy and at the midpoint at origin force resultants in the Z-direction are Q x and Q,y.

So, we are just putting force resultants and z we are only putting Z-direction related components, ok. Not x and y related components because they do not matter. If we are trying to find equilibrium in the Z-direction, so at a region if we have Q x and Q y, then on this surface again this is delta x and this is delta y, then here I have Q x plus del Q x over del x times delta x over 2 and on this side this is my other Q x. So, this is Q x minus del Q x over del x times delta x over 2.

On the other surface by Taylor series expansion and on this plane, I have Q y Q y plus del Q y over del y times delta y over 2 and on the other plane, I have Q i minus del Q y over del y times delta y over 2, ok. Is this clear?

Student: Direction will be negative.

Direction will be negative because whenever we have shears, if you have a shear here. Then on the other surface shear is always like that, right. The direction will be negative, ok. So, if we sum up all the forces in the Z-direction sigma of F z is equal to 0, then what do we get? We get Q x plus del Q x over del x times delta x over 2 times delta y, right minus Q x minus del Q x divided by del x times delta x over 2 times del y plus. We will write the components due to y Q y plus del Q y over del y times delta delta y over 2 times del x delta x minus Q i minus del Q y over del x: I am sorry del y times delta y over 2 times del x.

So, these are the forces due to Q s, these are the forces due to Qs, right. So, these are the forces due to these 4 Qs, but there is 5th force which is related to this external load Q.

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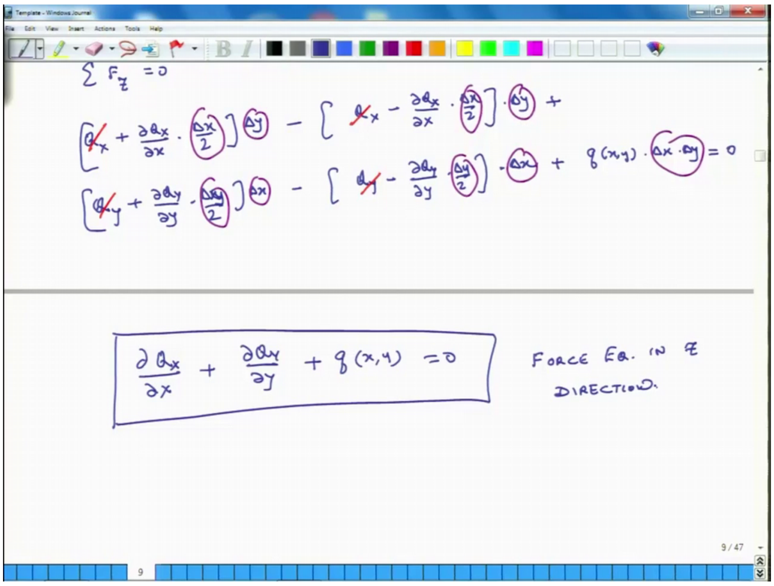

So, what is the force due to this? It is Q which is a function of x and y times. The area of the plate which is del x times del y and all of these guys, they add up to 0. So, these terms go away and also this term delta x delta y, they also go away. So, the equilibrium equation for Z-direction, it becomes del Q x over del x plus del Q by over del y plus Q of x y is equal to 0.

So, this is the equation for, this tells us about force equilibrium in Z-direction. So, we have developed 3 equations related to force equilibriums.

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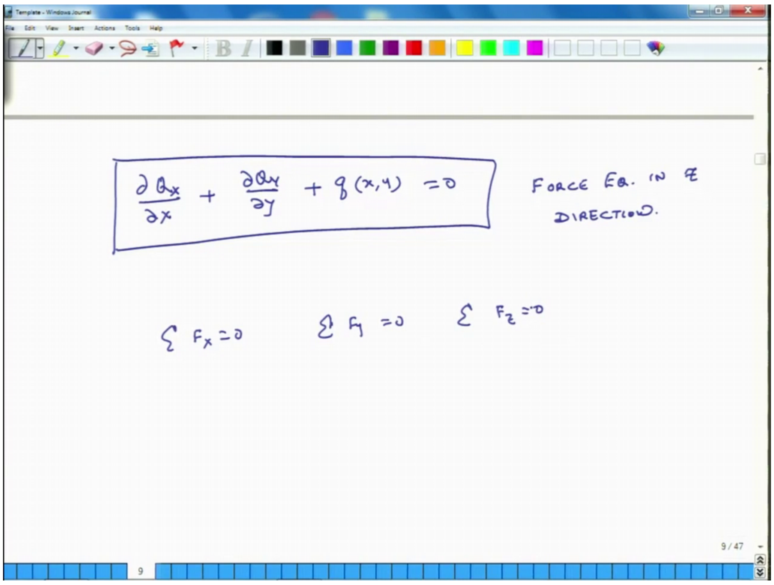

First one is, F x is equal to 0, second one is F y equals 0 and the third one is F z is equal to 0, and as I said earlier if there is a particle which is subjected to external forces and if it is a point object. Then the only equations we have to worry about is force equilibrium equations, but since this small element of plate is not just a point object, but it has a finite non-zero size. So, we also have to worry about momentum equilibrium equations. So, starting next week or not next week, starting tomorrow we will start looking at these moment equilibrium equations and we will also develop moment equilibrium equations related to sum of moments in X-direction, Y-direction and Z-direction is 0.

So, that concludes our discussion for today. We look forward to seeing you tomorrow once again in context of these Moment Equilibrium Equations.

Thank you. Bye.