Phase Equilibria in Materials (Nature and Properties of Material-II)
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Lecture – 19
Gibbs Phase Rule: Eutectic Point and Lever Rule

Welcome again, we start this lecture number 19 today on Phase Equilibria.

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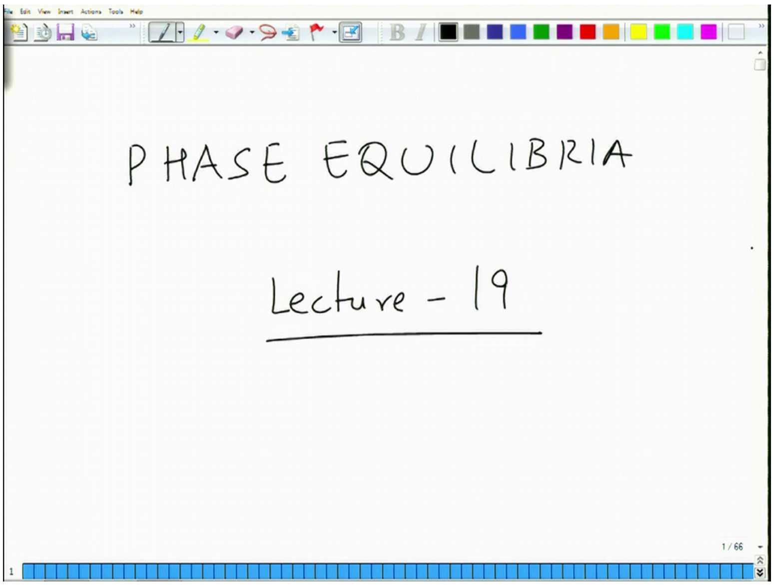

So, what we are going to do is that we will just recap the last lecture and before we move on to the contents of this lecture.

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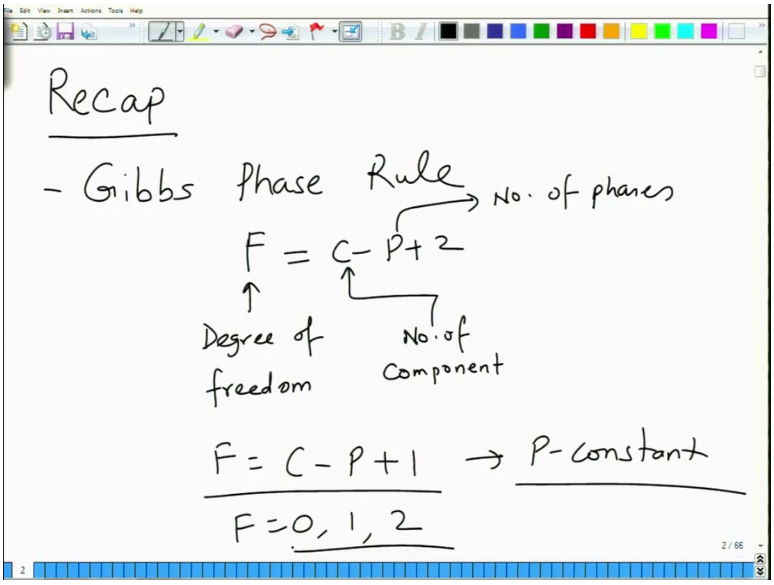

So, in the last lecture we derived, last two lectures actually we derived what we call as Gibb's phase rule. This is Gibbs phase rule allows us to correlate number of phases and number of components or the number of independent variables that are required to maintain phase equilibrium. So, you might have.

So, we saw examples in unary systems where component is 1 variable suppression and temperature. And then depending upon whether you are in two phase equilibrium or one phase equilibrium or three phase equilibrium you might have a situation where you can you may vary two parameters independently or you may not vary any parameter independently.

So, what this was basically? F is equal to C minus P plus 2; so, degree of freedom is F. So, degree of freedom is basically number of independent variable and this is number of components. So, these basically components means, free species. So, you can have element so, you can have compounds in pseudo binary diagram; this is number of phases.

So, this is C minus P plus 2 and this 2 comes from pressure and temperature. In many material systems as a general rule F is equal to C minus P plus 1 because pressure is constant. So, if you have pressure as a constant then this modifies to F is equal to C minus P plus 1.

So, this is the modified rule. So, this rule allows you to determine number of independent variables that can be present depending upon number of components, the number of phases. So, we look at the example of for instance in binary and unary systems for F is equal to 0, 1, and 2 and. So, what we saw is that when F is equal to 0, it means that system is invariant, which means you are standing at a point, the moment you move away from a point you are in.

So, in a unary system it meant three phase equilibrium and what it meant was that you did not have any degree of freedom which means three phase equilibrium exists only at a point. The moment you move away from the point the equilibrium goes away, you are in the two phase of one phase equilibrium.

Similarly, one phase equilibrium when you have big when you have degree of freedom as 1, then you are in then you are in two phase equilibrium and you are basically moving along a line. So, you change one parameter, the second one is automatically fixed.

So, you have only one independent variable, when you have F is equal to 2 which means we had one phase equilibrium and what that meant was that you can maintain a one phase equilibrium and you can while Simon while varying pressure and temperature independently. So, this was for unary systems similarly, we also did the same exercise for binary isomorphous system.

Today we will take examples of phase rule and eutectic system for instance.

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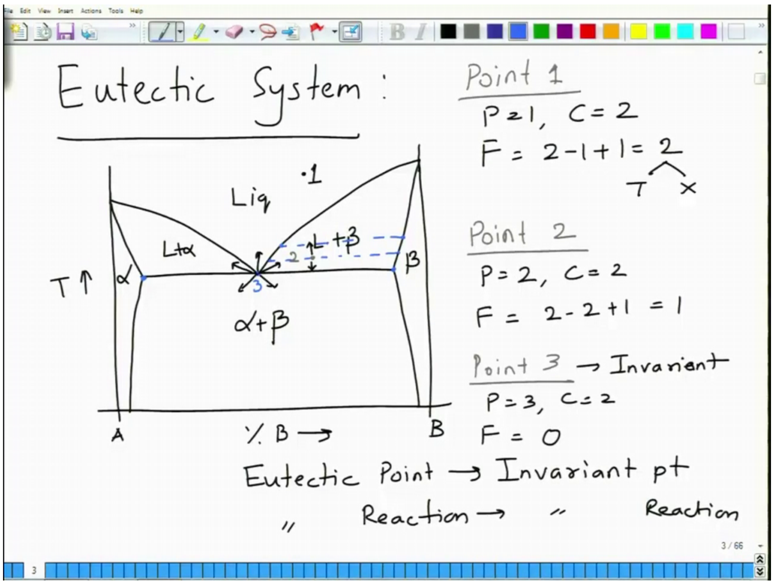

So, and this approach can also be used in peritectic in other systems. So, eutectic system is basically the phase diagram is something like. This is temperature, this is percentage B lets say and here you have liquid phase alpha, solid solution beta solid solution, two phase region. So, what you have here is basically if you look at this point for example, in this range this is 0.1, so, at 0.1.

So, point 1 we can see that we have single phase equilibrium. So, P is equal to 1, in a binary system C is equal to 2. So, F is equal to 2 minus 1 plus 1, this is equal to 2. So, when you can vary temperature and composition independently. So, two variables which means both temperature and composition can be varied independently and you will still have single phase equilibrium.

Now, when you go to point 2, let us say the point 2 is here. So, this is similar to what we did earlier. So, now, we have P is equal to 2, C is equal to 2 and F as a result becomes equal to 2 minus 2 plus 1 that is 1. So, you have only one degree of freedom whether you change composition or temperature.

So, when you change the temperature for instance you go here or here the composition is automatically fixed. So, for this point the composition is this when you go that way the composition is this ok. So, or even if you go to for example, this point the composition of phases remain the same. So, basically when you change temperature the composition is automatically fixed. So, you have only one degree of freedom, independent degree of freedom.

Now, you come to a point 3, which let us say we define as this point and this is the point at which you have liquid alpha beta coexisting which means P is equal to 3, C is equal to 2 and F becomes equal to 0. Which means the moment you move away from this point whether you go in this direction or in this direction or in this direction or in this direction or in this direction you will have 0. You will you will reach in one phase or two phase equilibrium which means you have no degree of freedom. So, this point is called as invariant point.

So, eutectic point is called as invariant point and eutectic reaction is called as invariant reaction ok. Invariant means there is no degree of freedom. So, whether you are at this point or you are at this point or you are at this point they are all triple points where degree of freedom is equal to 0 and these are called as independent, these are called as invariant points.

So, all these reactions so, I will not now go to peritectic etcetera.

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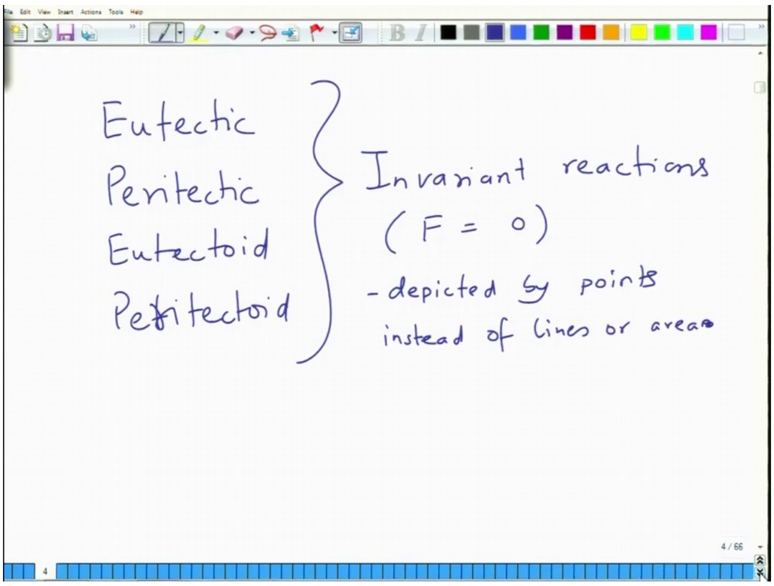

So, all the reactions eutectic, peritectic, eutectoid, peritectoid, they are all invariant reactions and what this means is that for. So, this is for this for a binary system where these reactions show that you have invariant reactions. For these F is equal to 0 which means they are basically you are just standing at a point where three phases coexist all right.

So, this is how the phase rule is applied to various diagrams. So, invariant reactions are F is equal to 0 and depicted by instead of lines or area.

Let us see what you can do with the phase diagrams. So, what is the use of phase diagram? So, first thing is phase diagram allows you.

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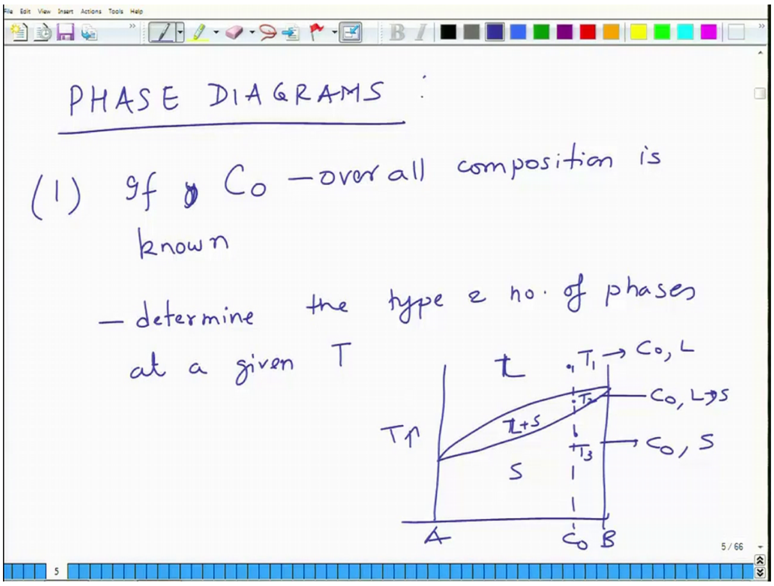

First thing that we have seen is if you know if let us say C naught, overall composition is known then you can determine the type and number of phases at a given temperature. So, this is the first utility of phase diagram. So, we can see that when you draw a phase diagram between A and B let us say you have isomorphous system.

So, you have temperature, liquid, liquid plus solid and solid. So, you can see that if you are at so, this is temperature axis. So, if you had a temperature T 1 you see it is C naught composition and liquid as a phase. When you are at a temperature T 2, your composition is C naught and your phases are liquid and solid two phase. When you are here at T 3 composition is same as C naught. So, you are standing at the same line C naught and your phases are solid.

So, this is the first information that you get from the phase diagram that for a given alloy composition what is the kind of phases you have and how many phases you get ok. So, the first thing you find out, second thing that you find out from.

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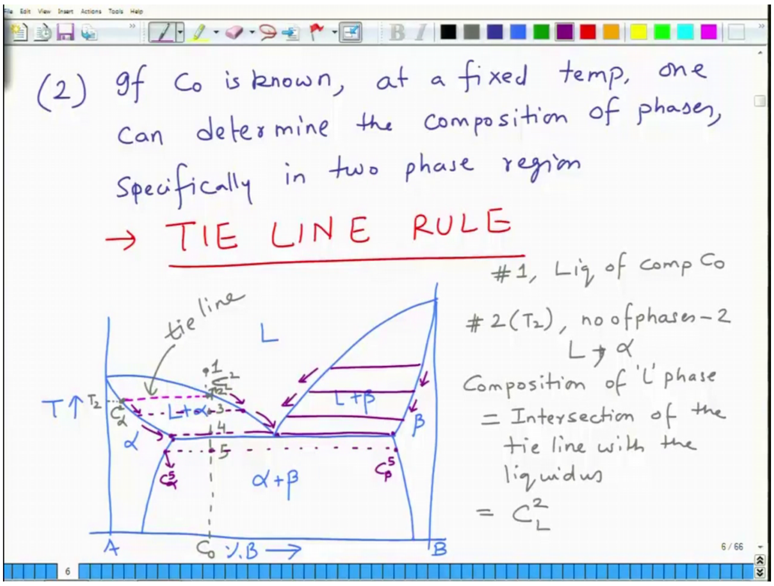

If you know, if C naught is known then at a fixed temperature one can determine the composition of phases and this is specifically for in two phase region. So, when you are in single phase region then the composition of phase is same as the alloy, but when you are in two phase region then it is different. So, this rule is called as tie line rule. So, what it means is that when you have a phase diagram like this. So, let us say you are at this composition ok. So, let us say the alloy composition is this, this is C naught ok.

Now, of course, if you are at this point 1 then its liquid phase of composition C naught. So, you have only one phase of composition C naught, but what will happen when you enter into this regime in the two phase regime? So, what is at this point? What is at this point? Let us say what is at this point? So, these are let us say point 2, point 3, point 4 and then of course, you will enter here point 5. So, when you are at point 2 you can see that you have just entered a two phase region in this case what you do is that you draw a horizontal line. So, this is the horizontal line parallel to the composition axis at that particular temperature.

So, let us say oh point 2 represent a temperature T 2 ok. So, this is T 2 temperature, in that case you can see that you have number of phases 2, which means you have liquid plus alpha, you have liquid and alpha two phases. The question is what is the composition of liquid phase? The composition of liquid phase is given by this horizontal line the intersection of this horizontal line with the liquidus.

So, this is the point this is let us say C L 2 and this is C alpha 2. So, basically intersection of the so, this is horizontal line is called a tie line. Intersection of the tie line with the liquidus boundary. So, this is C this is C L 2, this point. And the composition of so, again we are at point 2.

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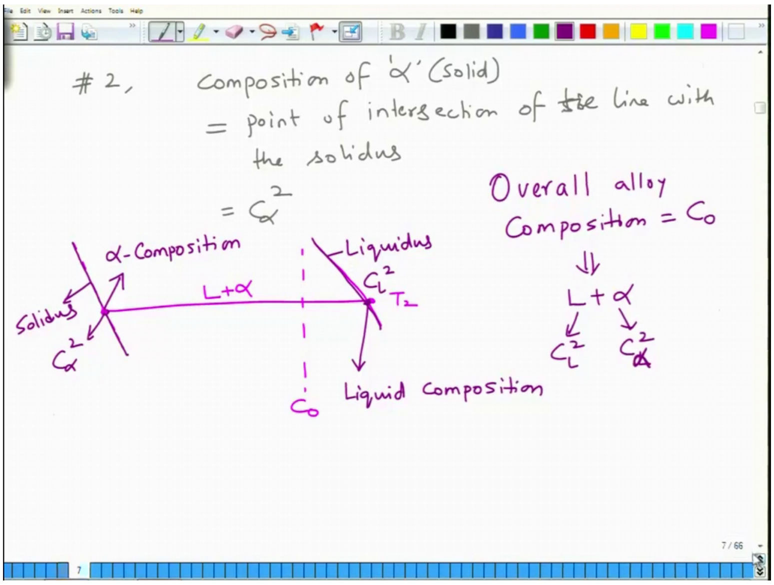

Composition of alpha which is the solid phase that is given as point of intersection of tie line with the solidus. So, in this case it becomes C 2 C alpha 2 which is equal to C alpha 2. So, when you have this as a tie line. So, this is the temperature T 2, this is the let us say overall alloy composition C naught, along this tie line you have two phases liquid of plus alpha. Let us say this is the liquidus, this is the solidus let me use a different colour, this is this is liquidus. So, this point is alpha composition and this point is liquid.

So, what you have here is overall composition is overall alloy composition is still C naught C naught is given a percentage of B or mole fraction of B, but you have two phase equilibrium, liquid plus alpha. Liquid of composition this composition let us say this is C L 2, C L 2 and alpha of composition C L C alpha 2 this is C alpha 2.

So, this is what it is so, you have two phase equilibrium in which you have two phases, but the two, but the two phases have different amount of solute in them. So, one has C naught percentage of solute, another has sorry one has C C L 2 percentage of solute, another one has C alpha 2 percentage of solute. So, this is how you can do. So, you can see that how the composition changes so, this is at T 2 and this will be at T 3. So, you can see that this point has moved here, this point has moved here, when you come to this point, this point has moved here and this point has moved here.

So, this is this is so, the liquidus is the composition of the liquid phase is moving towards the eutectic point as it decrease the temperature and composition of the solid phase is moving towards this point which is the maximum solid solubility of B in A is moving towards this point. Similarly, if you are in this regime your composition will change in this fashion. So, it will come here, it will come here until you reach this point so, at this point.

So, basically what you have is and when you come to point 5, at point 5 you have two phase region. So, this alpha will have so, you have alpha of composition this. So, this is C alpha, this is the beta 5 ok. So, this is what you can do with the tie line, you can determine the composition of two phases in equilibrium at a fixed temperature by drawing a horizontal line parallel to the to the composition axis. And look at the intersection of this look at the point of intersection of this line with the solidus liquid or solvers line depending upon the kind of phase equilibrium that you have.

So, if you have two phase equilibrium consisting of liquid and solid then intersection with liquid as gives the liquid phase composition and intersection with the solidus gives the solid phase composition. If you have two solid phases in the equilibrium then intersection with the solvers lines will give the composition of two phases. So, this is how you determine the composition of two phases in the in equilibrium in binary phase diagram, this is called a tie line rule alright.

So, now next, next thing is once you have determined what is the composition of these phases that this was the first objective, this was the second objective, this is the third objective.

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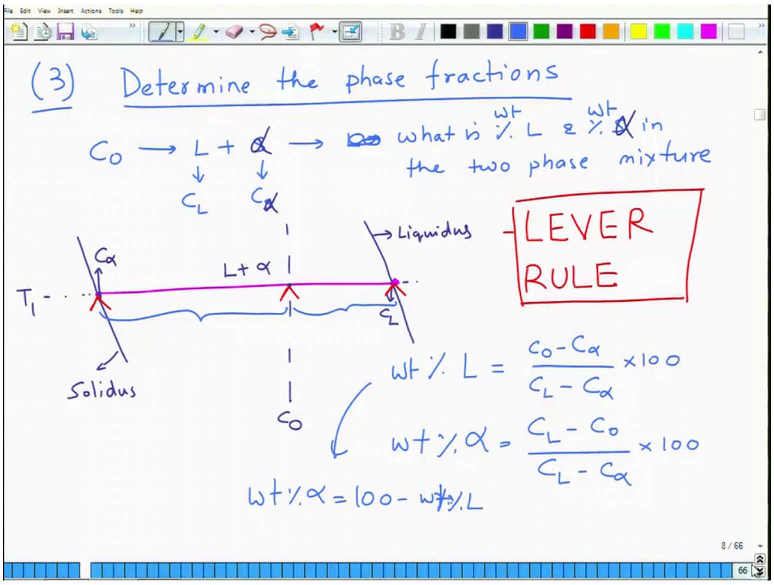

So, the third objective is to determine the phase fractions in two phase region. So, if I have alloy of composition C naught, if it is in two phase; let us say region liquid or solid, I know liquid is of compositions. So, I have I know that liquid is of composition C L and solid is of composition C S, but then what is the percentage liquid and percentage solid in the two phase mixture? And this we are talking in weight percent ok. In most cases we are looking at the phase diagram by depicting the weight percent of solute as well as weight percent of the phases.

Now, basically what we want to determine is let me draw the tie line again so, this is the tie line. So, this is the intersection with this is liquidus, this is solidus. We are in two phase region liquid plus alpha at a given temperature T 1, let us say T 1 and then let us say the alloy composition is C naught. So, this is the alloy composition C naught. So, and I know that I have a liquid phase of composition C L, solid phase of composition C alpha, I can say liquid plus alpha here.

Then what is the weight percent of alpha and what is the weight percent of a liquid? Now, this is given by a rule called as Lever rule. So, basically you can consider this tie line as a sort of a fulcrum of weights. So, the Lever rule says that percentage let us say you have two phases here. So, the perce weight percent of in this case the liquid will be equal to so, this is liquid plus solid.

So, weight percent of liquid is equal to the difference between these two that is per C naught minus C alpha divided by the total difference that is C alpha minus C liquid minus sorry C liquid minus C alpha into 100. And weight percent of alpha will be equal to C liquid minus C naught divided by C L minus C alpha into 100. So, this will be the in one case you are considering this divided by total, in the another case you are considering this divided by the total.

So, you can say that if you know weight percent of liquid; obviously, the weight percent of alpha will be equal to 100 minus weight percent of liquid which is same as what you have here. So, this is what is two phase equilibrium and you could determine the weight fractions of two phases in equilibrium with each other using this formula.

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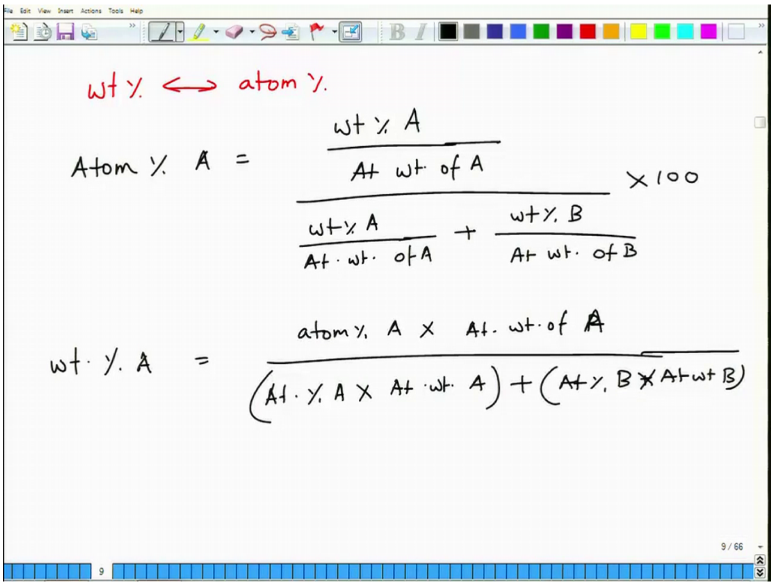

How do you convert the weight percent suppose you have weight percent, atom percent issues? Then atom percent is basically. So, if you have two multi phase multi component system, so, let us say atom percent of A is equal to weight percent of A divided by atomic weight of A divided by weight percent of A divided by atomic weight of A plus weight percent of B divided by atomic weight of B.

Similarly, you can do the same for B and multiply it by 100, similarly if you want to have weight percent of A from atom percent a mole percent of A then what you do is that you have atom percent of A multiplied by atomic weight of A divided by atom percent of A into atomic weight of A plus atom percent of B plus into atomic weight of B. So, if you let us say so, this is that this is how you convert the weight percent to atomic percent atomic percent to weight percent.

So, when you have a two phase equilibrium like this lets say.

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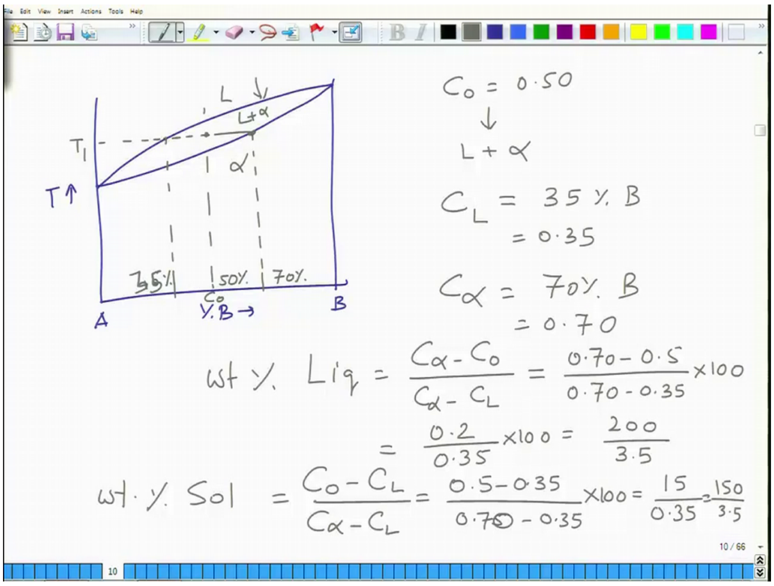

So, I am just going to give you an example ok, this is temperature axis, this is percentage B, let us say the alloy composition is 50 percent B. So, this is C naught is equal to 0.5 or 50 percent, C naught is in mole fraction that is a and then you take a fixed temperature T, T 1 let us say. So, I want to determine the.

So, you have two phases, liquid alpha, liquid plus alpha. So, two phases liquid plus alpha, let us say this point is equal to 70 percent and this point is equal to 40 percent, for now or maybe 35 percent 35 percent.

So, I can see that when you have two phases in equilibrium then C L. So, this is liquidus so, intersection with the liquidus will give me C L which is 35 percent B or 0.35. And C alpha will be equal to 70 percent B which is this point that is the solidus intersection with the solidus 0.70.

So, percentage, weight percent liquid will be equal to. So, this is C naught weight percent liquid will be equal to C alpha minus C naught divided by C alpha minus C liquid. So, this is equal to 0.7 minus 0.5 divided by 0.7 minus 0.35 into 100 which is equal to 0.2 divided by 0.35 into 100 or you can say it is equal to 200 divided by 3.5 ok.

So, this is 200 divided by 3.5 and weight percent of solid will be equal to C naught minus C L divided by C alpha minus C L that is equal to 0.5 minus 0.35 divided by 0.70 minus point into 100 and this is equal to 15 divided by, so, 15 divided by 0.35. So, you can say this is 150 divided by 3.5. If you add them up together you will get 100 percent.

So, this is what is the way method of determining the weight fraction of two phases. So, in this lecture we have discussed what is tie line rule, what is the lever rule to calculate the composition of phases in two phase equilibrium and the weight fractions. In the next lecture, we will do the derivation of lever rule and we will move on further to calculate to do some more analysis.

Thank you.