

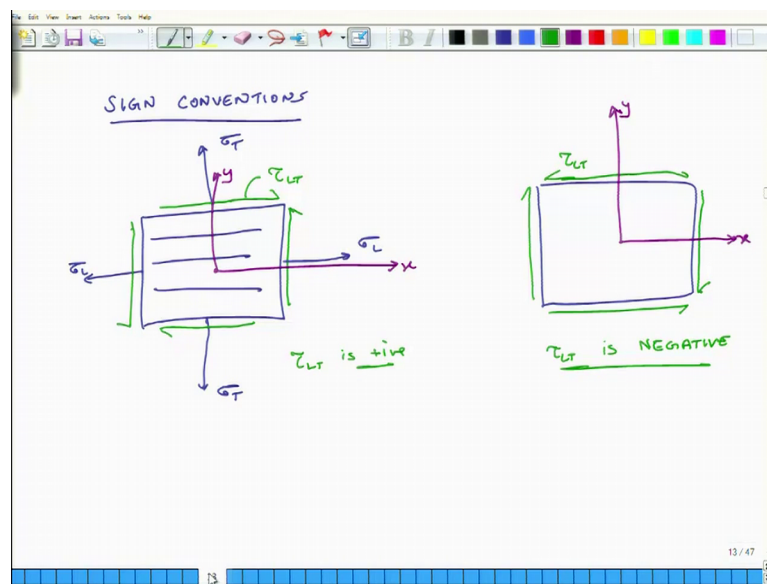
Advanced Composites
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Lecture – 22
Importance of Sign of Shear Stress

Hello. Welcome to Advanced Composites. Today is the fourth day of this course and today we will continue our discussion on failure in composite materials specifically at the fly level. And what we have discussed till so, far are three different failure theories which help us to understand whether a composite fly will fail. And these three theories are: maximum stress theory, maximum strain theory, and maximum work theory.

In this context it is important to understand how the sign of shear stresses influences the failure of a composite. So, this is what we plan to discuss today, but before we do that we should have a very clear understanding as to what are sign conventions are.

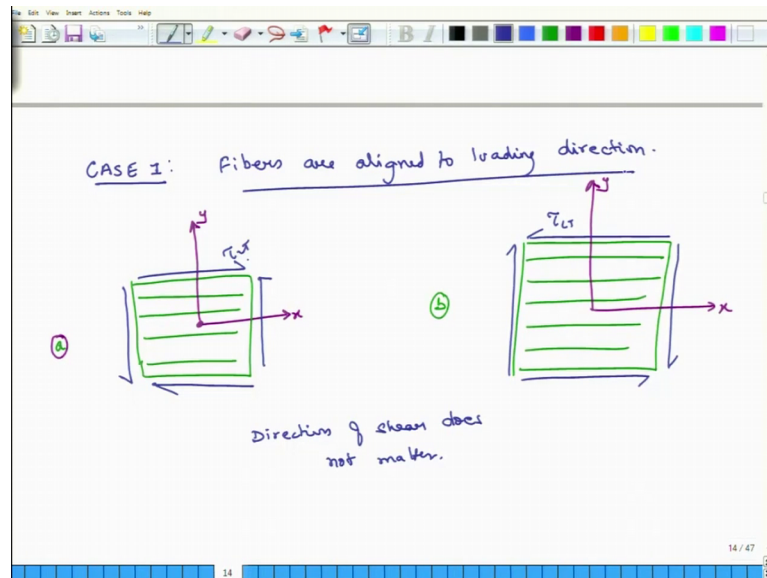
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So, sign conventions. So, if I have a composite material, I can subjected to tensile stress and in that case σ_L is positive. I can subject it to tensile stress in the transverse direction σ_T . So, this is the direction of the fiber and I can also subject it to shear stresses and this is one way I can specify the shear stress. So, this is τ_{LT} and in this case τ_{LT} is positive. This is our sign convention. What would be the case of a negative shear stress?

In case of negative shear stress, it will be placed like this. Now before that I wanted to we should specify what are access system is. So, this is a positive convention is this is positive x and this is positive y. Similarly if this is positive x, this is positive y, then this is tau x tau LT and tau LT is negative; tau LT is negative. So now, let us move on and let us understand, what happens when the direction of shear stress it gets reversed.

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So, we will consider two cases. Case 1: fibers are aligned to loading direction; fibers are aligned to the loading direction. And here in case 1 we will have two different cases. So, this is case 1 a - fibers are aligned to the loading direction. So, these are the direction in which fibers are running and this is the case of a positive shear stress and this is my axis system. This is the case of this the access system.

So, this is case a; and then b - we apply a negative shear stress fibers are still aligned to the loading direction. So, this is my x axis, this is the y axis, but here I am applying negative shear stress. The in these cases, so, when fibers are aligned to the loading direction, then in this situation the direction of shear does not matter. It really does not matter direction of shear. And you can see that mathematically, so, this tau LT and this is tau LT.

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TSAI-HILL CRITERIA OR MAX. WORK THEORY

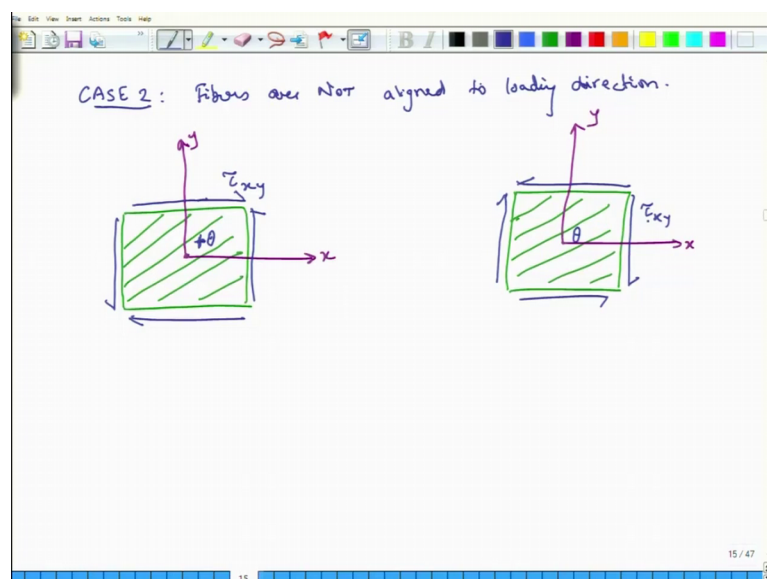
Failure will not occur if:

$$\left(\frac{\sigma_L}{\sigma_{LU}}\right)^2 - \left(\frac{\sigma_L}{\sigma_{LU}}\right)\left(\frac{\sigma_T}{\sigma_{LT}}\right) + \left(\frac{\sigma_T}{\sigma_{LT}}\right)^2 + \left(\frac{\tau_{LT}}{\tau_{LTU}}\right)^2 < 1$$

| | | |
|------------------------|----------------------|---------------------|
| $\sigma_L = 37.1$ ✓ | $\sigma_T = -12.1$ | $\tau_{LT} = -57.5$ |
| $\sigma_{LU} = 1062$ ✓ | $\sigma_{LT} = 31$ | $\tau_{LTU} = 72$ |
| $\sigma_{LU}' = 610$ ✓ | $\sigma_{LT}' = 118$ | |

You can see that mathematically because if you plug this, in this case you put plug tau LT into this equation, all these terms will be 0. Because there is no sigma L, there is no sigma T, there is no sigma T only tau LT is there. And whether tau LT is positive or negative, it does not matter everything its squared. So, the failure criteria does not change. So, direction of shear stress does not matter, in case one when fibers are aligned to the loading direction.

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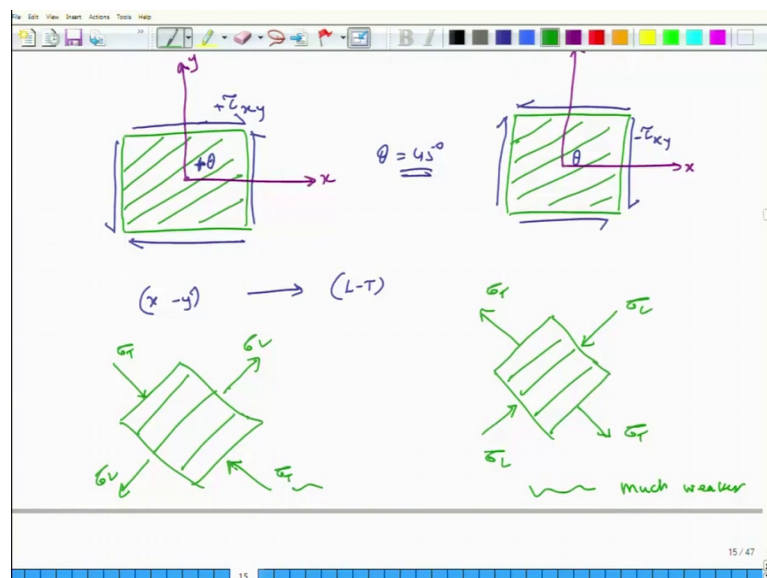


The second case is when fibers are not aligned to do loading direction. Fibers are not aligned to loading direction. The fibers are not aligned to loading direction, then what does it mean? So, we will draw a picture. So, these are two cases and fibers are in this case running like this. So, the angle is let us say, positive theta and in the second case also fibers are running in the same direction. So, here also the orientation of fibers is at positive theta with respect to the x axis. This is my x axis, y axis, y axis. But the difference in these two cases is that in one case, I have a positive shear. This is tau LT and in the other case I have negative shear. In this case, the direction of shear whether it is positive or negative, it will go with which will matter significantly.

Now in this case if we have to predict the failure of the composite, we can x; excuse me sorry. So, this I should not call it tau LT, I should call it tau x y. Why should I call it tau x y; because I am applying tau with respect to x and y direction.

In the other case the L direction and the x direction they were same. So, tau x y was same as tau LT, but in this case I have to do I call it tau x y right. So, here tau x y is positive and here tau x y is negative based on our convention. Now if we have to predict failure in this case, what do I have to do?

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From x y plane, I have to go to LT plane. I have to compute stresses in the LT plane. Right now in the x y plane, sigma x is 0 sigma y is 0 and tau x y is not 0. But for computing failure, I have to first go to LT plane. So, I have to compute sigma L sigma T

and tau LT ok. And let us assume that this theta in both the cases is 45 degrees that makes our things simpler. So, if that is the case, so if I go from this x y plane to LT plane, what happens in the first case? The fibers are oriented like this and if I do the computation I will find that there is a compressive stress in the transverse direction. So, sigma T is compressive and longitudinal stress is tensile, when I do the calculation in terms of tau x y hm. In the second case, fibers are still in this direction. The situation changes, here sigma L becomes compressive and the transfers stress the transfers stress becomes tensile.

Now, clearly the second case, the composite will fail early because the composite is extremely weak in the transfers' tensile direction. If you subject cir composite to tensile forces in the transverse direction, it fails at a very early stage. So, this is much weaker relative to the first case.

So, the overall point is that if the fibers are aligned to the loading direction that is if the LT plane and x y plane are aligned to each other, then the direction of sign does not matter. But if the LT plane and the x y plane are not identical if the access system of LT access system and x y access systems are not identical, then the direction of sign makes a very significant difference. So, this is something we will also do through an example.

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EXAMPLE :

Diagram: A square element with fibers oriented at 45° to the x-axis.

Given

| | |
|---------------------|----------------------|
| $\sigma_{LV} = 500$ | $\sigma'_{LV} = 350$ |
| $\sigma_{TV} = 5$ | $\sigma'_{TV} = 75$ |
| $\tau_{LTV} = 35$ | |

Q: At what value of τ_{xy} will composite fail if:

- (a) τ_{xy} is positive
- (b) τ_{xy} is negative

So, we will do this through an example. We will do this through an example. So, we will have, we have a material where the fibers are oriented like this and this is my x axis this

is my y axis. So, this theta we say is 45 degrees, theta is 45 degrees. And so, this is a single layer and strengths of this material are 500 mpa sigma L U prime is 350 mpa sigma T U is 5 mpa and sigma T U prime is 75 mpa and tau L T U is 35 mpa.

So, this is given. And then the question is at what value of tau x y will composite fail? If and their two conditions a tau x y is positive and b tau x y is negative. So, these are the two conditions. So, at what value of tau x y will the composite fail, if tau x y is positive? And the other one is if the shear is negative?

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Handwritten notes on a digital whiteboard showing stress transformation equations and a table for positive and negative shear.

Equations at the top:

$$\sigma_L = \tau_{xy} \sin 2\theta \quad \sigma_T = -\tau_{xy} \sin 2\theta \quad \tau_{LT} = \tau_{xy} \cos 2\theta$$

| | | | |
|---------|--------------|--------------|---|
| + SHEAR | $+\tau_{xy}$ | $-\tau_{xy}$ | 0 |
| - SHEAR | $-\tau_{xy}$ | $+\tau_{xy}$ | 0 |

Below the table, under the heading "POSITIVE SHEAR", is the following equation:

$$\left(\frac{\sigma_L}{\sigma_{LU}}\right)^2 - \frac{\sigma_L}{\sigma_{LU}} \frac{\sigma_T}{\sigma_{LT}} + \left(\frac{\sigma_T}{\tau_{LT}}\right)^2 + \left(\frac{\tau_{LT}}{\tau_{LU}}\right)^2 = 1$$

So, we make a table. So, whether it is positive or negative, we have to calculate sigma L and signal L is tau x y; sin 2 theta, sigma T is minus tau x y sin 2 theta and tau LT is tau x y cos 2 theta and we will make this table.

So, in the first case, it is positive shear and in the second case it is a negative shear. So, theta is 45 degrees right. The only thing where various is whether tau x y is plus or negative that is all ok. So, this is plus tau x y, this will be minus tau x y and this will be 0. And in the other case this is minus tau x y, this is plus tau x y and this is still 0. So, for these two situations, now we explore the failure stress. So, positive shear and what do we do? We use the maximum work criteria.

So, what is it what does it say? It sigma over sigma L U whole square minus sigma L by sigma L U sigma T by sigma L U plus sigma T by sigma T U whole square plus tau LT

by τ_{LTU} whole square at. So, when it is failing when the composite fails, this becomes 1. So, our condition for failure is this is equal to 1. So, we putting these values.

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The image shows two handwritten equations on a whiteboard background, representing failure conditions for positive and negative shear.

POSITIVE SHEAR

$$\left(\frac{\sigma_L}{\sigma_{LU}}\right)^2 - \frac{\sigma_L}{\sigma_{LU}} \frac{\sigma_T}{\sigma_{LT}} + \left(\frac{\sigma_T}{\sigma_{LT}}\right)^2 + \left(\frac{\tau_{xy}}{\tau_{xyf}}\right)^2 = 1$$

$$\left(\frac{\tau_{xy}}{500}\right)^2 - \frac{\tau_{xy}}{500} \times \frac{\tau_{xy}}{350} + \left(\frac{\tau_{xy}}{75}\right)^2 + 0 \rightarrow \tau_{xyf} = 75.4 \text{ MPa}$$

NEG. SHEAR

$$\left(\frac{\tau_{xy}}{350}\right)^2 - \frac{\tau_{xy}^2}{500 \times 350} + \frac{\tau_{xy}^2}{5^2} + 0 = 1$$

$$\tau_{xyf} = 5 \text{ MPa}$$

So, when it is positive shear, then it is τ_{xy} divided by what σ_L is positive σ_L is positive. So, σ_{LU} is what is the value of σ_{LU} 500 right and this whole thing is square minus τ_{xy} by 500 times τ_{xy} sigma T. Now sigma T is negative sigma T is negative, this number is negative right. So, it means it is compressive it is values is still τ_{xy} .

So, τ_{xy} and then the corresponding strength is.

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350 right 350 plus sigma T which is again τ_{xy} divided by sigma T U which is and this, this again compressive; so, 750 75 plus 0.

So, this if you solve for τ_{xy} , you get τ_{xyf} at which it will fail. This comes out to be 75.4 mpa using this thing. And if we are talking about negative shear, then the numbers change. So, this is how the equation looks like τ_{xy} by 350 minus τ_{xy} square by 500 into 350 plus τ_{xy} by 5 square plus 0 is equal to 1. And this gives us a τ_{xyf} is equal to 5 mpa. So, this is very significant difference between the failure strength of the material, if I have a positive shear or a negative shear.

So, this is what I wanted to illustrate in this example and we will close this discussion today. And tomorrow we will start discussing failure of the laminate, because still so far we have only discussed how a particular layer fails. Now we will start discussing about failure of the laminate. So, that is pretty much it for today and we will look forward to seeing it to you tomorrow.

Thank you.