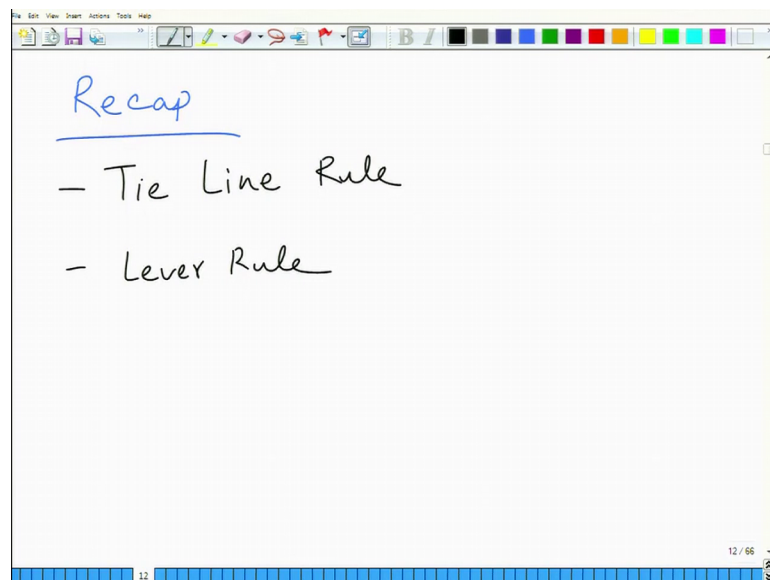


Phase Equilibria in Materials (Nature and Phase Properties of Materials - II)
Prof. Ashish Garg
Department of Materials and Metallurgical Engineering
Indian Institute of Technology, Kanpur

Lecture – 20
Phase Fraction Calculation in a Phase Diagram

So, welcome again we now begin lecture 20. So, we are midway of the course now. So, in this lecture we will again delve on the composition weight Fraction Calculation etcetera. So, first let me just recap the last lecture.

(Refer Slide Time: 00:31)



So, in the last lecture we talked about, the Tie Line Rule which allows us to determine the compositions of two phases in equilibrium in the two phase region in the binary phase diagram. So, what you do is that you basically draw. So, if you know the overall all composition and if you know the temperature and at a fixed temperature you draw a horizontal line. This horizontal line intersects the two phase boundaries on either side. So, if you are in two-phase region let us say liquid plus solid then intersection of this horizontal line with the liquidus will give you the composition of liquid phase.

And intersection of this line with the solidus will give you the composition of solid phase in equilibrium with that liquid. So, you will have two phases in equilibrium whose composition can be known. Then we looked at Lever Rule, lever rule is basically a rule to calculate the weight fraction of two phases in equilibrium with each other. So, once

you know the two phase composition of two phases you can also calculate the weight fraction of two phases in equilibrium with each other.

(Refer Slide Time: 01:45)

Lever Rule

$T = T_1 - \text{fixed}$

Overall Composition = C_0

- $\alpha + \beta$ region

$\alpha \rightarrow C_\alpha$

$\beta \rightarrow C_\beta$

wt fraction of $\alpha \rightarrow f_\alpha$

" " - $\beta \rightarrow f_\beta$

$\Rightarrow f_\alpha + f_\beta = 1$

So, in this lecture what we will do is that we will derive the lever rule, how does the lever rule come about. So, let us say we have a two phase equilibrium. So, this is C_0 naught this is let us say C_α and this is C_β two phases in equilibrium and this is the tie line basically ok. Now, at a fixed temperature of course T_1 , so, temperature let us say is T_1 that is fixed overall composition C_0 naught.

Now, it is a two phase equilibrium which means you have alpha plus beta region, alpha phase of composition C_α and beta phase of composition C_β . So, let us say for overall for this overall composition and the weight fraction of alpha is f_α and the weight fraction of beta is f_β such that $f_\alpha + f_\beta$ is equal to 1 because the mass is conserved.

(Refer Slide Time: 03:35)

Since ^{total} amount of elements is also conserved:

Amount of B in alloy
= amt of B in α + amt of B in β

$$C_0 = f_{\alpha} \cdot C_{\alpha} + f_{\beta} \cdot C_{\beta} \quad \text{--- (2)}$$

from (1) $\rightarrow f_{\alpha} = 1 - f_{\beta}$

$$C_0 = (1 - f_{\beta}) C_{\alpha} + f_{\beta} C_{\beta}$$

Now, amount of solute in any phase is also conserved. So, since amount of elements is also conserved which means let us say amount of. So, total amount of element is also conserved amount of be in a alloy let us say is equal to amount of b in alpha plus amount of b in beta. So, if amount of b in alloy is given as C_0 which is the overall composition which is nothing, but when you are going from a to b it is percentage amount of b. So, amount of being alloy is C_0 so amount of being alpha will be equal to fraction of alpha phase multiplied by the composition of alpha phase in percentage b let us say that is C_{α} .

And then we have fraction of beta phase into composition of beta phase that is in C_{β} . We know that f_{α} is equal to $1 - f_{\beta}$ ok. So, this is the second equation the first equation was this equation and the second equation is this equation. From the first equation you can write this as a result so you can write C_0 is equal to $1 - f_{\beta}$ into C_{α} plus f_{β} into C_{β} if you work it out. So, you have to rearrange it in such a fashion so that you can determine f_{β} . So, f_{β} can be found as from this expression f_{β} will be equal to so if you just have to rearrange this.

(Refer Slide Time: 05:43)

The image shows a whiteboard with handwritten equations in purple ink. The equations are:

$$f_{\beta} = \frac{C_0 - C_{\alpha}}{C_{\beta} - C_{\alpha}}$$
$$\% \text{ wt } \beta = \frac{C_0 - C_{\alpha}}{C_{\beta} - C_{\alpha}} \times 100$$
$$\therefore f_{\alpha} = \frac{C_{\beta} - C_0}{C_{\beta} - C_{\alpha}} = \underline{1 - f_{\beta}}$$

The whiteboard also shows a toolbar at the top with various drawing tools and a status bar at the bottom with the number 15.

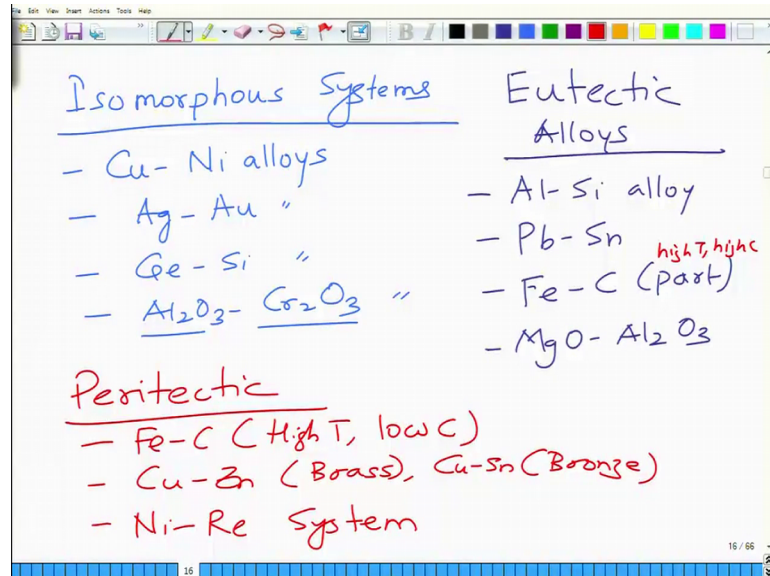
So, that you have a common factor of beta. So, it will become f beta into C alpha minus C beta which is and equal which is equal to C alpha minus C naught or C naught minus C alpha. So, f beta will turn out to be C naught minus C alpha divided by C beta minus C alpha. So, percentage of beta will be equal to C naught minus C B divided by C beta minus C alpha into 100. Similarly you can work out f alpha which is equal to C beta minus C naught divided by C beta minus C alpha which is equal to basically 1 minus f.

So, this is how you derive the lever rule which is essentially based on the principle of mass conservation. So, basically the mass of in a two phase mixture the weight fraction for a given composition is equal to this they add up to one. So, for 100 gram you will so for hundred so consider you have hundred grams. So, 100 gram would have certain amount of alpha and certain amount of beta. So, you can calculate the weight fraction. So, did they do it those weight fractions mud must add up to 1? Similarly, the amount of solute is also conserved. So, whatever the phase fraction may be the total amount of b that is present in both the phases must be equal to what is present in that alloy.

So, that is what that is how you determine the phase fraction. So, this is what these are the few things that that are important about the phase diagrams to quantify various things. So, you can determine composition of the alloy, if you know the composition of the alloy and temperature you can determine composition of phases, and you can

determine their phase fractions. Let us first look at the examples of common eutectic phase diagrams and common phase diagrams that are present in our system.

(Refer Slide Time: 07:55)

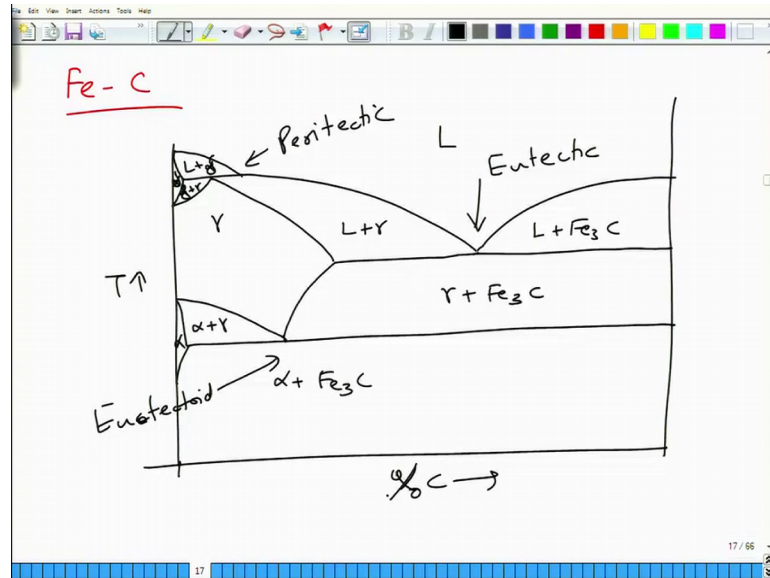


So, basically isomorphous systems you have copper nickel alloys, then you have silver, gold you have germanium, silicon and then you can have a pseudo binary system such as Al_2O_3 Cr_2O_3 where this is one component this is another component. So, all of these make nice isomorphous perfectly soluble systems. If you look at the eutectic systems and the eutectic your first very common alloy is the aluminum silicon alloy. Then you also have lead tin alloy which is used as a solder, part of iron carbon phase diagram shows this kind of eutectic reaction in the pseudo binary system you can have MgO Al_2O_3 system and so on and so forth.

So, these are eutectic alloys and then third one let us say peritectic systems. In case of peritectic the melting point difference is quite high. So, part of iron carbon system high temperature part. High temperature low carbon part in case of this is high temperature high carbon part. So, high temperature low carbon part shows peritectic reaction similarly copper zinc system that is brass copper tin system bronze. So, there is a substantial difference between the melting point of two elements that is why they show peritectic reactions. You can have nickel rhenium system and various other systems where you have large differences in the melting point they show peritectic. Often in many alloys the phase diagram is not only my eutectic or peritectic or isomorphous it is a

mixture of your reaction part of the phase diagram would be so you can have several reactions taking place in single phase diagram.

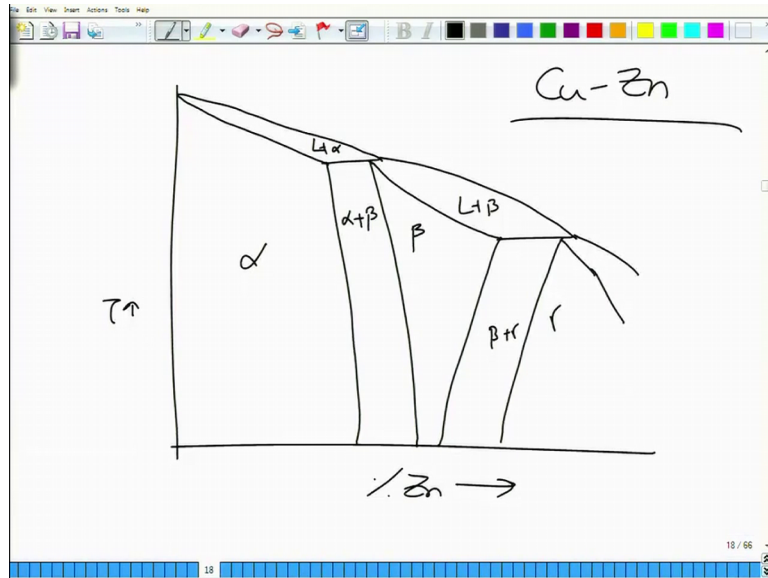
(Refer Slide Time: 10:27)



So, for example, you can have a phase diagram like this. So, your iron carbon phase diagram as I do earlier also is you have peritectic reaction on this side and then you have this eutectic reaction and then you have eutectoid reaction. So, this is what you will have an iron carbon diagram you will have liquid phase. So, you will have gamma alpha liquid plus Fe_3C , liquid plus gamma plus Fe_3C .

This will be liquid plus alpha, alpha plus gamma this is alpha plus so this is no sorry this is not alpha this delta and this is alpha this is alpha plus Fe_3C . So, you can see that this is eutectic, this is peritectic and this is eutectoid. So, this is iron carbon system you have others you have several other systems which are very complicated for instance you can have copper zinc system.

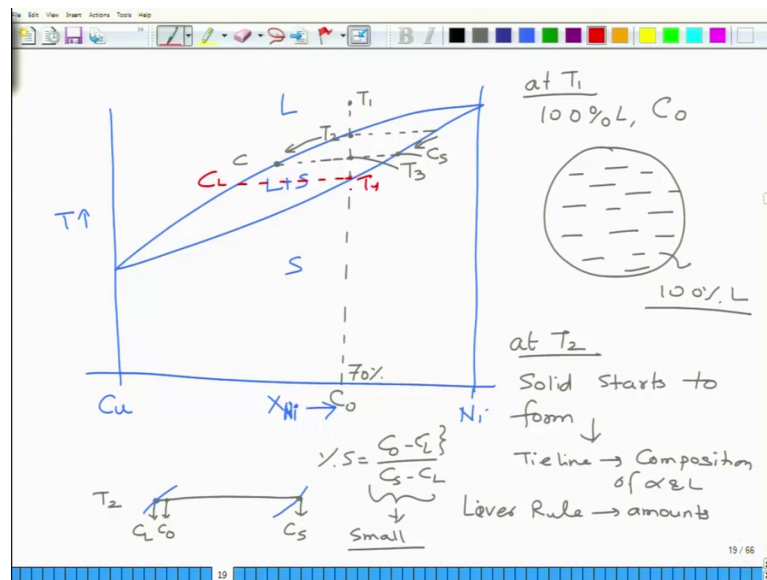
(Refer Slide Time: 12:33)



In copper zinc system I will come to them later on, but in copper zinc system you have first peritectic reaction here a second peritectic reaction here. So, it goes to a C peritectic reactions ok. Then and so on and so forth so this is temperature this is C u this is percentage zinc. So, you have alpha liquid plus alpha you have alpha plus beta you have beta phase then you have liquid plus beta and you have beta plus gamma then you have gamma region and it just keeps moving forward.

So, you have lot of these peritectic reactions one after another. So, this is Cu Zn system this is how you can have multiple phase diagrams. Now, what we want to first do is that we have to first understand how phases develop in a in a phase diagram. Then how can we calculate their fractions in a given phase diagram.

(Refer Slide Time: 14:03)



So, now, let us first let us do an analysis of copper zinc system let us say sorry copper nickel system. So, you have Cu you have Ni system picture this is X Ni ok. We know this is liquid this is solid and in between we have liquid plus solid. So, let us say we have alloy of composition C naught.

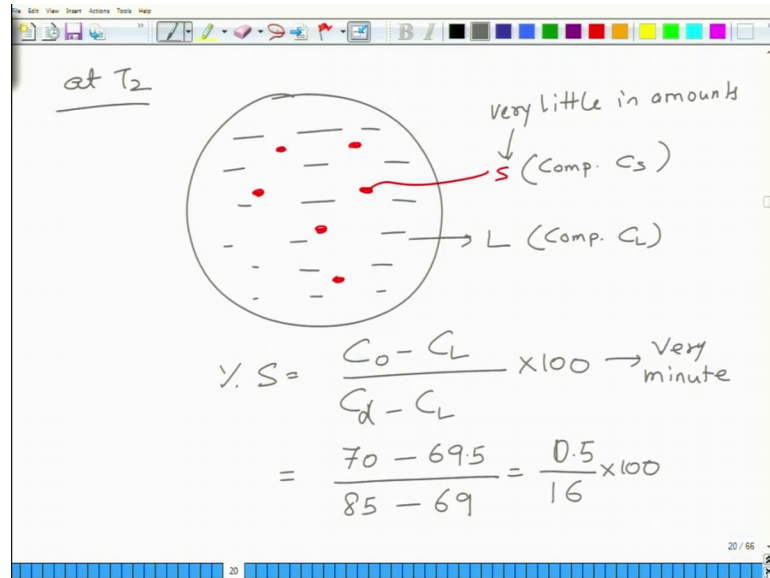
So, what happens at this point let us say we start from this temperature T₁. So, at T₁ we have 100 percent liquid of composition C naught. So, what is it going to look like it is going to look like something like this if you look at it the. So, you are if you look at it under microscope for instance you will have hundred percent liquid nothing else you come to this point now across the liquidus boundary at T₂ when you come to T₂ you start forming little bit of solid.

So, solid starts to form so which means some solid and the compositional amount of that solid can be given by lever rule. So, you can apply tie line rule and lever rule to calculate the composition. So, tie line will give you composition of alpha and liquid and lever rule will give you amounts. So, let us say we are let us zoom this portion at T₂. So, at T₂ we are something like this so this is at T₂ the our overall composition is this is C naught this is solidus so, this is C₁ C_s and this is C_l.

So, since the solid has just about to just begin to form the difference between C_l and C naught is very small. So, percentage so as a result percentage solid which is equal to C

naught minus C_1 divided by C_α minus C_1 . Since this is very small it is extremely small. So, you have mostly liquid with some particles of solid forming.

(Refer Slide Time: 17:15)



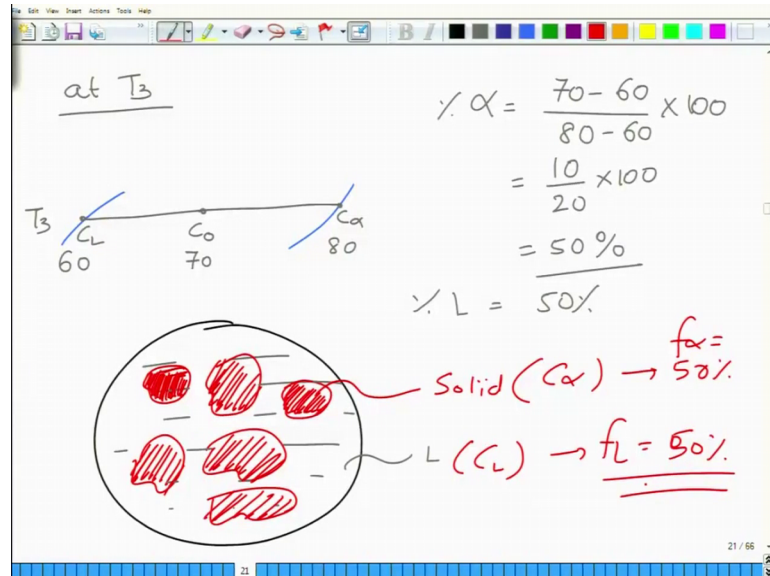
So, what you have here is at T_2 you have mostly liquid phase with tiny particles of all solid forming somewhere. So, this is solid so this is solid and this is liquid and solid of composition C_s and liquid of composition C_L , but very little in amount just a few specs just a few pieces here and here. Now, as you go further so basically percentage solid you can work out as C_0 naught minus C_1 divided by C_α minus C_1 in 200 and this will be very minute.

So, let us say for an example if C_0 naught was you know if C_0 naught was 70. And if C_1 was 69 and C_α happens to be let us say 85 minus 69. And you can see that this is 1 by 16 into 100. So, it is a very little amount it is only 5 6 percent or so 6 something percent. And if it is let us say 69.5 it is even lower. So, it would be 0.5 divided by 16. So, you see it would be 1 by 32 basically ok. So, which is a very small amount of solid.

Now, let us do one thing let us go to point three, T_3 at point T_3 . So, when we go to point T_3 you can draw another tie line so this is let us say 70 percent. At point T_3 you have formed more solid and solid composition is given by so you form more solid. So, this is solid composition which has moved along this line and this is the liquid composition which is moved along. You can see that as you have formed more solid the liquid has become depleted in solute. And solid also has become depleted and solute as

compared to the solid which had formed earlier, but the solid composition overall is higher than the overall composition of the alloy.

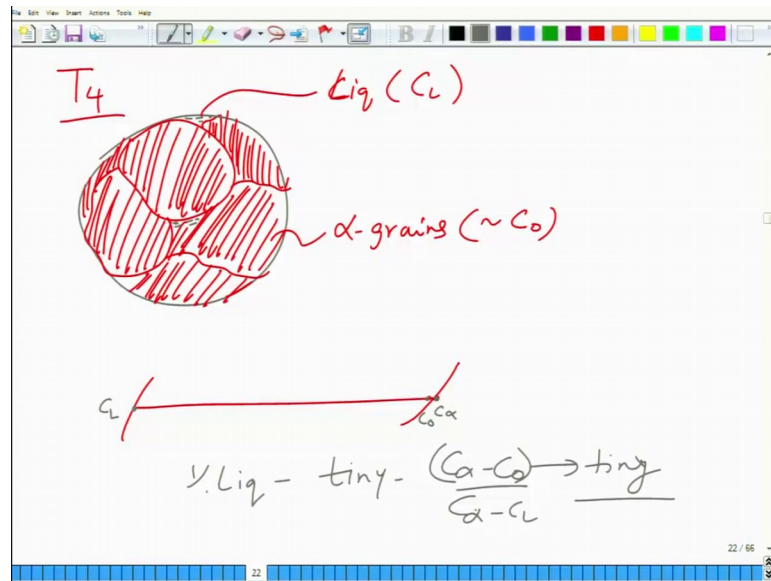
(Refer Slide Time: 20:25)



So, at T_3 if this is the zoomed up view this is at T_3 and this is C_α and this is C_L . So, this was 70 let us say this works out to be now first we took we took 85 let us say this turns out to be 80 ok. If this is 80 and if this turns out to be was saying it was 66 let us say I do not know 60. So, if this is 60, 70, 80 then percentage alpha in this case is 70 minus 60 divided by percentage solid as alpha is same as solid. So, pardon me for swapping alpha with solid so this is 10 divided by 20 in 200. So, it is 50 percent and as a result percentage liquid is also 50 percent.

So, how does it look like in the microstructure so you have you have you have liquid and these solid particles which are present earlier they have now grown bigger. So, now these solid particles have grown so this is solid. So, solid of composition C_α liquid of composition C_L which is about 80 and 60 and now solid is about 50 percent f_α and f_L is about 50 percent also so solid has grown in mass the moment you reach T_4 .

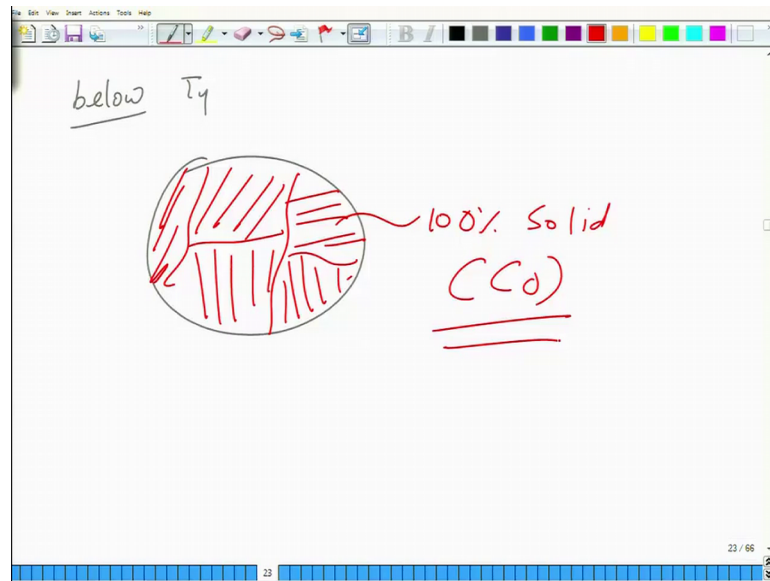
(Refer Slide Time: 23:03)



T 4 is a temperature which is here just above the solidus. So, when you are a T4 then liquid composition is this solid composition almost same as the alloy composition you have very little liquid left. So, what you have here is at T 4 is situation like this. So, most of it is solid the solid will impinge into each other. So, it is mostly solid with a few pockets of liquid left here and there. So, you might have some liquid here you might have some liquid here and mostly it is solid. So, these are called as grains of solid. So, this is solid grains alpha grains alpha grains of composition nearly C naught and you have some liquid C liquid of composition C l.

So, if you look at the phase boundary now this is your liquid solidus this is your liquidus you are here T4. So, this is your overall composition C naught this is your C alpha and this is your C l. So, your C percentage, percentage liquid will be tiny because C alpha minus C naught divided by C alpha minus C l this is very tiny. So, what you are left with is basically nearly 100 percent solid with very tiny amount of liquid present solid. And solid composition is nearly same as alloy composition and liquid is highly depleted in salute with composition C l and the moment you cross over below T4.

(Refer Slide Time: 25:23)



You will have 100 percent. So, you will have hundred percent solid of composition C naught. So, basically what you have done is you have started from this point T1 where you had a 100 percent liquid gone through formation of solid in sequential fashion with decrease in the amount of liquid slowly and gradually until you cross point to T4 below which its 100 percent solid. So, this is how the microstructure develops and the phase fractions change we will do further analysis based on these in different systems and take it further.

Thank you very much.