

**Phase Equilibria in Materials (Nature and Properties of Materials - II)**  
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**Lecture – 37**  
**Ternary system with three phases**

So welcome again to everyone. This is lecture number 37 in the course Phase Equilibria. So, what we will do is that we were discussing in the last few lectures about ternary phase diagrams. We will just briefly recap some of the past work and before we move on to the new contents.

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Recap

- Ternary Phase Diagrams
  - Compositions
  - Tie Line
  - Application of lever rule
- Phase Rule for ternary systems
 
$$F + P = C + 1 = 4$$

<u>Invariant pt</u>	← F = 0	→ 4-phase equilibrium
<u>Line</u>	← F = 1	→ 3-''
<u>area</u>	← F = 2	→ 2-''
2-1)	← F = 3	→ 1-''

So, basically we introduced ternary phase diagrams and in those diagrams we looked at a few things like you know; how to determine the composition? What is the tie line? What is the line of fixed composition? What is the horizontal line parallel to any particular sides of the triangle?

So, basically to turn it that determine it is represented by these equilateral triangles and where 3 sides represent the pure components A, B and C. And you can determine the lower composition by reading from the phase diagram. You can make alloys with fixed composition of say one element, in this case C you can have alloy of composition with fixed A by B ratio or B by C ratio or C by A ratio.

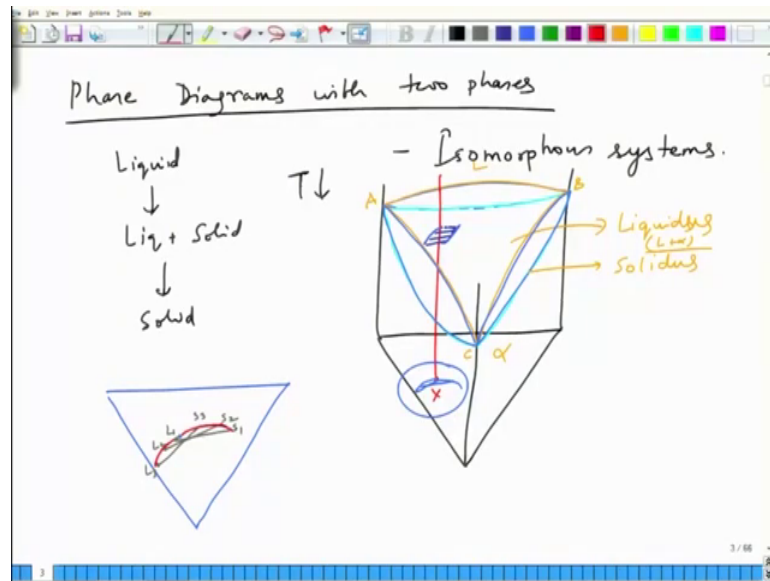
And then we also introduced the concept of so first thing was compositional, composition, then we looked at the tie line, we looked at application of lever rule and so on and so forth. And then we introduced the application of phase rule. And phase rules for ternary systems are slightly different, because you have one extra component.

So, if you have this  $F + P = C + 1$  then since you have  $C$  as 3. So, this is 4 and depending upon the number of phases that are present you can have degree of freedoms which are ranging from 0 to 3. And so, for example, you can have a 4 phase equilibrium.

If you have 4 phase equilibrium then degree of freedom will be 0, for 4 phase equilibrium and if you have degree of freedom as number of phases as 3 then it would be 1. So, this would be 3 phase equilibrium and degree of freedom will be two if you have two phase equilibrium and it would be 3 if you have one phase equilibrium. So, it would be either liquid or solid state this would be liquid plus solid, solid plus solid this could be liquid plus solid plus solid or solid plus solid plus solid and this could be liquid plus 3 solids.

So, this is how so this is basically the invariant point in a 3D, 3 dimensional phase diagram of a ternary system. Whereas, in this case this  $F$  is equal to 1 will give you a line variation, this would give you an area variation, and this would give you a 3 dimensional variation of degree of freedom. So, number of independent variables is different in case of different phase equilibria.

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And then we also introduced, so we first initiated with phase diagrams with two phases. So, those phase diagrams basically have liquid, going to liquid plus solid and then you have solid as you go down in temperature ok. So, I mean examples of such systems could be you know isomorphous systems. Isomorphous systems where A B B C and C A all 3 of them make isomorphous phase diagrams and eventually what you obtain in the case of a ternary system is something like this. So, you have.

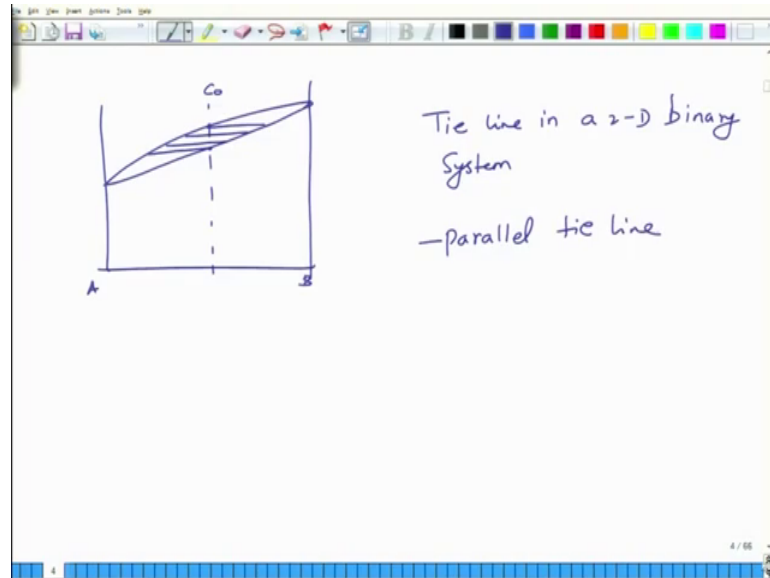
So, let us say you have phase diagrams like these. And so, the top surface consisting of these liquidus will be the liquid surface liquid a surface which is a curved surface let us say A B C ok. And then you have this bottom surface which is the surface which is depicted by this turquoise color this is the solidus.

So, this is the liquidus, and this is solidus and the difference and in between two you will have a single phase region. So, between these two surfaces these are curved surfaces you will have two phase region let us say you start with liquid you end up with liquid plus alpha, and between the two you will have liquid plus alpha region in between these two surfaces.

So, when you cool an alloy for example, of a composition X you start with a alloy of composition x. This X starts from liquid state then it slowly starts converting into solid. And as more and more solid forms just like in a binary solid solution the solid and liquid

compositions they change along the tie line. And finally, the solidification is over into a single phase.

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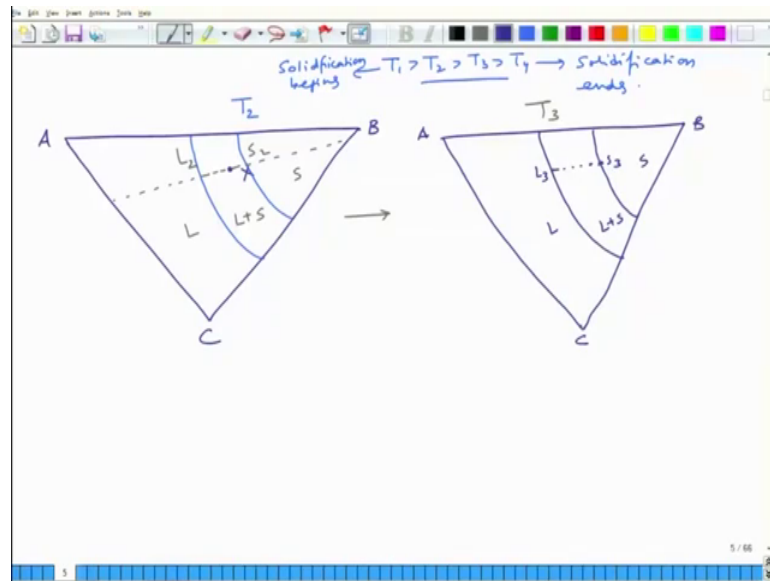
However, unlike a binary system, so in a binary system what happens is that if you recall in a binary system as you, so I am just going to go across. So, this is a binary system let us say A B. So, as we decrease the temperature for a given alloy the tie lines vary like this as you reach so they start to end up the solidification.

So, this is composition C naught. So, tie lines in a binary system they are flat and they are parallel to each other, whereas, in a ternary phase diagram the tie lines need not be parallel to each other. So, because it is a because solidification liquids are curved surfaces and the profiles may not be exactly parallel to each other.

As a result what will happen is that you will have liquidus going something like this and then correspondingly you will have some solidus that will go in another fashion. And the tie lines between these two they may not exactly be parallel to each other and they may change directions as you go from section to section.

So, if you if you draw 3 dimensional sections of so, this is tie line in a 2 D binary system ok. And they are parallel tie lines whereas, when you come to ternary system.

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So, if I draw for instance again so let us say this is the. So, this is A B and C let us say. So, this is let us say the alloy composition  $x$ . So in the first instance what happens is that that your liquidus and solidus are something like this alright. So, this is let us say at a temperature so we went through last time temperature  $T_1$ ,  $T_2$ ,  $T_3$ ,  $T_4$  it is added somewhere. So, you have temperature  $T_1 > T_2 > T_3 > T_4$  it is added four.

So,  $T_1$  is the temperature at which solidification begins, and  $T_4$  is solidification ends ok. So,  $T_2$  and  $T_3$  are the intermediate temperatures. So, if  $T_2$  and  $T_3$  are intermediate temperatures then the you can draw the tie lines the tie lines would be for this composition let us say this is the tie line ok, and this falls on this line ok. So, this is your liquid  $L_2$ . And this is your  $S_2$  composition corresponding to this is solid state this is liquid state and this is in between liquid and solid state.

Now, as you go to  $T_3$  another temperature as you go down in temperature the liquidus and solidus moves. So, more and more liquid the solidus comes closer to  $x$ . So, this is your  $X$  let us say and the solidus is somewhere here liquidus is somewhere here. And so, your liquid region is shrinking a little bit and your tie line may not be the exactly what you see the tie line could be this tie line ok. So, this is  $L_3$ ,  $S_3$ .

So, these tie lines may not be exactly parallel to each other rather and the paths may be different, but they do not within at the same at the same temperature tie lines do not

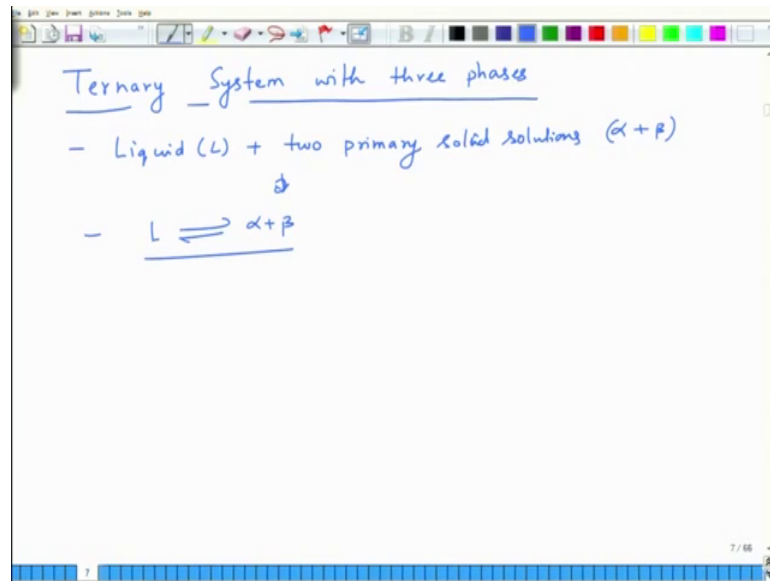
crisscross each other. So, tie lines are there could be a certain angle to each other, but they do not go crossing each other. So, this is how tie line orientation varies in a ternary phase diagram as compared to and the ends of tie lines are determined by free energy considerations. What is the equilibrium composition of solid and what is the equilibrium composition of liquid? So, you will have to do that one has to do that exercise. So, you can do that manually, but nowadays software's are used to calculate these free energy composition diagrams to give you the composition of phases in equilibrium. So, this is how the solidification will proceed.

So, if you go back to previous diagram in this your if you if you are plotting the composition for example, if you are plotting the cooling of liquid. So, liquid goes this way and the solid corresponding solid will be at this point and if you plot it them on to two D plane the liquidus liquid will show a tragic a path like this the solid composition show path like this. So, you will have these are tie lines, this is first tie line, this is the second tie line. So, if I just zoom it up let us say I zoom it up a little bit this particular part. So, this is the composition X and within this composition you have this projection of liquid compositions and projection of solid compositions.

So, the first liquid that formed here is somewhere here. So, this corresponds to so let me just make a little bit of modification its does not go like this, it goes more like this. So, the first solid and that forms is somewhere here and this is corresponding to this tie line the second one will form somewhere here and this is corresponding to this one.

So, this is L 1, S 1 this is L 2, S 2 and this would be let us say the L 3, S 3 and so on and so forth; that is how the liquid and solid curves will change. So, this is solid composition curve there is a liquid composition curve projected on the 2 D plane within the ternary phase diagram. So, this is how you can monitor the change in composition upon solidification of an alloy within the within a two phase within a ternary system consisting of two phases.

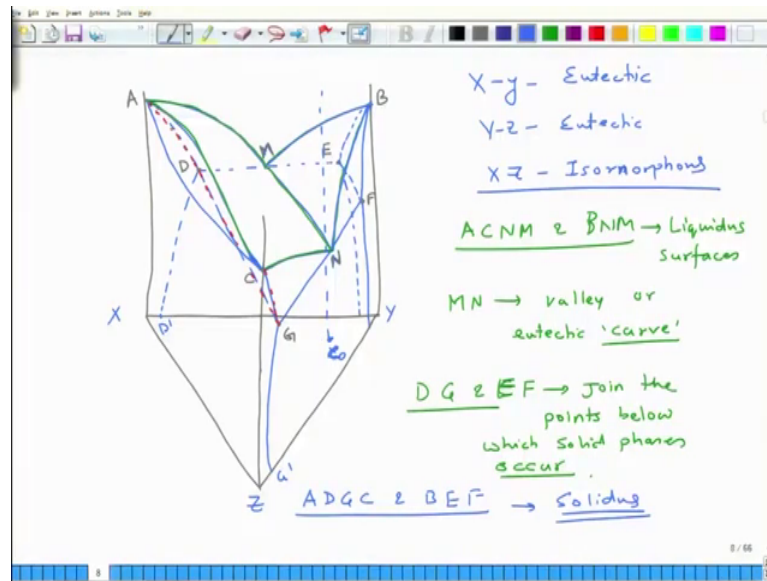
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Now, let us look at another ternary system with 3 phases. So, an example of such a system could be something like what I am showing you here. So, I am not going to go through all of them, but what we are going to do is that we have basically what it means is the liquid L plus 2 primary solid solutions.

So, this could be you know alpha plus beta alpha and beta and it gives basically it gives rise to and you have a leak eutectic reaction liquid giving rise to alpha plus beta. You can have peritectic also, but we are going to discuss the eutectic in this case for the sake of simplicity.

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So, the phase diagram looks something like this ok. This is the phase diagram, and this right. And let us now build a phase diagram on the on this side we have phase diagram which is like this let us say, alright. On the other side, we have another phase diagram which is of this coin let us say right. And then we have phase diagram on this side which is something like this.

So, if I take components sorry just. So, this if I take component  $X$   $Y$  and  $Z$ , then between  $X$   $Y$  I have a eutectic reaction, and between  $Y$   $Z$  I have a eutectic and between  $X$   $Z$  I have isomorphous; isomorphous system. So, we have various surfaces here let us say I identify them as  $A$   $B$  and let us say what is  $C$ , this is  $C$  and then we say this is point  $D$ , we said this has point  $E$ , and this is point  $F$ , and this is point  $G$  right and we have somewhere point.

So, we can draw few lines here. So, this could be for example, point  $M$  this could be  $N$  ok. So,  $M$   $N$  and we can draw some internal boundaries as well. So, internal boundaries could be for example, drawing this point to this point. Similarly we can draw this point to this point ok. And we take another  $Y$  composition for instance some plane  $X$  this is the  $C$  naught let us say the composition.

So, basically what we see is that these curves that we have for example, in this case  $M$   $N$ . So, this you have various things here. So, what is the liquidus surface here? The liquidus surface would be  $A$   $M$   $B$   $N$  and what is the  $C$ ? So, this is the so if you look at the top part



which is the liquidus surface. Let me just draw the liquidus using a different color in this case.

So, this would be these are the bounds of liquidus alright. So, these are the bounds of liquidus and in between you have this. So, you have one region which is A A C N M, another region which you have is B and M; so a C N M and B N M. These are the liquidus surfaces two segments and they are joining each other at N M. Now in this case we have shown N M to be going down, but it is possible that an NM achieve a maximum as well.

Now, in between somewhere you have these other surfaces as well. So, you have so this is this M N. Let us say is in this case is the sort of valley or eutectic in this case it would be curve for a ternary system.

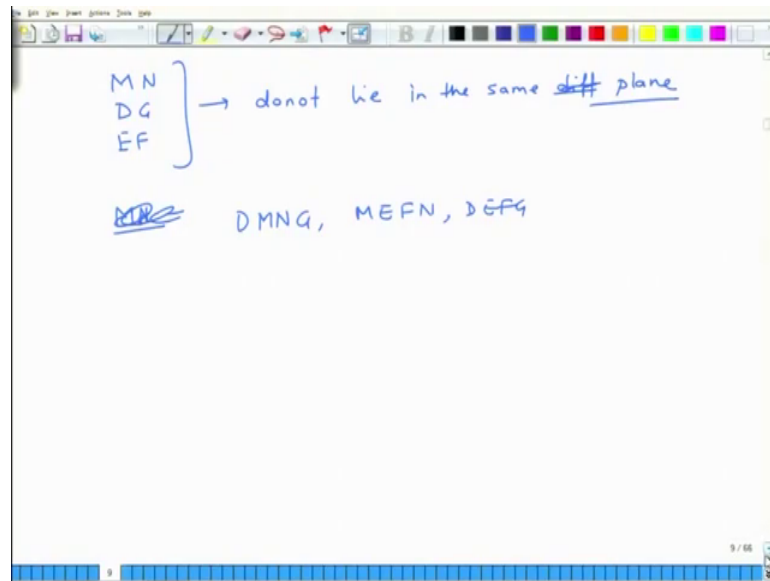
And then we have other curves like D G and E F and. So, D G and E F they join the points below which solid phases occur ok. So, these are the points of so this basically they are the compositions of alpha and beta phases that you might have in the eutectic reaction D G and E F. So, this for example, would be let us say alpha this would be alpha, this would be beta. So, they join the compositions and then surfaces another surface that you have is A D.

So, you have another surface which is A. So, you have A D G and then you have C ok. So, you have surface A D G C and another surface you have B E F. On the other side B E F and these are the surfaces which they form the solidus. So, they basically separate the two phase liquid which they could plus solid region with from the solid region.

So, basically these are the various surfaces and boundaries which separate liquid plus solid solid plus liquid liquid plus solid from the liquid liquid plus solid from the solid regions. And in between you have these solvers. So for example, this would be the solvers boundary, this would be the solvers G.

So, if you plot for example, here D prime and G prime. So, D prime, G prime G would be the solvers it separates a solid from each other. So, that is how we describe a ternary phase diagram consisting of these two eutectic and regions.

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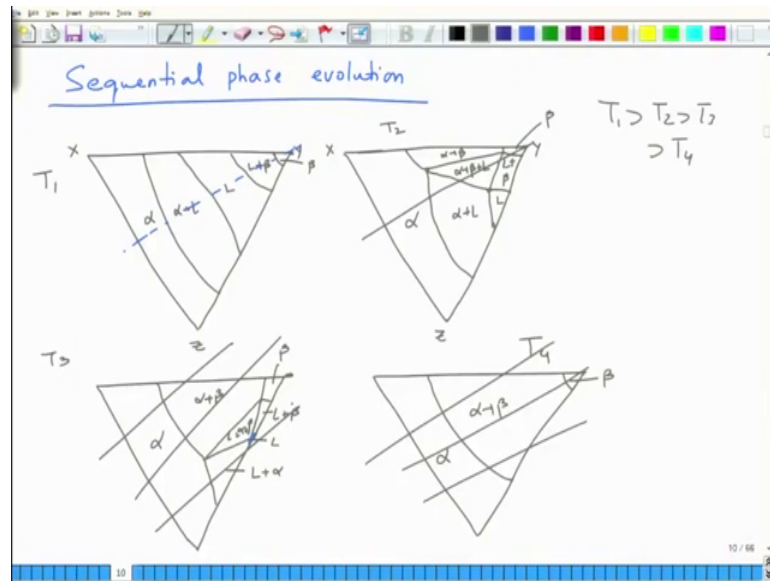
Now, we see that the curves for example, MN DG as well as EF. So, you can see MN this is MN BG and EF they do not necessarily lie in the same plane. They will probe most likelihood they will not lie in the same plane if the eutectic temperatures are different they are in and the curve MN in this case.

So, the curve MN this lies above the surface this D E F G sorry D E F g. So, this D E F G is the surface which separates a solid region from the two phase region. So, MN lies above this alright. And then you have so basically there are 3 curves you know D M N G, M E F N, and D E F G.

So, you have D M N G and then you have M E F N and then we have D E F G and these are the 3 surfaces which basically enclose the 3 phase space consisting of liquid plus two solids. And that is how you be essentially draw the compositional landscape of a composition temperature landscape of a ternary phase diagram.

Now, how does it look like for you? If you take a alloy of a given composition how does it for example, look like at different temperatures. So, when you draw for example, you take an alloy of a composition X C naught in this case and as it goes through the solidification, as it goes to a solidification at various temperature how do the sections vary?

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So, sequential variation phase evolution; so variation in types of phases as you go on decreasing the temperature so you may have various scenarios. So, for example, in this case one scenario could be you know used. So, it is not going to be very exact as compared to the as according to the phase diagram that is drawn, but this is just a schematic ok.

So, for a given composition let us say at a temperature  $T_1$  you are you have a section like this. So, of course, we are not going to go for composition which is going to go for that sections; so let us say we have something like this; so, this is at a temperature  $T_1$ , this is let us say X, Y and Z.

So, you have alpha alpha plus liquid you have liquid plus beta and then you have beta. So, basically you are at a temperature where you are in partial partially solidified state. So, that you have on the boundaries you have alpha plus beta regions. So, if you take a for example, at section along this if you take section along this starting from pure Y to let us say some ratio of X and Y you will have a binary phase diagram just like what you see.

So, it will be a binary phase diagram between pure Y and some ratio of X's to Z. As you decrease the temperature further to  $T_2$ ; let us say the phase boundaries will move because the phase fields the width of the phase fields changes ok.

So, if you go to  $T_2$  it might change you know something like what I am showing you here. Let us say you have a situation like this. So, this is again X Y Z let us say this is phase field beta, this is phase field liquid plus beta. So, this is alpha essentially alpha field phase field is broadened this is liquid plus alpha this is liquid this is alpha plus beta and this is alpha plus beta plus liquid. So, these are different phase fields you can see that this is a point which is 3 phase point ok.

So, if you fixed, so you have only one degree of freedom when you have only one degree of freedom you can only change the composition or the temperature. So, if you have fixed the temperature and you can vary the composition along the line; so, this is for instance in this case the point at which you have three phase equilibrium that is existing.

So, as you decrease the temperature further you might have you will have broadening of these phase fields solid solid phase fields. For example, at a temperature  $T_3$  you might have a situation where alpha has broadened further and liquid has really shrunk. So, this is for instance beta, this is alpha plus beta, this is alpha some somewhere in between you have tiny phase field here as liquid. Then you have liquid plus alpha you have liquid plus alpha plus beta and then you have liquid plus beta.

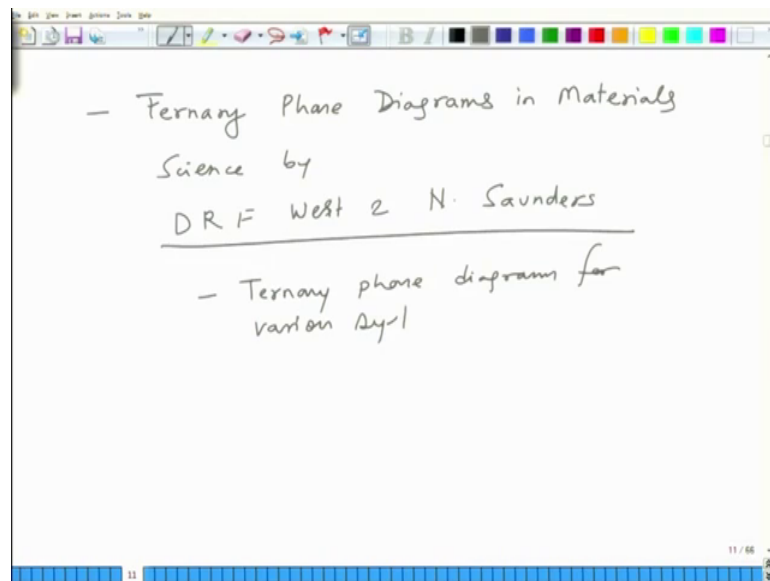
So, this very tiny phase field here is liquid and when you come to very low temperature below the solidus. So, here you are below the solidus you are now slowly approaching the solidus surfaces. So, your liquid phase field is shrinking and solid phase field is expanding whether its alpha beta it depends upon what kind of phase diagram you have.

So, eventually what you have is you have alpha plus beta and then beta. So, this alpha phase you can see that sequentially you started from liquid with surrounded by liquid plus solid surrounded by terminal solid solutions. As you decrease the temperature further to  $T_2$ , so,  $T_1$  is greater than  $T_2$  is greater than  $T_3$  is greater than  $T_4$ . So, this is  $T_4$  ok. So, you can see that alpha plus beta, two phase field has started forming the liquid shrunk and there is an increase in the areas of two and three phase fields regions with expansion of some expansion in the single phase field regions. And as you went down in temperature further your two phase fields and solid phase fields have increased in size and liquid phase field a shrunk in size and as you come down to  $T_4$  the solid phase field has expanded.

So, this is how the micro structural evolution takes place in a ternary phase diagram. You can draw these tie line you can draw these lines like sections you can take isopleths and evaluate the phase revolution sideways looking at in a binary system.

So, this is how the phase phases will evolve in a ternary system. So, we will not get too deep into ternary systems. We will close this discussion on ternary systems here and I would suggest that for further reading on ternary phase diagrams.

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As I suggested you should do you should read the book on this ternary phase diagrams in material science by D R F West and N Saunders. This is a very good book it has it has a lot of ternary phase diagrams for different alloy systems for various systems and it gives a very good flavor it also has some exercises you can do and work out.

So, we will close here this lecture. What we have looked at is the ternary phase diagram consisting of 3 phases of course you can go for the 4 phases it gets more complicated. We will probably touch upon that in the next lecture, and then we will move on to the next topic.

Thank you very much.