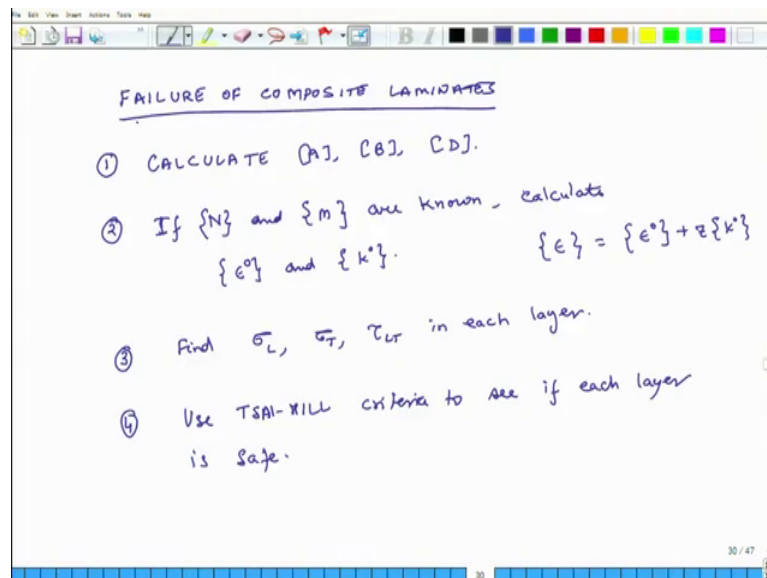


**Introduction to Composites**  
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**Lecture - 72**  
**Failure of Composite Laminates**

Hello, welcome to Introduction to Composites, today is the last day of this particular course; and what we plan to do today is have a very [vocalized- noise] brief overview of how to predict failure of composite laminates. That is one thing we will do and then we will have a formal overview of whatever we have discussed in this course.

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So, we will start our discussion by talking about failure of composite laminates failure of composite laminates. So, the purpose of this discussion is to develop an approach that, if we follow it in a methodical step by step way we know that at what load which particular layer is going to fail, ok. And once that layer fails then what happens to the composite if I keep on increasing load further; this is what I want to discuss and the way to address this question is by following a step by step process.

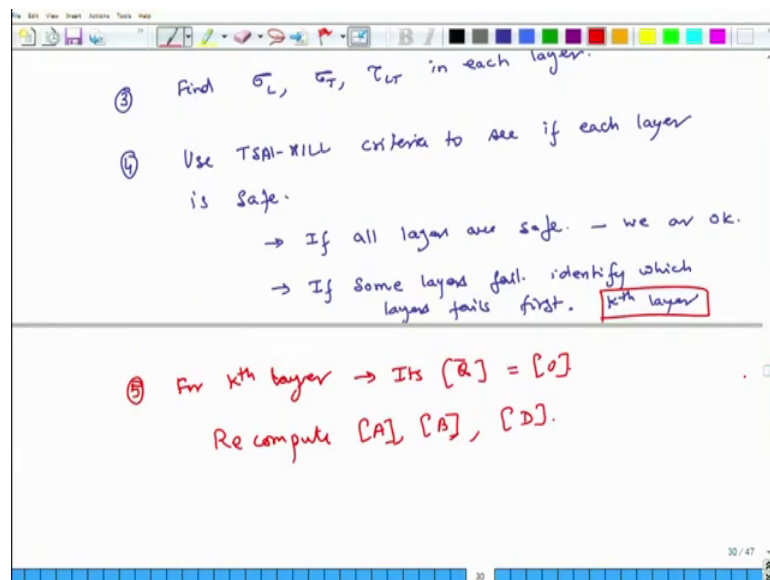
So, predicting failure means, first calculate A, B and D in each for the entire laminate that is the first step we have to calculate A, B and D. Second, if N and M are known, then calculate epsilon and curvatures in each for the laminate. We have to find the mid plane strains and mid plane curvatures. And this is only if we know N and M, if we do not

know  $N$  and  $M$  then we cannot solve this problem of failure. So, we have to know  $N$  and  $M$ . Once, we calculate the mid plane strains and mid plane curvatures; then, we find  $\sigma_L$ ,  $\sigma_T$  and  $\tau_{LT}$  in each layer.

How do we find it out? We, for once we know  $\epsilon_0$  and curvature, for each layer we can calculate strains in the in each layer. Because a strain vector is equal to mid plane strain plus  $Z$  times curvature. So, for each layer  $I$  can calculate strains in the  $x$  as the  $Y$  and shear directions using this relation. And then, using this thing I can transform these strains in the  $L T$  coordinates; once I have converted them in the  $L T$  coordinates using the  $q$  matrix, I can calculate  $\sigma_L$ ,  $\sigma_T$ ,  $\tau_{LT}$  in each layer. So, I have to do layer by layer approach, ok.

Then, the next approach method step is that, we use some criteria for instance the Tsai Hill Criteria. So, in this case we will just say, use Tsai Hill Criteria to see if each layer is safe. So, we have to do it at every single layer, we have to apply Tsai Hill Criteria to compute  $\sigma_L$ ,  $\sigma_T$ ,  $\tau_{LT}$  for each layer. And then, for each layer we have to use Tsai Hill Criteria to make sure that whether it is safe or not.

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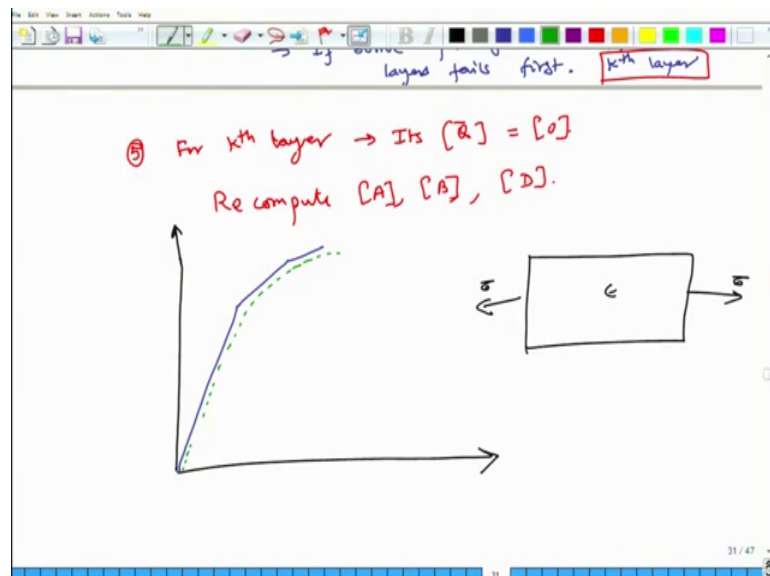
And then, if all layers are safe; then we are ok, then we are ok. Otherwise, if some layers fail; if some layers are failing then what do we do. Then what we do is, so, we will see that the Tsai Hill Criteria will be violated by a maximum extent; on some layer in some other layer it may not be violated by that big rate an extent.

So, the layer and which it is failing first that is the layer we will consider that it will break first, where it gets violated first. If some layers fail, what we have to do is; identify which layer fails first, ok. So, let us say that is the Kth layer; let us say this is the Kth layer it is failing first. Then, we go back and for Kth layer. So, we know that Kth layer is going to fail first. So, we say as I keep on increasing the load, Kth layer is going to fail first. But, I am still increasing my load what happens next.

Once the Kth layer fails, what does it imply; its  $q$  bar matrix it becomes zero. Because, once it has failed it cannot take any more load. So, if its  $q$  bar matrix becomes zero, and then we recompute  $A$ ,  $B$ ,  $D$  for the remaining laminate and we go back; we go back, we again find  $\sigma$   $L$   $\sigma$   $T$   $\tau$   $L$   $T$ . And in this way, what we will find is that we will see progressively as I keep on increasing my load; one first one layer will fail if I keep on then another layer will fail. So, in this way I will be able to predict the sequence of failure of layers in a step by step process.

So, it takes a lot of time, but if you follow this method religiously and in a very disciplined way you will be able to predict how the laminate is going to fail as I keep on increasing the load on it, ok. And in terms of its stress strain response, the failure.

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Suppose, I have a plate and I am just pulling it. So, I am applying some  $\sigma$  and because of this there is some strain in it. If you plot its stress strain response and suppose there is no coupling between bending and extension; then initially, it will be a straight

line. And we are assuming that, it is a linear elastic system. And then the first layer fails, then the first layer fails what happens; one of the  $q$  bar matrix becomes zero, and because of this its overall stiffness becomes a little less, so, the slope changes. And then the second layer fails; then the slope changes further. And then the third layer fails, and after a certain point this slope becomes either zero or negative.

And the moment that happens, it means that the plate cannot take any further load and the whole thing will collapse. Now, this is using principles of linear elastic theory; in reality the curve is something like this something like this. But, the blue line will give you a very good idea as to how layers are going to fail in a series of progressions. So, this is the basics of progressive layer failure layer by layer failure in a composite laminate.

So, this is what I wanted to talk about at least in context of failure we will not develop any detailed mathematical models for this; but, this will give you some basic idea. And it will get you going in case you really want to figure out how these layers fail in a progressive way. And that concludes our discussion for this course, this has been a twelve week course and over this course we have discussed a series of topics.

And we started our discussion by talking about composites, what are composites, what are different types of composites? Their classifications, then we also discussed different types of fibers, matrix, matrices; their strong and weak points, their manufacturing methods, different types of additives and failures which going to composites. And once we have discussed each of these different materials, we started discussing different manufacturing methods for different types of composites. And once the manufacturing was over then, we started discussing how to predict the properties of uni directional lamina if we know the individual properties of fiber and matrix materials.

So, we develop models which could which now help us, compute  $e_L$ ,  $T_g$ ,  $L$ ,  $T$  Poisson Ratio  $\nu_{LT}$  and also the failure properties  $\sigma_L$ ,  $u$ ,  $\sigma_T$ ,  $u$  and so on and so forth. So, that was for unidirectional composites if the direction of force and the material axis were mutually aligned. Then, we make this a little more general, that if these this alignment was not present then, what do we do. So, in that context we started looking at the elasticity tensor; and from 81 constants we were able to show that, for a in plane

isotropic condition or for the situation when we have a plane stress state the number of constants required for a uni directional laminate is four.

And using that simplification we developed different stiffness matrix, matrices  $q$  bar matrix and using that we developed relations for A, B and D matrices. Which influence the mid plane strains and mid plane curvatures, if a composite plate is loaded by force resultants and moment resultants. And then, we have discussed different lamination sequences and today we talked about and very basic progressive failure theory for laminated composites.

So, this is pretty much the conclusion of our discussion on in this course, we will have a formal exam on the dates which you already are aware of; you already know the format of those exams. So, please prepare for those again all these exams will be multiple choice questions types, but you will have to do some detailed calculations for solving the problems which you will see in the examinations. So, please do review everything what I have taught you and if you do detailed review and work diligently I am certain that you will do well in your examination.

Some of you may also be interested in an advance level composite course. So, I just wanted to tell you that next semester I will be offering an advance level composite course, which will be a development on whatever we have discussed till so far. Specifically what I will cover in that course will be three or four important things; one is that, what are the differential equations which govern the behavior of composite plates. Till so far, we have assumed that if the value of N and M is known then I can calculate mid plane strains and mid plane curvatures; and that basically assumes that, external force and moment resultants are known and they are constant.

But if it is a variable loading condition then, what do we do. So, in that context differential equations become important. So, I will talk about those, and then we will solve some real life problems involving beams, plates for different lamination conditions, foundation conditions, loads, temperatures so, that is one part. And then, we will also do the same thing on plates; we will also see at what loads plates buckle, how can we predict, bending of a plate which is loaded uniformly on its entire surface area and so on and so forth.

The second important theme which I will cover will be short fiber composites and there we will discuss both oriented composites as well as randomly oriented composites. So, that is another one and then of course, we will discuss in detail about this progressive failure theory for laminates. So, and we will also talk about high growth thermal and temperature induced stresses and strains in composites.

So, these are three or four broad themes which I will cover in advance composites and of course, we will start with having a good review; so, that people who are new to the course will also not feel handicapped. So, that concludes our discussion for today, I hope that you had a wonderful and enjoyable as well as an educational experience in this course and I wish you best for your future and look forward to seeing you in the next semester as well.

Thank you.