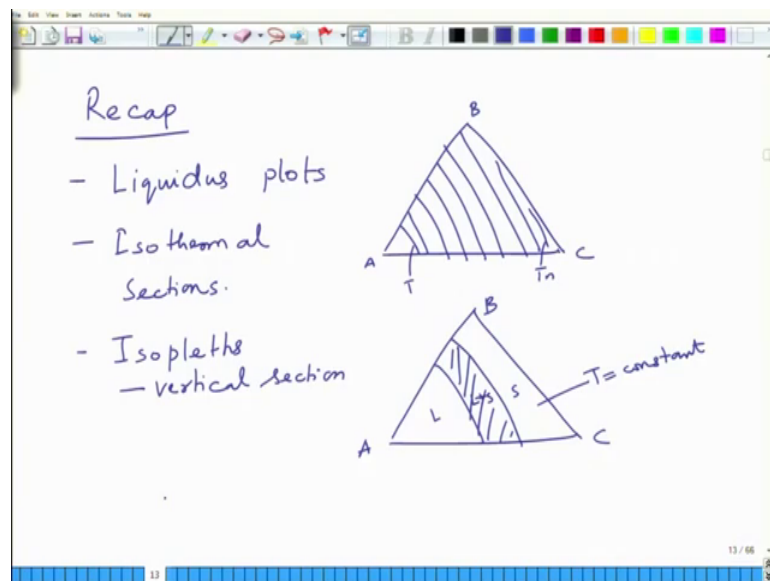


**Phase Equilibria in Materials (Nature and Properties of Materials - II)**  
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**Lecture – 36**  
**Ternary system with two phases**

So again we began a new lecture, this is lecture 36. So, Phase Equilibria course.

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So, let me just give you a brief recap of previous lecture. So, in the previous lecture we looked at for instance drawing the vertical sections. So, we look at the better ways to define that rephase diagram. So, we look looked at what is the liquidus plot; liquidus plots are basically the isothermal contours of the liquidus surface found in the liquidus surface which are projected in tori.

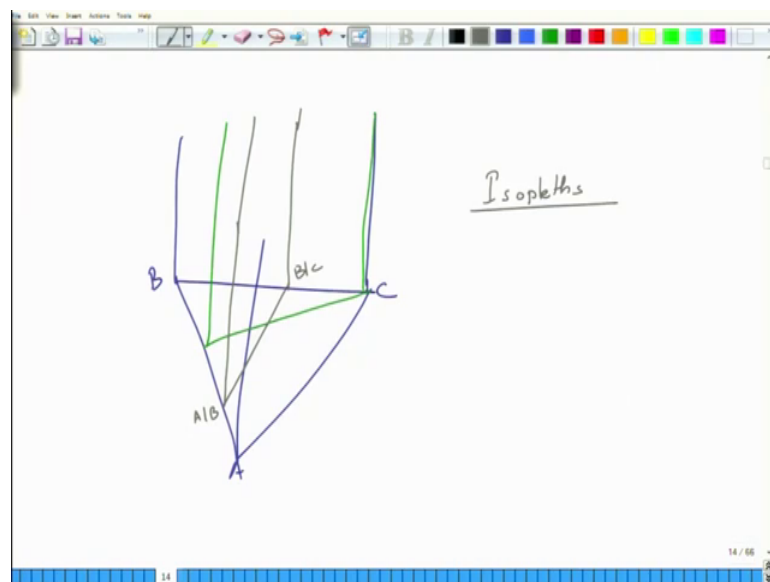
So, they are plots like these so, for example for isomorphous system you has situations like these. So, these are various temperature profile  $T_1$  to let us say  $T_n$  as you go from A B C and they basically determine the liquidus surface of the ternary phase diagram. Then similarly you can have solidus plots as well, then we looked at isothermal sections.

So, isothermal sections are nothing, but sections drawn at constant temperature unlike the previous one so, you can have for instance the phase fields. So, if you go to binary if you if you see ternary isomorphous system at high temperatures you might have liquid.

So, whereas somewhere in between you will have liquid, liquid per solid and solid so, something like that. So, this is at a constant temperature ok. So, this is liquid plus solid region.

And then we also looked at isothermal, isopleths and these isopleths are nothing, but vertical sections. So, basically they are they are drawn vertically parallel to temperature axis and intersecting 2 parts of the phase diagrams to an horizontal plane. So, essentially they can go from so, if they go from A to B then it is nothing, but a binary phase diagram for A to B. But, what you want to do here is for example, you want to go from pure A to let us say so, let me just draw isopleths.

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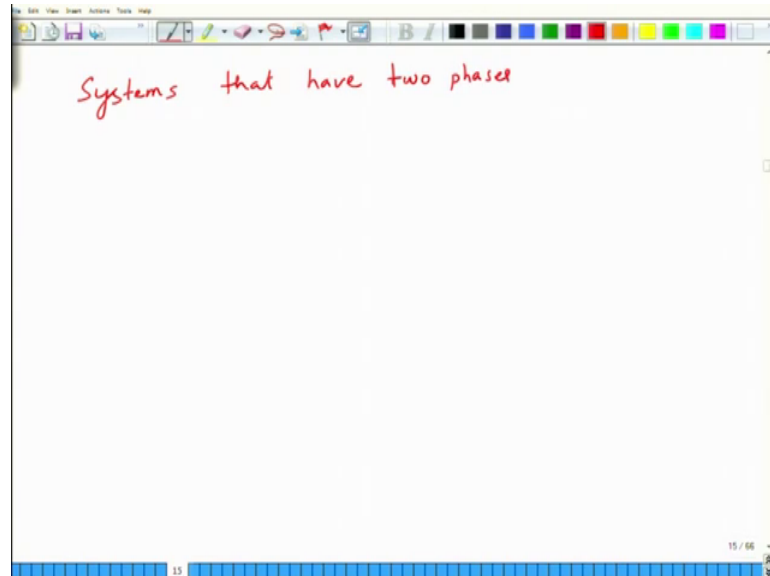


So, essentially these A B C A C A B and B C are nothing, but isopleths but there vertical they are binary phase diagrams. Here you want to go for example you want to draw this section vertical section ok. So, this is isopleth you may want to draw another one which is let us say like this. So, these are the isopleths which give you evolution of phases as a function of temperature for given ratio.

So, you can see that this has certain A by B ratio; this has certain B by C ratio. So, you are moving from certain A by to certain B by C. And in between for example, you will encounter certain specific compositions and these basically give you evolution of phases at the function of temperature along this line. So, along this line of composition you see the phase evolution as a function of temperature. So, these are nothing, but isopleths

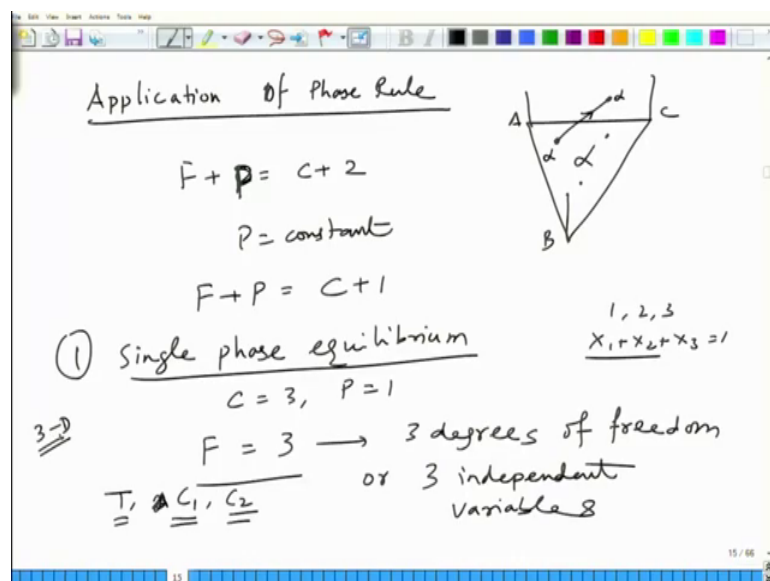
alright. So, in this lecture what we will do is that in this lecture we will look at some ternary phase diagrams of specific types.

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So, for example, we begin with systems that have two phases. By the way another thing which I will come back to this little later, another thing that I skipped is application of phase rule.

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So, we know that F plus P is equal to C plus 2. And, when pressure is constant then this becomes F plus P is equal to C plus 1 where, F is the number of degrees of freedom. P is

the number of phases and  $C$  is number of components. So, you can have various situations in a ternary phase diagram. So, for example, you can have situations where you have single phase equilibrium that is very rare, but you can see intermetallic systems it may occur. So, if you have situation like this then  $C$  is equal to 3 and  $P$  is equal to 1 as a result  $F$  is equal to 3.

So, you have 3 degrees of freedom here basically, there are 3 independent 3 degrees of freedom or 3 independent variables. So, what variables could it be? So, you have the variables are  $T$  and  $A$ . So, you can say  $C_1$  and  $C_2$  because, if you have 3 components 1 2 3 you know that  $X_1$  plus  $X_2$  plus  $X_3$  is equal to 1. So, if you define 2 the third one is automatically known. So, the 3 variables that you have available are composition of one component, composition of second component and the temperature. These are 3 variables that you have which you can vary independently in a single phase field.

So, if you have for example, a situation like this let us say this is isothermal section of A B C in alpha phase region. So, you can vary so, you can whether you are at this point whether you are at that point or this point does not matter you are always in single phase regime. So, this is like changing the composition. So, if you go to two different temperature as well there also if it is alpha phase field and you can vary temperature as well as composition simultaneously and you will still have single phase field.

So, it is like going from one point to another point in 3 D and still going from alpha to alpha alright. So, this is single phase equilibrium in a ternary region. You can have two phase equilibrium, if you compare this to binary by the way in case of binary you will have for a single phase equilibrium, you will have 2 variables. That is temperature and composition and that composition is basically percentage of one component and second one is automatically known.

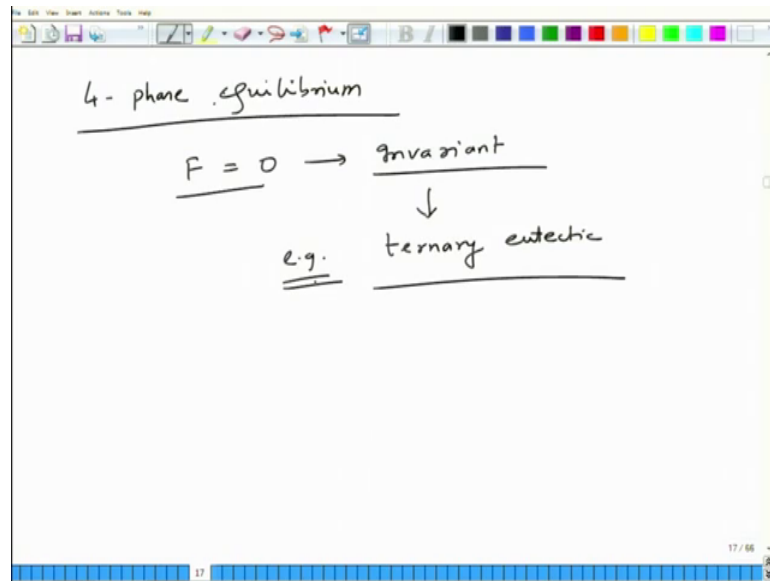
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The image shows a whiteboard with handwritten notes. At the top, it says "2-phase equilibrium" with a horizontal line underneath. Below this, the equation  $F = c + 1 - p$  is written, followed by  $= 3 + 1 - 2$  and then  $= \underline{\underline{2}}$ . To the right of the number 2, there are two arrows pointing to "T, composition of one component" and "T = fixed,  $x_1, x_2$ ". Above the second arrow, the text "2-D" is written and underlined. Below the 2-phase section, it says "3-phase equilibrium" with a horizontal line underneath. Below this, the equation  $F = c + 1 - p$  is written, followed by  $= 3 + 1 - 3 = \underline{\underline{1}}$ . To the right of the number 1, there is an arrow pointing to "one independent variable" and "T or  $x_1$ ". To the left of the number 1, the text "1-D" is written and underlined. At the bottom right of the whiteboard, there is a small number "16/06".

Now, if you compare if you now go to 2 phase equilibrium which is possible for example, in ternary eutectics even in case of isomorphous where you have liquid plus solid regime. So, you have  $F$  is equal to  $C$  plus  $C$  plus 1 minus  $P$ . So,  $C$  is 3 plus 1 minus 2 so, you have 2 degrees of freedom. So, what are the 2 variables you have? 2 variables that you may have is one is temperature and second is composition. Composition of one component, but a better one would be you fix the temperature of one component ok; temperature is fixed and you vary the composition  $X_1$  and  $X_2$ . So, you vary two compositions  $X_1 X_2$ , but you remain in an isothermal plane.

So, this is two phase equilibrium; similarly in case of 3 phase equilibrium  $F$  is equal to  $C$  plus 1 minus  $P$  and this will be 3 plus 1 minus 3 1. So, you have only one variable one independent variable and that could be either temperature or  $X_1$  that is it, mostly it is temperature that we talked about.

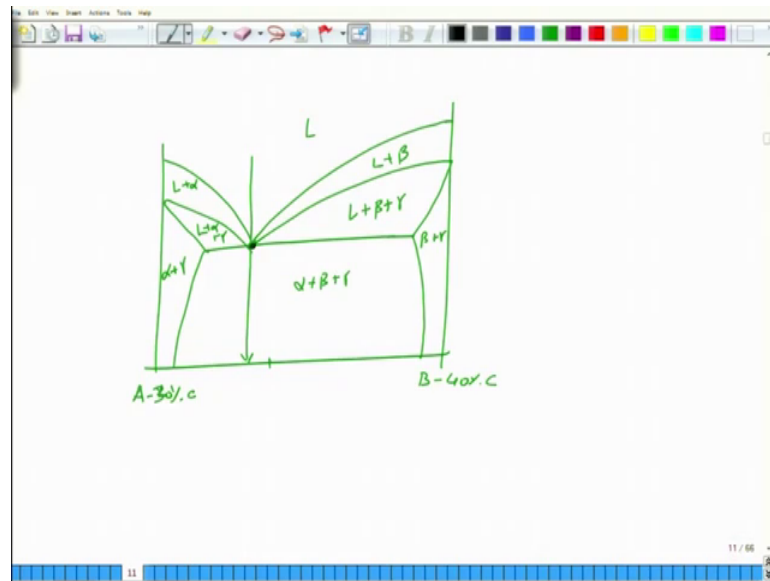
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And then you have 4 phase equilibrium which is not present in binary systems. So, if you recall that in case of binary system, if you have two phase equilibrium then your degree of freedom is 1. So, if you move temperature compositions automatically fixed or if you change composition temperature, is automatically fixed. Similarly in 3 phase equilibrium whereas, in binary system it is invariant which means you have no degree of freedom which is like a point in a binary phase diagram. In this case it is a line you can see that it is there is 1 degree of freedom

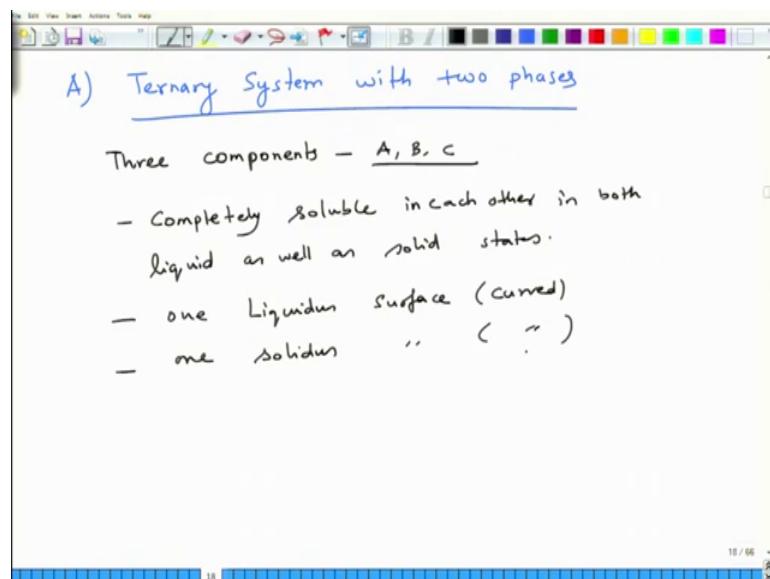
So, in this case this is you can say a 3 dimensional change, this is a 2 dimensional change you can move along the surface. This is the 1 dimensional change and in the 4 phase equilibrium you have F being equal to 0 which means it is invariant and this could be situation like you know ternary eutectic. So, we saw for example, somewhere here you have ternary eutectic in isopleth.

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So, this is the isopleth you have. So, at this point liquid and alpha plus beta plus gamma they coexist. So, this is a 4 phase equilibrium which is not found in binary systems, but in ternary system. So, as a result you have we can say example is ternary eutectic. So, these are the 4 possible scenarios that you will encounter in case of ternary systems as far as the application of phase rule is concerned alright.

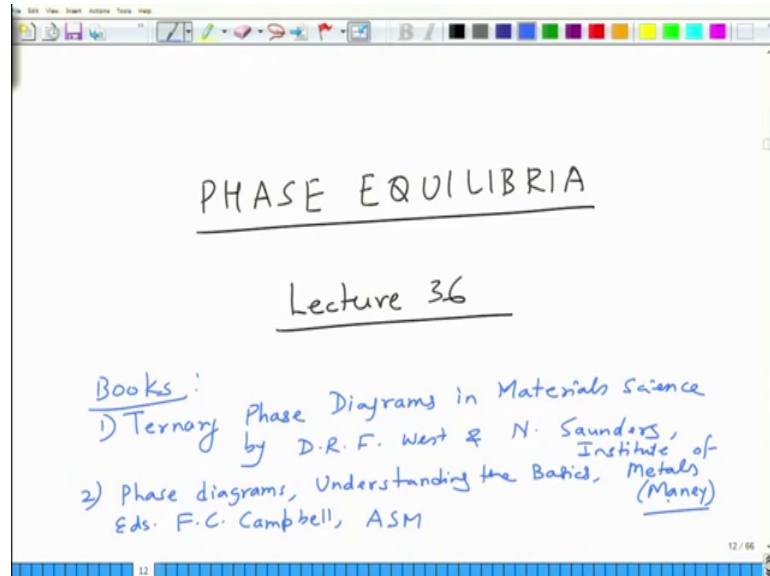
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So, now let us get moving to some ternary example, systems examples and we will first start with ternary system with two phases; by the way one thing which I forgot to

mention there are 2 books you can refer to. So, let me just write it in the beginning of the lecture.

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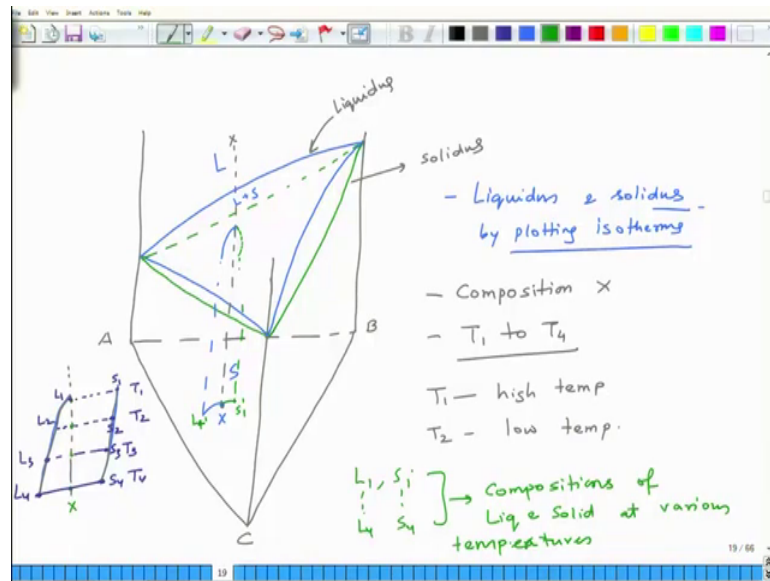


So, the books for ternary phase diagram understanding could be: the first book is Ternary Phase Diagrams in Material Science by D. R. F. West and N. Saunders. It is a very nice book, comprehensive book. And, the second book that you may consider which has only a chapter on phase this thing ternary phase diagram which is Phase diagrams, Understanding the Basics. This is by edited by F. C. Campbell and this is from ASM international. And, this is from Institute of Metals UK, Institute of Metals I think it is UK and it is basically you can say Maney m a n e y by Maney publications.

So, these are the 2 books that you can refer to while reading about ternary phase diagrams, what I am teaching in these lectures is mainly adapted from those 2 books. So, let us begin with the ternary system that contains two phases and this two phases basically. So, you have 3 components A B and C, the condition is they are completely soluble in each other in both liquid as well as solid states. So, you will have one liquidus surface basically curved surface and you will have one solidus surface which is again curved. So, let us see how do we draw this, it is a little complicated. So, bear with me if I do not do a very good job at drawing it ok.



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So, we start from let us say A B and then we draw the vertical lines here, these are the vertical lines right and let us say we have a situation like this. So, this is the liquidus let us say this is a liquidus surface and the solidus surface would be let us say this, this is the solidus and this be for the backside. So, this is A B and C so, this is how the phase diagram will look like. So, you have liquidus, this is solidus. So, phase fields will be you have liquid here, in between these two solidus and liquidus you will have liquid plus solid. And, then somewhere in between you will have only somewhere below the surface solidus you will have only solid.

Now, let us say in this case when you determine the liquidus and solidus. So, if you determine liquidus and solidus. So, why how you do is that you basically by plotting isotherms as I showed you in the last class. So, if you draw these isotherms they allow you to determine for a given composition what is the start of solidification, what is the end of solidification? So, in some cases what may happen is that, this liquidus and solidus may also so, touch each other at certain point. And, this could be a maximum point for example, you take 2 surfaces which may touch like this or they it could it could make a valley where there is a there is a dip somewhere in between. So, there could be a maximum and minimum.

Now, the process of solidification let us say you take a composition X. So, you take a composition let us say X alright and let us say so, you take a composition X which cools

from temperature  $T_1$  to  $T_4$ . So,  $T_1$  is high temperature and  $T_2$  is low temperature. So, the way it goes is that at temperature  $T_1$  so, if you draw on the side let us say; so, this is what you have. So, on the side of it at  $T_1$  you have a composition liquid here, solid at somewhere let us say this is solid. So, this is  $L_1 S_1$  at temperature  $T_1$ , then as you keep solidifying it the liquid will slowly convert into solid. So, this is  $L_2 S_2$ , then it will again go if you go to lower temperatures so this is  $T_2$ , this is  $T_3$  and this is  $T_4$  alright.

So, you have  $L_3 L_4 S_3 S_4$  and these 4 lines can be joined together like this. Similarly, these 4 lines can be joined together in this fashion. So, when you look at this in the 3 D you start for example, for a point this and the liquidus let us say move in this fashion on this surface. Correspondingly, you will have some movement of a similar movement on the solidus which can be able to project like this and this is nothing, but what I am projecting on the left hand side here. So, this is the liquidus surface so, this one is the liquidus  $w_1$  let us say which moves its composition. And, this is the change in the solid composition as a function of just like a binary phase diagram you are looking at in ternary, but you cannot visualize this in 3 D very easily.

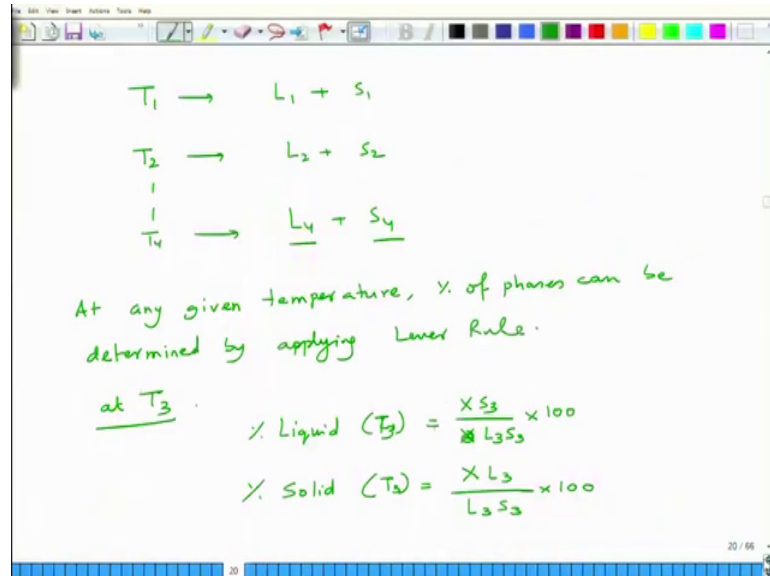
So, you need to draw a 2 D projection. So, this is how the liquidus composition will move and this is how the solidus composition will move. As we know that you know when you form solid, solid will liquid will reject solid as a result there will be change in the composition. And, as you keep on forming more and more solid there will be subsequent changes in the liquid composition as well as solid composition.

And, as far as it is equilibrium cooling you allow slow conditions to prevail as a result liquid and solid achieve equilibrium composition. So, if you take the projection of these lines on the 2 D plane you will see this is for example, X the liquid composition for instance may move in this fashion whereas, the solid composition may move in another different fashion.

So, this could be the solid so, this is X this is for example, movement of liquid. So, this is  $L_4$  prime and this would be  $S_1$  prime. So, these are the contours that have been projected on 2 D in the basal plane of this prism and these are the changes in the composition. So, these lines which are from  $L_1$  let us say this is X and these are compositions ok. So, these lines which connect  $L_1$  to  $S_1$  through X so, this is also X,

this is also X, this is also X. So, these points first of all L 1 and S 1 L 4 to S 4 they are compositions of liquid and solid at various temperature temperatures ok.

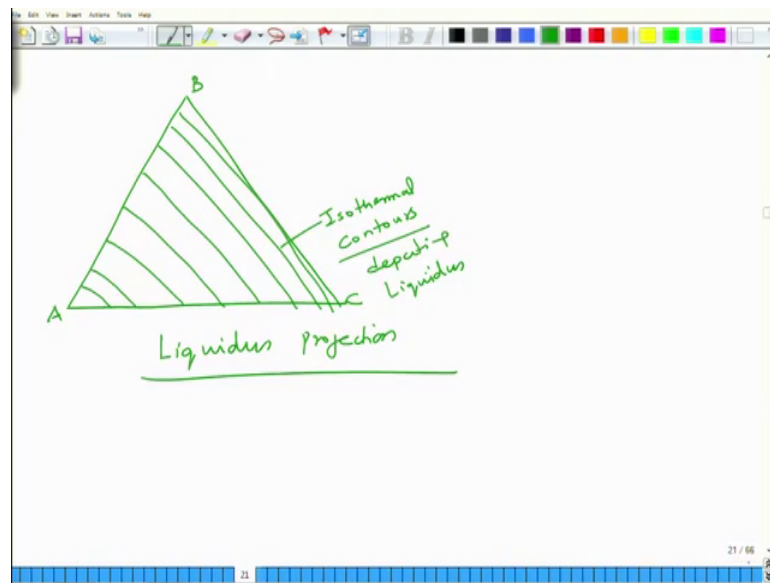
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So, at temperature T 1 you have liquid L 1 plus solid S 1 in equilibrium alloy is composition X, but liquid of composition L 1 and solid of composition S 1 are in equilibrium, at temperature T 2 L 2 plus S 2. So, liquid composition as a L 2 and solid composition S 2. So, as you come down T 4 you have L 4 plus S 4 situation. So, liquid of composition L 4 and solid of composition S 4. So, the way you determine their so, you apply now the lever rule. So, at any given temperature if you want to determine the percentage of phases can be determined by applying lever.

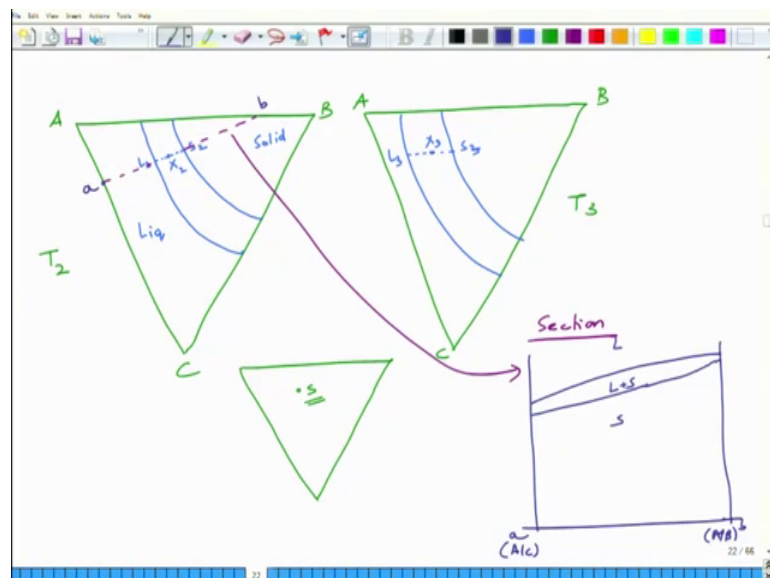
So, here for example, if you take a temperature let us say so, at T 3 if you go back to previous number this is of liquid composition is L 3, solid composition is 3 and alloy composition is X percentage of liquid at L 1 ok, at temperature T 3 would be X S 3 divided by X sorry divided by L 3 S 3 into 100. Percentage solid will be equal to at T 3 will be equal to X L 3 divided by L 3 S 3. So, the key thing is that these tie lines have to pass through X. However, what you notice that because of the 3 diagram and these are curved surfaces, these tie lines are not necessarily parallel to each other. They are they may be in different directions depending upon the waves modification prevail proceeds.

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So, for example, if I take before I do that let me just draw the liquidus plot. For example liquid as we have seen how will it look like, if you draw the liquidus for such a system from. So, this is let us say A B and C the liquidus may vary something like this, you will have these isothermal contours. So, these are isothermal contours depicting liquidus. So, this is basically a liquidus projection. Now, you go to let us say we take so, let us we take a few isothermal sections.

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So, let us say we take this first isothermal section which is something like ok. So, this is let us say at temperature  $T_2$  and then we take another one or I can maybe so, this is at  $T_3$  that is better. So, let us say this is A B and C; this is also A B and C. So, in case of  $T_2$  let us say the phase fields are the phase fields are something like this right. So, this is L this is liquid, this is solid and we are here at some point X here. And, in this case the tie line is something like this. This is let us say  $L_2$ , this is  $S_2$  when you move to  $T_3$  which is lower temperature, the solid regime little bit expands.

Liquid regime shifts to the left, the X point is somewhere here which is same as X will not change, but the direction of this line. So, X may remain same, but you can see that this is in this direction whereas; this could be in this direction depending upon the nature of the surface. But this X will always follow fall on the same always fall on the tie line. So, this will be let us say point  $X_2$ , this is  $X_3$  same as X basically my composition. So, essentially these tie lines may not necessarily remain exactly parallel to each other as you cool the alloy. So, eventually as you get down to lower temperatures so, as you go down to lower temperature if it was a cross section you will have not very good.

So, you will have alloy X which is completely solid. So, your this would be completely solid at this point your this thing might have moved somewhere here. But does not matter your alloy has completely well you will not have for this composition for isomorphous system you will not have this thing. So, you will have complete solidification by this point.

So, now what I wanted to say was basically that you can determine the composition of two phases in equilibrium by using the tie line rule, applying the lever rule. You can determine the phase fractions and you can visualize the unification using by drawing the isothermal sections and then looking at how the compositions are changing for the two phases.

So, if you want to for example, draw the isothermal section what you do is that let us say if you want to draw, let me choose this point if I draw the isothermal section so, not isothermal section, but the vertical section that is isopleth. So, if I draw the isopleth, isopleth will look something like this. So, you will have so, this is liquid, this is solid, this is liquid plus solid. So, this is from let us say a b, this is from a b. So, it is not pure a to pure b or pure b to pure c, it is from some composition a which has certain A by C

ratio and this is another composition  $b$  which is A by B ratio and you can see the phase boundaries are not closing. So, it is not complete phase diagrams. So, it is a section vertical section or isopleth that is taken wrong that in parallel temperature axis.

So, what we have done in this lecture is we have just looked at some graphical description of a ternary phase diagram consisting of two phases. So, A and B and C are completely miscible in both solid and liquid state. We have seen how to draw the how one can determine the phase compositions and phase fractions by drawing the tie line. And, you can visualize the solidification phase changes by drawing the isothermal sections and drawing isopleths. So, in other two other ternary systems and before we finish this discussion.

Thank you.