

Advanced Composites
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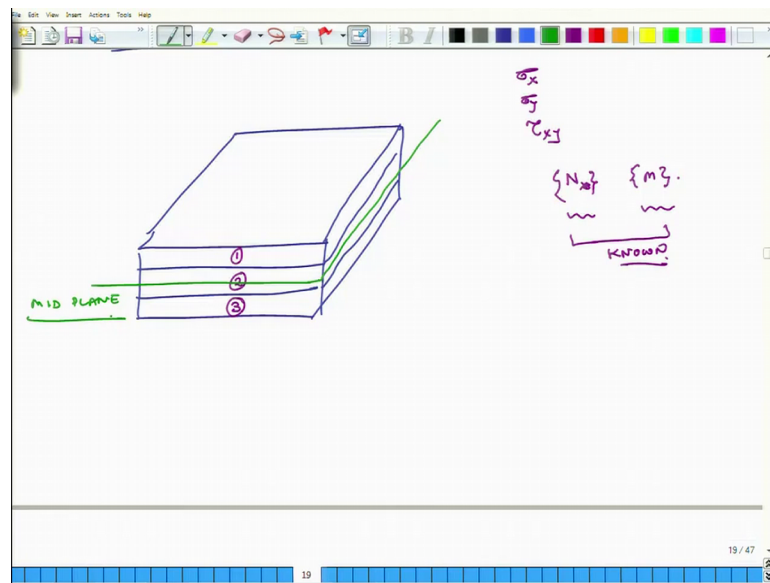
Lecture – 23
Failure Initiation in Composite Laminate

Hello, welcome to Advanced Composites. Today is the 5th day of the ongoing 4th week of this course. And today and tomorrow we will have a detailed discussion on failure of composite laminates. Till so far we have only discussed how a particular layer of a composite fails. But what happens when the composite at a laminate level is subjected to a stresses and strains then how do we predict its failure. So, this is what we will discuss today.

And also if time permits we will start discussing failure of composite laminates in a progressive way, because when we have a composite laminate what happens is that as I keep on subjecting to more and more stresses and strains, quite often there will be one particular layer which will fail first. Then it will stop taking any further load and all that extra load will go to other layers, and then some other layer will fail and this process continues till the entire composite fails.

So, how does this failure happen and this failure process that is why it is known as a progressive failure. But before that we will first start trying to figure out how do we predict the failure of the first layer because if we understand the failure of the first layer then that will teach a sufficient enough material. So, that we can go on and predict the failure of second layer, and third layer, and 4th layer and so on and so forth.

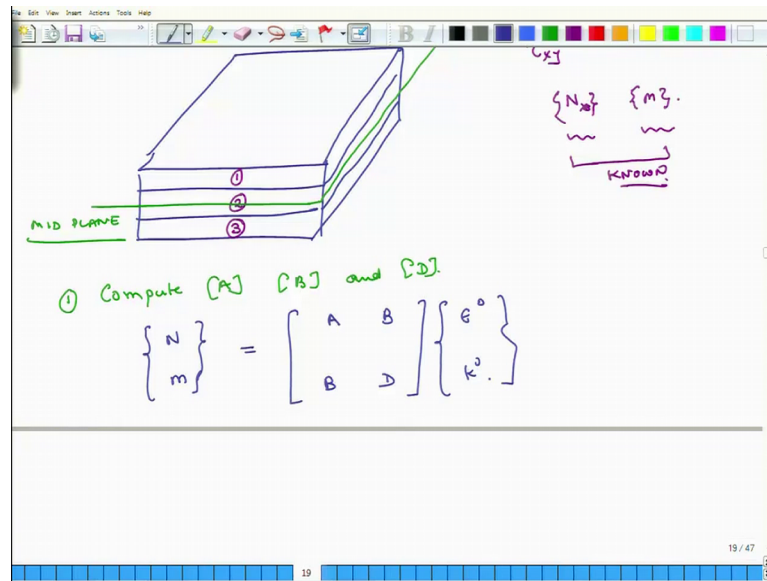
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So, the theme of today's discussion at least for starters with the failure of the first layer in a composite laminate, failure initiation in composite laminate. So, suppose I have a composite actually and it is not just a single layer, but a laminate. So, let us say it has three layers. So, this is layer number 1, layer number 2, layer number 3 and this composite is subjected to some several external stresses and strains. So, it may be σ_x , σ_y , τ_{xy} and so on and so forth.

But at the laminate level whenever we analyze composites we never worry about σ_x , σ_y and τ_{xy} at the laminate level, we always talk about force resultants and moment resultant. So, if I have to predict the failure of a composite laminate I should know N and M , which are the force resultants and moment resultants acting on all the edges of the plate. If I do not know then I cannot predict the failure of the composite. So, so this we assume that it is known, we assume that this is known. Now, if this is known then first thing is I identify a mid plane of the plate. So, this is the mid plane, ok.

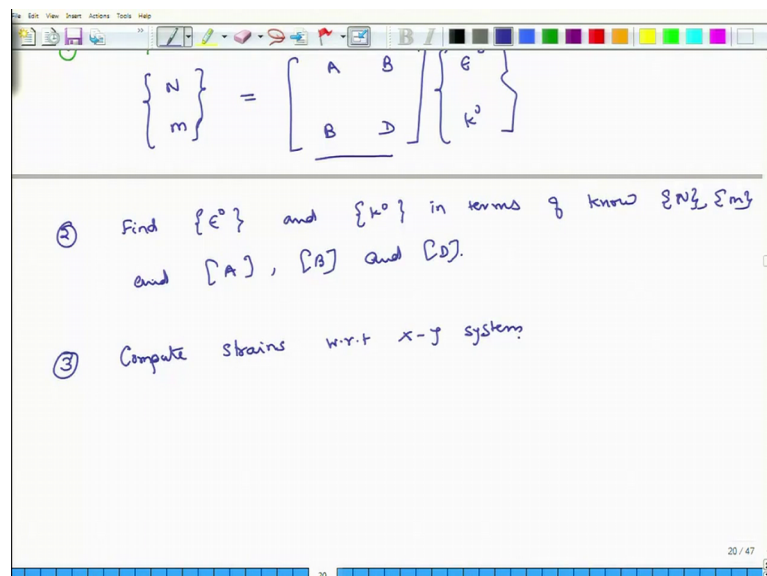
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And then the first step I do is I compute A, B and D matrices of the laminate. At this state nothing in the laminate is broken, so we compute the A, B, D and no traces of the laminate and then what we do is we calculate the mid plane stresses, mid plane strains and mid plane curvatures of the plate.

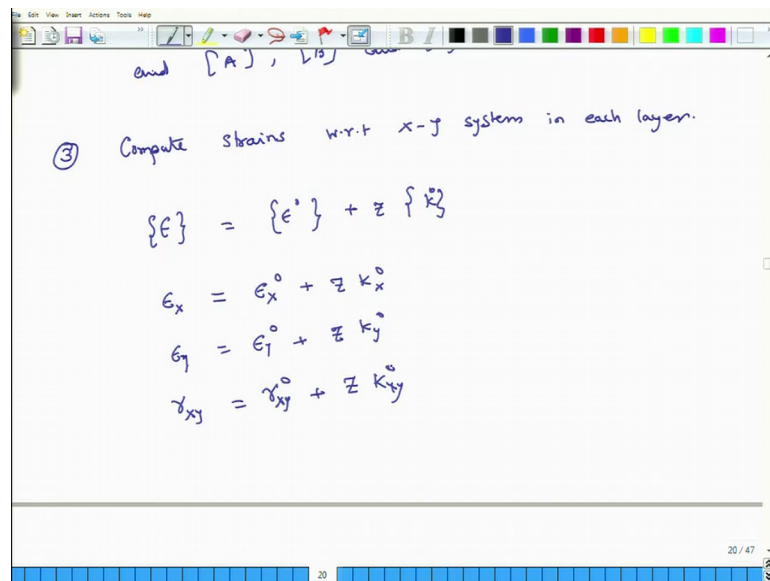
So, we know that the relation is N M equals, A, B, B and D. And this is the mid plane vector, force mid plane strain vector and this is the mid plane curvature vector, ok. So, we first compute A, B, D.

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Then the second step we do is find epsilon naught and k naught in terms of known N, known M, and A, B and D matrices. Basically we have to invert the 6 by the 6 by 6 matrix multiply on the other side and we will get strains and curvatures. Third we have to compute the strains with respect to xy system. So, when I am talking about A B D matrix I still have my x and this is my y direction, ok. So, with respect to these x and y coordinate system I computed strains in each layer.

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and $[A], [B], [D]$

③ Compute strains w.r.t x-y system in each layer.

$$\{\epsilon\} = \{\epsilon'\} + z \{k'\}$$

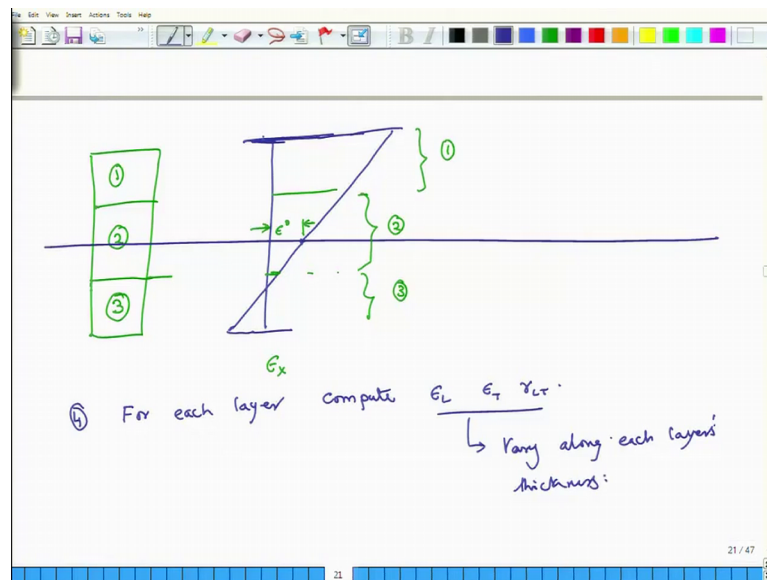
$$\epsilon_x = \epsilon_x^0 + z k_x^0$$

$$\epsilon_y = \epsilon_y^0 + z k_y^0$$

$$\gamma_{xy} = \gamma_{xy}^0 + z k_{xy}^0$$

Now, we had explained earlier that. So, how do we compute the strains in each layer see epsilon is equal to epsilon naught vector plus z times curvature vector, ok. So, specifically epsilon x will be epsilon x naught plus Z times K x naught epsilon y equals epsilon y naught plus Z times K y naught and gamma xy equals gamma xy naught plus Z times K xy naught.

(Refer Slide Time: 08:04)



So, again we look at our laminate and suppose it has 3 layers, layer number 1, layer number 2, layer number 3, then what we will do is we will across the thickness we will find out how is the strain varying. So, we will draw 3 in this case we will draw 3 graphs. So, let us assume that ϵ_x , ϵ_y and γ_{xy} are all positive. If they are positive then this graph will look something like this.

So, if it is positive then when Z is equal to 0 the strain will be nonzero when Z is equal to 0 because if z is equal to 0. So, at mid plane the strain will be nonzero and it will have some slope. So, it may look like this. So, this is ϵ_x , ok. Now, we have to draw 3 such graphs, one for ϵ_x , one for ϵ_y and one for γ_{xy} , ok.

And we have to get an idea that. So, this is, so this is my region for layer 1, this is my region for layer 2, and this is my region for layer 3. And we have to draw similar graphs for all the strains for a ϵ_x , ϵ_y , ϵ_z , γ_{xy} , γ_{yz} , γ_{zx} . So, this is only for ϵ_x similarly I have to draw similar strain graphs for all the things, ok. So, once we have this understanding then the 4th step is for each layer, for each layer compute ϵ_L , ϵ_T and γ_{LT} , ok. You have to compute strains ϵ_L and ϵ_T , γ_{LT} for each layer and these strains will also vary because ϵ_x , ϵ_y and γ_{xy} are changing. So, ϵ_L , ϵ_T and γ_{LT} will vary along each layers thickness.

(Refer Slide Time: 11:19)

→ Vary along each layers thickness.

⑤ For each layer compute σ_L σ_T τ_{LT}

$$\{\sigma\}_{LT} = [Q] \{\epsilon\}_{LT}$$

⑥ For each layer, identify the location where LHS of ~~work~~ MAX. work criteria is at its peak.

Fifth, so once we have calculated epsilon L, epsilon T and gamma LT, then what we do is for each layer compute sigma L, sigma T and tau LT. Using what? Using this relation sigma in LT frame equals Q matrix for that particular layer times strain vector, ok. So, this has to be done at each layer, this has to be done at each layer, ok.

Sixth, for each layer identify the location where LHS of, left hand side of work maximum work criteria is at its peak. What does this mean?

(Refer Slide Time: 13:11)

⑥ For each layer, identify the location where LHS of MAX. work criteria is at its peak.

LAYER ②

⑦ If this value exceeds 1, then that layer will fail.

What it means is that suppose this is layer number 2, suppose this is layer 2, ok. Now, if this is layer 2, the strains maybe will be varying, the strains will be varying along the thickness, strain x may be doing this, strain y might be doing, may be I do not know it may be it may be constant. Strain x , γ_{xy} may be doing this it depends on curvatures and all that, and because these 3 difference strains are doing all sorts of things the only thing which they are doing constantly is that they are linearly varying along the thickness that is all.

So, there is no guarantee whether the location of maximum σ_L , maximum σ_T , and maximum τ_{LT} will simultaneously exist at a particular point on the thickness of each layer. So, you have to scan the thickness of each layer and then identify where the left hand side of the maximum work energy criteria which we have discussed is at its peak, and then if this value exceeds 1, then that layer will fail, ok, then that layer is going to fail.

So, you have to do this layer by layer and you have to make sure that for each layer what is this maximum value of lhs of the work in a work maximum work criteria. And see if that value is equal to 1 or if it exceeds 1 then that layer will fail. And if it does not fail then you are fine, but if it meets 1 or if it exceeds 1 then that layer will fail. And if there are several layers which are failing then the value the layer which has this value at its maximum that is the layer which will fail first. So, that is how you identify how this whole thing works, this is important to understand.

(Refer Slide Time: 15:55)

EXAMPLE

Diagram of a 3-layer composite: Top layer (3 mm, 45°), Middle layer (6 mm, 0°), Bottom layer (3 mm, 45°). A green arrow labeled 'x' points to the right from the middle layer.

Q matrix for 0° layer:

$$[Q]_0 = \begin{bmatrix} 20 & 0.7 & 0 \\ 0.7 & 2 & 0 \\ 0 & 0 & 0.7 \end{bmatrix}$$

Q matrix for 45° layer:

$$[Q]_{45} = \begin{bmatrix} 1182 & 662 & 567 \\ 662 & 861 & 567 \\ 567 & 567 & 662 \end{bmatrix}$$

Load vector:

$$\begin{Bmatrix} N_x \\ N_y \\ N_{xy} \end{Bmatrix} = \begin{Bmatrix} 1000 \\ 200 \\ 0 \end{Bmatrix} \text{ N/mm.} \quad \{M\} = \{0\}$$

Whether the composite will fail?

So, we will do an example. So, the example is that we have a composite and it is a composite with 3 layers, the top layer is 3 millimeters thick, bottom layer is 3 millimeters thick and middle layer is 6 millimeters thick. And the top layer is having an orientation of 45 degrees, bottom layer is also having an orientation of 45 degrees, and the middle layer is having an orientation of 0.

So, my mid plane passes through the 0 degree layer this is the x axis, ok. And we will also define Q matrix for each layer. So, for the 0 degree layer it is 20, 0.7, 0; 0.7, 2 0; 0, 0, then 0.7. So, this is for 0 degree layer. And Q for 45 degree layer is something different. So, it is 1182, 662, 567; 662, 861, 567; 567, 567, and 662.

And this composite, so we said that we want to find when this composite is whether this composite is going to fail or not hm. So, whether it is going to fail we have to know what are the external what, force and moment resultants on this system. So, that is given. So, N_x , N_y , N_{xy} and that equals 1000, 200 and 0 Newtons per millimeter, and no external moment resultants on the system. So, this is the problem and then we have to figure out whether the composite will fail, whether the composite is going to fail.

So, that is what we have the problem definition as; and we will continue this particular problem tomorrow because it will take some time. And then we will see whether this composite fails, and more importantly we will learn how to solve this problem. So, with that we conclude for today and I look forward to seeing you tomorrow.

Thank you.