Introduction to Composites
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Lecture – 71
Quasi-Isotropic Laminate: Part-II

Hello, welcome to Introduction to Composites course today is the 5th day of the last week of this course, and today we will continue our discursion about Quasi-Isotropic Laminates. And specifically what we will cover in today's lecture is that, suppose I have a composite laminate which has glass fibers randomly oriented; so, that means, that in all directions the fibers are oriented with equal amount of distribution.

So, we would except because fibers are oriented in all directions, that this type of a plate will have Quasi-Isotropic material properties because, it will have same number of fibers in all directions. So, the question is that how can we predict the Young's modulus, the Shear modulus and the Poisson's ratio for such a randomly oriented fiber randomly oriented laminate using whatever we have learnt till so far so, that is the question.

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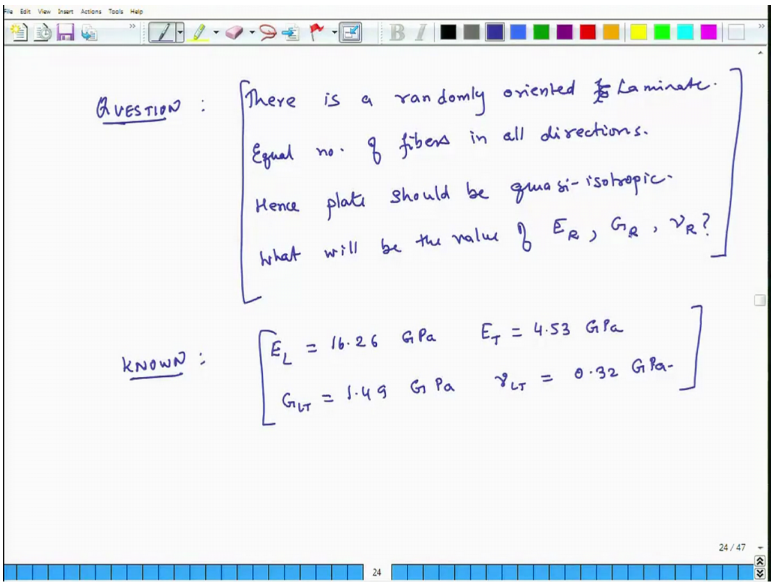

So, the question is that there is a randomly oriented laminate, so, randomly oriented laminate equal number of fibers in all directions. Hence, plate should be Quasi-Isotropic. What will be the value of E R? So, ER means Young's Modulus for this randomly oriented plate, GR which is the Shear Modulus for this randomly plate which has randomly oriented fibers and, nu R which is the Poisson ratio for this plate which has randomly oriented fibers. So, this is the question, ok; and we know everything about the materials, so, we know what kind of fibers it has. So, we say that it has glass fibers; we know the Young's modulus of glass fiber we know the matrix material Young's modulus and so on and so forth, but this is the overall approach question.

So, what is known, what is known? That, suppose using the same fiber which I am using to make this plate, I make a unidirectional laminate then for that unidirectional laminate E L is equal to 16.26 GPa suppose, I know this. And I also know the Transverse modulus of a unidirectional plate made of the same material and the Transverse modulus is 4.43 GPa 4.53 G LT is 1.49 and Poisson ratio is 0.32 GPa.

So, this is what we know. So, the question is that can we now from these data; so, there is a unidirectional laminate made from the same fibers and these are the properties from this information can we find the Young's modulus, Shear modulus and Poisson ratio for this plate which has randomly oriented fibers that is what we are interested in knowing. So, how do we do this?

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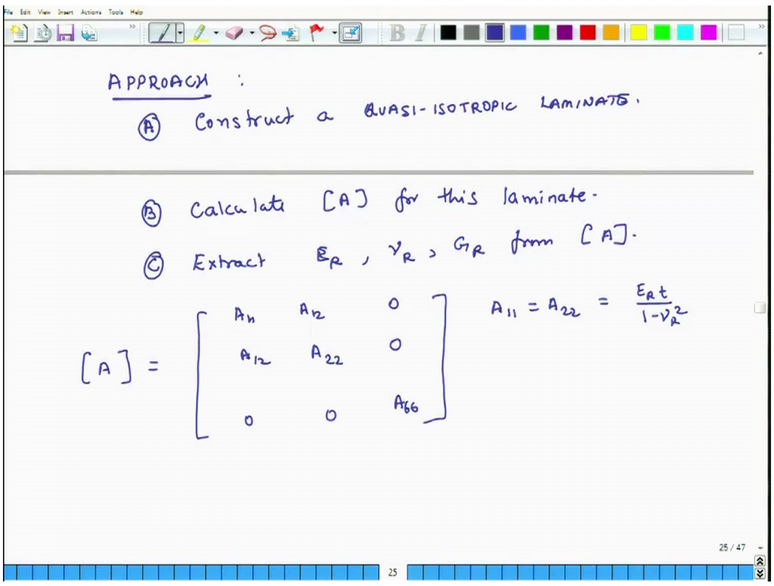

So, our approach A first thing is, construct a Quasi-Isotropic Laminate. We construct a Quasi-Isotropic Laminate, and the more the number of layers we put in this Quasi-Isotropic Laminate the better it will be because fibers will be spread more or more uniformly in all directions. But, we can always start in today's class we will just use 3, because, that requires less computation. So, we construct a Quasi-Isotropic laminate.

B, calculate A matrix for this laminate, and C, extract E R, mu R and GR from A. And how do we do this, we will explain that. So, first I generate a Quasi-Isotropic Laminate, let's say it is 0 plus 40 0 plus 60 minus 60, suppose it has just 3 layers. So, it is 0 plus 60 minus 60; for this kind of a laminate I can calculate it is A matrix. And from the elements of that A matrix I calculate E R, mu R and G R, and the procedure for that I will explain that.

So, we know that A is equal to A 11, A 12, A 12, A 22, 0, 0, 0, 0, A 66 for a Quasi-Isotropic Laminate this is the case; and for Quasi-Isotropic Laminate, A 11 and A 22 is same, right. Actually, I will not write it in the matrix I will write it separately. So, for Quasi-Isotropic Laminate A 11 is equal to A 22, and if it is an Isotropic material I can also write it as ER t times 1 minus nu R square, ok. How did I get it? Remember for an Isotropic material system we had said that, the elements of A matrix are given here. So, here if E is ER that is all I am doing.

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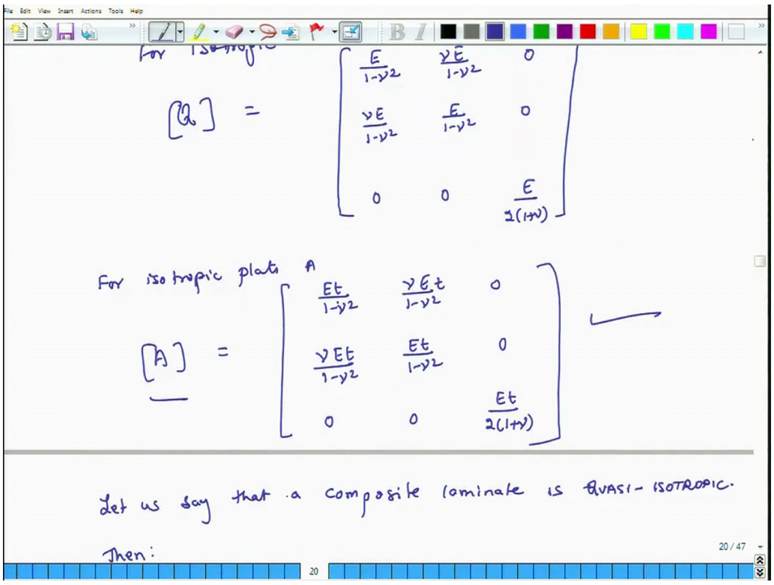

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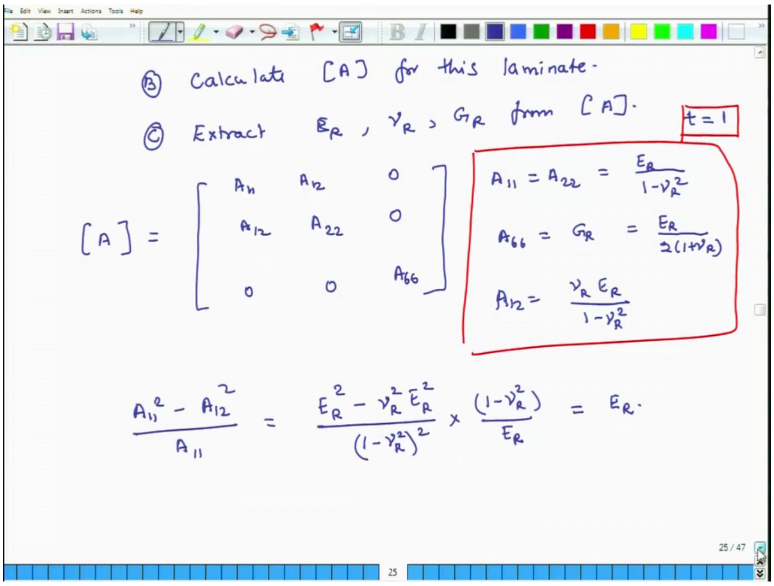

So, that is where am getting this ER from; so, if the, so, this is there ok. Similarly, A 66 is equal to GR times t and that equals ER times t divided by 2 into 1 plus nu R, ok. I make one simpler when one assumption here also just to make mathematics simple, that we assume that the overall thickness of the randomly oriented composite is 1 1. So, if that is 1 then this becomes this t goes away. We can keep t, it will get cancelled out later; but just makes our calculations little simple that's all. So, I am just making assumption that t is equal to one. So, the randomly oriented mat is of uniform thickness and its thickness is 1.

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No no, if there is a randomly oriented laminate it has only 1 play, because everything is randomly oriented. So, the overall thickness of that randomly oriented laminate is unity and the what will be it's A 11, it will be ER times ER times t t is 1 divided by 1 minus nu R square and so on and so forth, ok. So, GR time's t, t is one. So, if this t goes away and A 12 equals nu R ER divided by 1 minus nu R square, ok. So, so, these are the relations for randomly oriented play right and I can calculate the values of A 11, A 22, A 66 and A 12 using the Quasi-Isotropic Laminate which I am thinking about ok.

So, from these relations then I can back calculate ER, nu R, GR and these things, so, let's see how we do that. So, from these we see that A 11 square minus A 12 square divided by A 11, what does it come out to. This is equal to ER square minus nu R square ER square divided by 1 minus nu R square the whole thing square into 1 minus nu R square divided by E R, ok. How did I get? I am just substituting A 11 as ER by 1 minus nu R square A 12 as ER nu R divided by 1 minus nu R square. I am just substituting this here that's all. So, if I simplify this comes out to be ER. So, if for a Quasi-Isotropic Laminate I know A 11 and A 12, I can calculate its ER.

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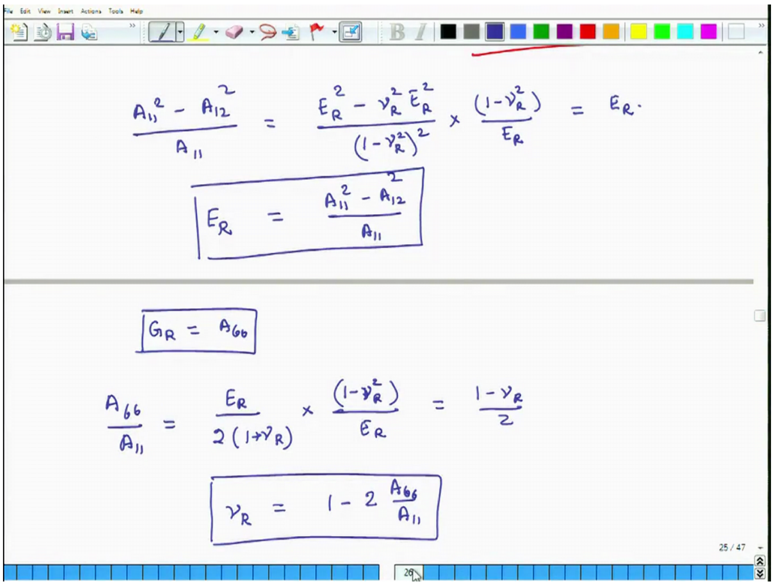

I can calculate ER for the randomly oriented mat by this relation, ok. So, if for a Quasi-Isotropic Laminate I know A 11, A 12 then, I can say that A 11 square minus A 12 square divided by A 11 will be the Young's modulus of a randomly oriented mat, ok. Next we will see how does G work out. So, GR is equal to A 66 that is direct because, thickness we have assumed as unity ok.

So, this is the second relation and the third relation is for Poisson's ratio. And what do we do? A 66 divided by A 11, is equal to what, A 66 is ER divided by 2 into 1 plus nu R. And what is A 11? A 11 is ER over 1 minus nu R square. So, this is on the denominator, ER cancels out. So, what we are ending up with this, 1 minus nu R divided by 2. So, Poisson's ratio for a randomly oriented mat is equal to 1 minus twice of A 66 divided by A 11.

So, this is the overall method. So, the question. So, the once again; what is the method? First, we find out the A matrix for a Quasi-Isotropic Laminate and the laminate should have total thickness of unity. The total thickness of laminate should be unity; and then for corresponding to that a matrix, if I use these relations then I can calculate ER GR and nu

r. So, with this background what we will do is, we will actually calculate for a material system whose unidirectional lamina has these properties ok.

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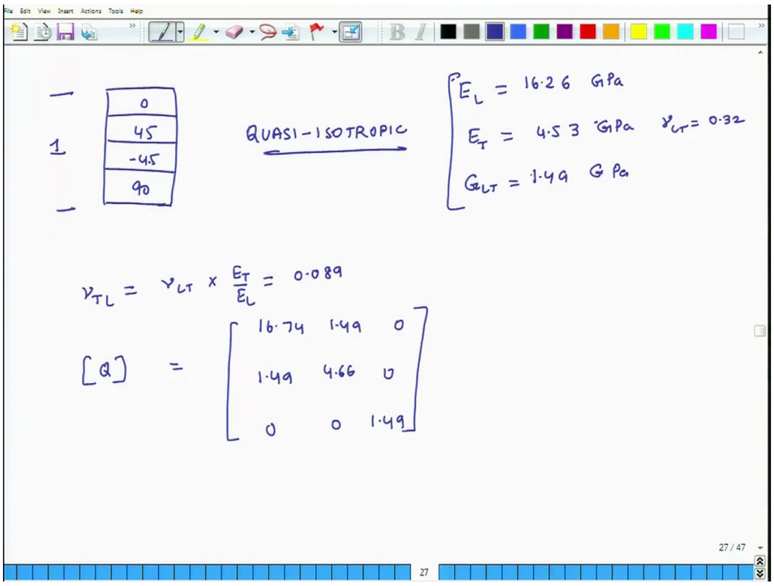

So, what we do is we construct a laminate, and we will say that it has just four layers: 0, 45, minus 45 and 90, ok. So, it may not be symmetric, but if I just reflect it on the other side I can make a symmetric laminate. So, that's not the thing point, ok. So, this is the Quasi-Isotropic Laminate, because it has more than 3 layers 3 or more than 3 layers and the angles are in steps of 45 degree, ok. And for this we know that EL is equal to 16.26 GPa, E t is 4.53 GPa, nu LT is 0.32 and G LT is 1.49 GPa; so, this is what we know.

So, now, what we do is first we find out the A matrix for this lamina, and overall thickness is unity. So, each thickness each layer is 1 4th thick, ok. So, first is, we find nu TL. Nu TL is what? Nu LT into ET over EL and that comes out to be 0.089. So, now, I know all the five parameters. So, I can find out the Q matrix for this material and Q matrix is equal to. So, I will directly write down the relation numbers.

So, this is Q matrix this is 16.74, 1.49, 0, 4.66, 1.49, 0, 0, 0 and this is 1.49; so, this is the Q matrix. Now, this has 4 layers 0 degree, 45 degrees, minus 45 degrees and 90 degrees. So, we have to find Q bar for each of these layers, ok; because to find the Q matrix A matrix I have to find first Q bar and then multiply by individual thicknesses.

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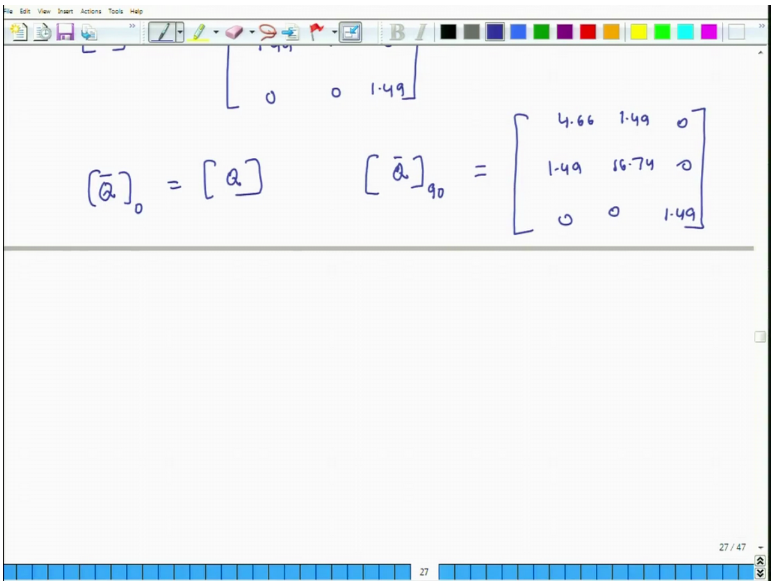

So, Q bar for 0 degree is same as Q matrix, ok; for 90 degrees thing, Q bar for 90 degree it is we just replace 16.74 by 4.66 and vice versa. So, it is this 4.66, 1.49, 16.74, 1.49, 0, 0, 0, 0 and 1.49 ok. And for 45 degrees we actually use the relations which we have explained in the class.

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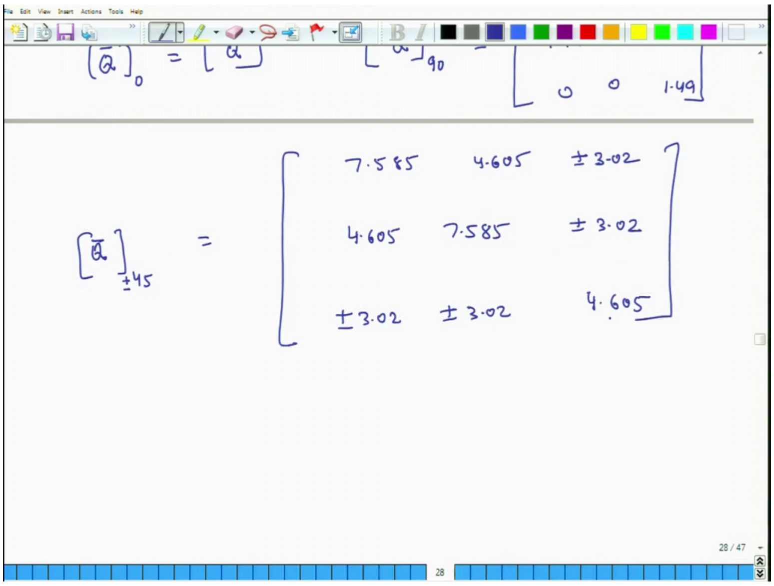

So, Q bar for, 45 degrees this works out as; so, this is for both plus and minus. So, wherever I have a plus sign so, this is 7.585, 4.605 and this is plus minus 3.02. This is also plus minus 3.02 and this is 4.605, 4.605, 7.585 plus minus 3.02 plus minus 3.02. So, this is the Q bar matrix for 45 degrees.

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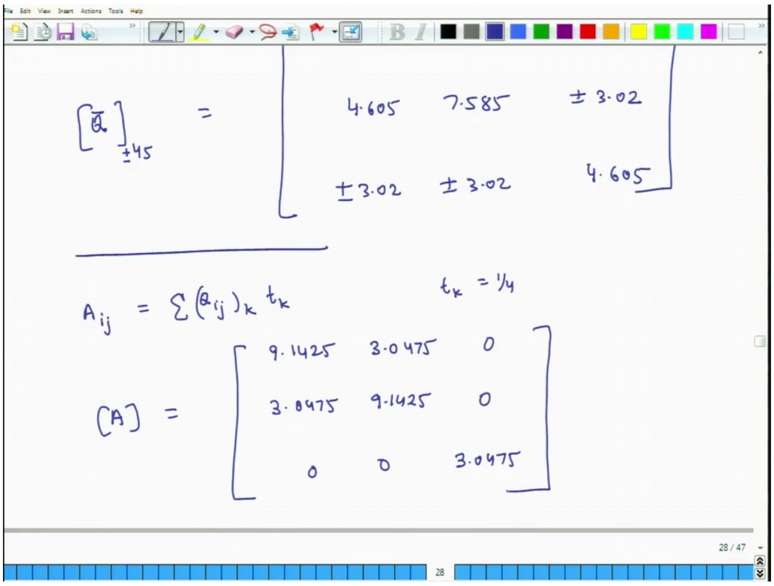

So, A ij is what, is Q ij for the kth layer times thickness of the kth layer tk and we sum it right. And what is tk? tk is equal to 1 by 4th because the overall thickness is 1. So, basically I add up all the 4 Q matrices and I get and divide them by 4, I get A matrix. So, essentially A is this matrix, 9.1425, 3.0475 and because there is a plus 45 and minus 45, so, A 16 is 0, 3.0475, 9.1425, 0, 0, 0 and the last number is 3.0475. So, I have found the A matrix for the Quasi-Isotropic Laminate; and now using these relations these 3 relations I can calculate the Young's modulus, the Shear modulus and the Poisson's ratio for the randomly oriented mat.

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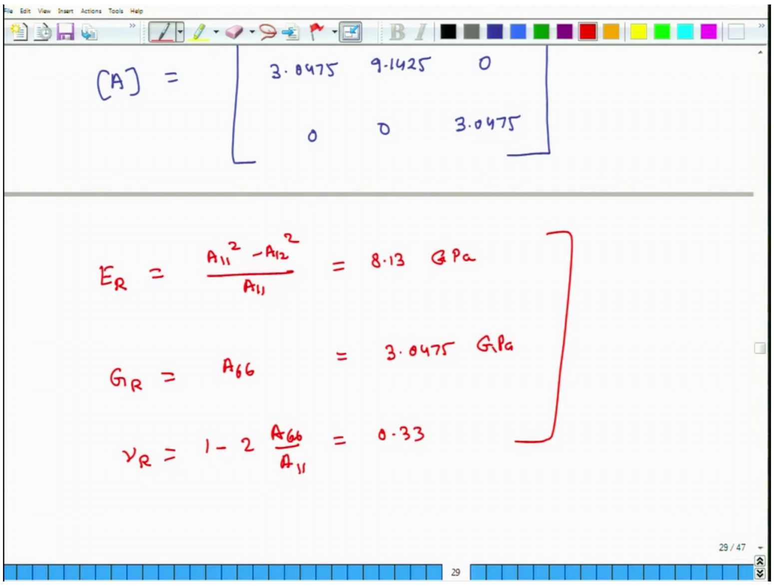

So, I find that ER Equals A 11 square minus A 12 square, divided by A11 and I plug in the all the numbers I get 8.13 GPa. GR is what? Is same as A 66 and that is equal to 3.0475 GPa and nu R equals 1 minus 2 A 66 by A 11 and this comes to be 0.33.

So, these are the material properties of this randomly oriented mat and I can make it a little more accurate if I have more layers in the system. The more the number of layers I have in the system, the more accurate these estimates of ER GR and nu R become. So, the point here is that this is the very useful method not only to predict behavior of oriented laminates, but even if the laminate has randomly oriented fibers. There also we can use these principles to predict the Young's modulus, Shear modulus and Poisson's ratio of randomly oriented materials. And these types of materials are used in a large number of applications boards, fiber glass, tanks and things like that. So, you can predict their properties using this kind of an approach.

So, this concludes our discussion for today, tomorrow we will have some discussion on failure of laminates and then we will also do an overview of the overall course and with that we will close our discussion for this course. So, that is what we plan to do tomorrow and till then have a great time and I look forward to seeing all of you tomorrow.

Thank you.