Introduction to Composites
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Lecture – 68
Simplification of Stiffness Matrices: Part- I

Hello, welcome to Introduction to Composites course. Today is the second day of the ongoing week. And today what we plan to we plan to discuss is three types of couplings.

We have already discussed this earlier and today we will extend that discussion and we will try to see, how these three types of couplings; specifically the coupling between extensional strains and shear strains, the coupling between extension and bending, which is attributable to the B matrix and the coupling between bending and twisting which is attributable to elements D 1 6 and D 2 5; how these three individual couplings they can be minimized and in that context we will also explore specific lamination sequences.

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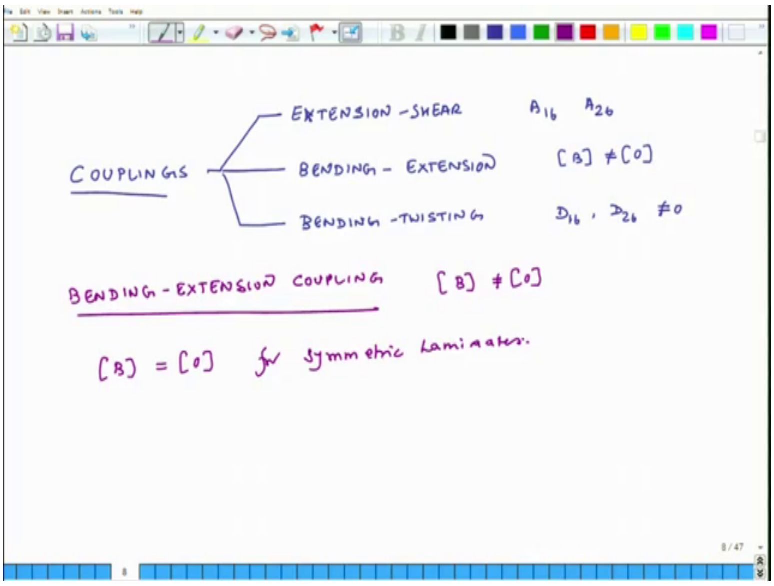

We had talked about three types of couplings and the first coupling we discussed was extension shear coupling.

So, in extension shear coupling this is attributable; so this type of coupling what does it imply? It implies that if I pull a laminate it will also generate shear strains or if I apply shear it will also generate extensional strains. And we had seen it, that this is attributable to elements A 1 6 and A 2 6. So, if we are able to remove these elements make them 0, then this coupling will not be present; and that is something we would like to have because typically, if I have a laminate and if the fibers are like this and if I am pulling it and if it is also experiencing shear, then the chance of it failing will be higher, because the material is weaker in the shear direction because of the structure of the system.

So, we do not typically like to have extension shear coupling. The second coupling we had discussed was bending extension coupling. Now, this coupling we said that it exists, if the B matrix that is the coupling matrix, coupling stiffness matrix is not equal to 0. So, again we will explore methods through which we can make this matrix 0, and if we are able to achieve it then this coupling can be eliminated. And the third coupling we had discussed was bending twisting.

So, you bend something and it where you try to bend something, but it not only bend it also tries to twist and vice versa. And this happens when D 1 6 and D 2 6 are not equal to 0. So, again we will explore; how we can reduce or eliminate the effects of these terms also. So, we will start with the simplest one. So, we will start with bending extension coupling. Bending extension coupling and we had said that this coupling exists, when B is not equal to 0.

So, how do we make B to B 0 and we had explained earlier and now we will discuss the details that; B is 0 for symmetric laminates, that is if there is a layer oriented at positive angle theta at a particular distance below the mid plane and if there is a similar layer above the mid plane oriented in the same direction, then that kind of a laminate is a symmetric laminate. So, we will discuss this further.

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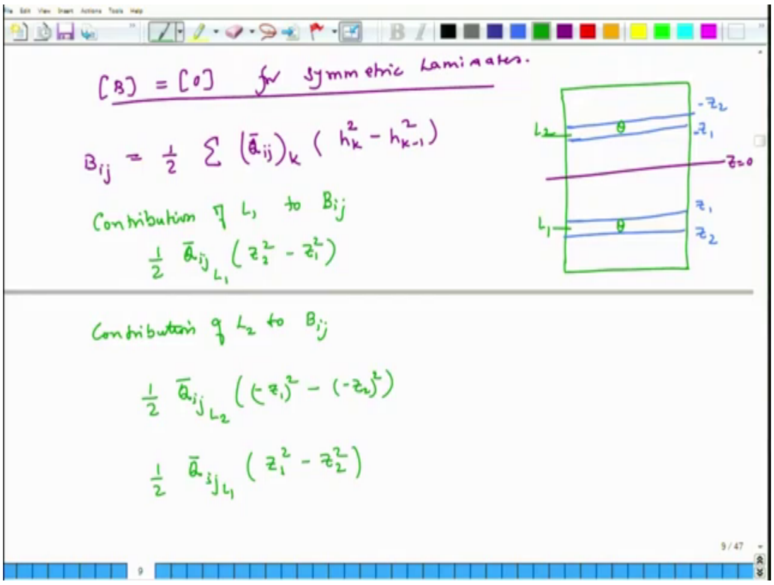

So, we know that Bij equals half of sum of Qij bar for the kth layer times hk square minus hk minus 1 square ok. So, this is the definition for the ijth element of the B matrix.

Now, consider a laminate, let us say this is my mid plane. So, z is equal to 0 and let us say there are two layers: there is one layer and there is another layer; both these layers are equidistant from the center, which is z is equal to 0. So, let us say this coordinate of the bottom layer is z 1 and z 2, then the coordinate of the top layer will be z 1 minus z 1 and minus z 2 and we say that, because the laminate is symmetric; the orientation of this layer is theta and the orientation of this layer is also theta.

So, let us see the contributions of these two terms into the B matrix. So, contribution of; so let us say this is yeah. So, the contribution of these two layers, so let us say this is L 1 and this is L 2. So, contribution of L 1 to Bij, what will it be? It will be half Qij bar for this layer and let us say this is; so this is L L 1 we are talking about L 1 times z 2 square minus z 1 square ok and contribution of of L 2 to Bij.

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So, this for L 1; so it is z 2 minus z 1. So, contribution of L 2 to Bij, what will it be? It will be half Qij bar for layer L 2 times hk square minus hk minus 1 square. So, hk is minus z 1 minus z 1 square minus minus z 2 square. So, this is equal to half and because the material is same L 1 and L 2 the material is same. So, Qij L 1 is same as Qij L 2. So, I can just call it Qij L 1, because the material same and in the parentheses I am left with z 1 square minus z 2 square.

So, what we see is that the contribution of L 1 is just the negative of contribution of L 2. So, when I add up the; these two contributions they will sum and they will become 0. So, likewise if I have for every layer on the top I have a similar layer on the bottom side at the same distance, then the overall contributions from all these layers will add up to be exactly 0. So, that is why; the B matrix is equal to 0 for symmetric laminates; this is very important ok.

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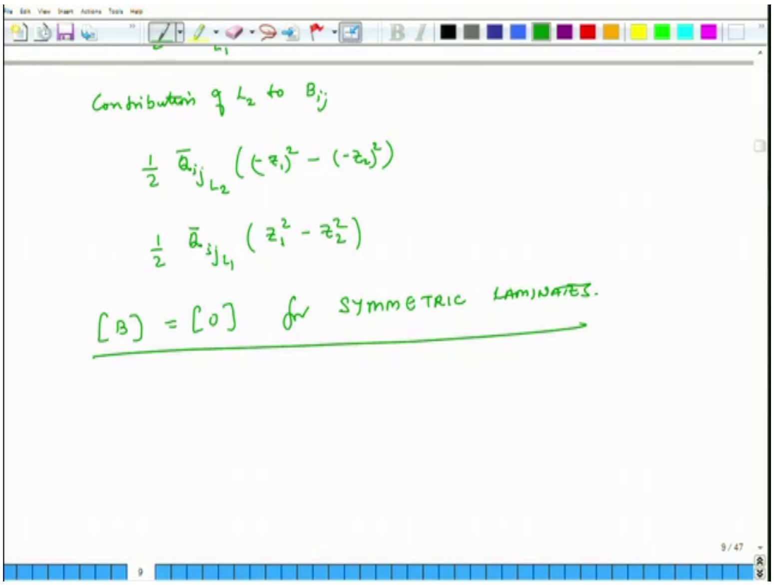

So, we will like to have, if we want to eliminate bending twisting or this bending extension coupling right; then what we have to ensure that whatever laminates we use, they are symmetrically oriented with respect to the mid plane. If they are symmetrically oriented with respect to the mid plane; then if I bend it will only bend it will not extend, if I extend it will only extend it will not bend this is very important.

Now, this is also important symmetric laminates are also desired even from a manufacturing standpoint. Because while manufacturing what happens is that I put all the layers on top of each other and then a heat the whole thing and then when I heat it this epoxy melts and then I add some additives and it starts curing, and when it starts curing it starts to solidify and as it solidifies once it has solidifies, then it starts cooling down. So, in the solid state it is flat and as it starts cooling down it starts generating internal stresses because of mismatch of expansion coefficients.

And, if the matrix B matrix is not 0, then not only it will contract it will also try to bend. So, what we would we would ideally like to expect a flat plate, but when I take it out after it has cooled, we will see that the plate has bent and it is totally distorted. So, if the matrix lamination sequence is symmetric we will still get a flat plate, but if the matrix lamination sequence is not symmetric; we may like to have a flat plate, but we will not really have a flat plate; it will be actually a bent and a curved in a verb plate.

So, even from a manufacturing standpoint it is desirable to have B matrix as 0, because that ensures the decoupling of bending and extensional responses and it also ensures that we get much better controlled geometries during the manufacturing process of the composite laminate. So, this is about bending extensional coupling.

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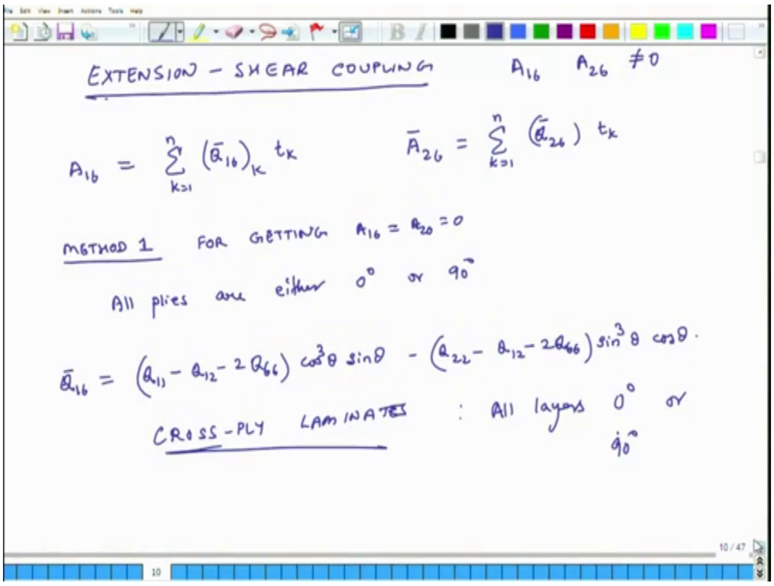

The next thing we will discuss is extension shear coupling extension shear coupling. Now, we said that this kind of coupling happens; when A 1 6 and a 2 6 are not equal to 0. So, if I want to eliminate this coupling, then what do I do? I have to make sure that A 1 6 and A 2 6 should be 0. Now, A 1 6 is what? It is equal to sum of Q 1 6 bar for kth layer times thickness of kth layer and k is equal to 1 to n. And similarly A 2 6 is equal to k is equal to 1 to n, Q 2 6 bar tk.

Now, there are several ways we can ensure that A 1 6 and A 2 6 are 0. So, there two or three approaches so method 1 method 1 for getting A 1 6 is equal to A 2 6 is equal to 0 this is the first method; what is the first method? We ensure that all plies are either 0 degrees or 90 degrees in orientation, all plies are either 0 degrees or 90 degrees in orientation, because if all plies are either 0 or 90 degrees, then for each ply Q 1 6 bar and Q 2 6 bar will be 0, why? Because Q 1 6 bar is equal to Q 1 1 minus Q 1 2 minus 2 Q 6 6 times cosine theta, sin theta minus Q 2 2 minus Q 1 2 minus 2 Q 6 6 sin cube theta cosine theta.

So, if theta is either 0 degrees or 90 degrees, Q 1 6 bar is 0 same thing is true for Q 1 Q 2 6 also. So, if all the layers in the laminate are either 0 degrees or 90 degrees, then the laminate will have exactly A 1 6 and A 2 6 will be exactly 0; so, that is 1. So, such laminates are known as cross ply laminates. They may or may not have B to be 0, but they are cross ply laminates. So, in cross ply laminates all layers are either 0 degrees or 90 degrees ok; these are cross ply laminates. You can also have a unidirectional system where all layers are 0 degrees that is also there. So, this is first step first method.

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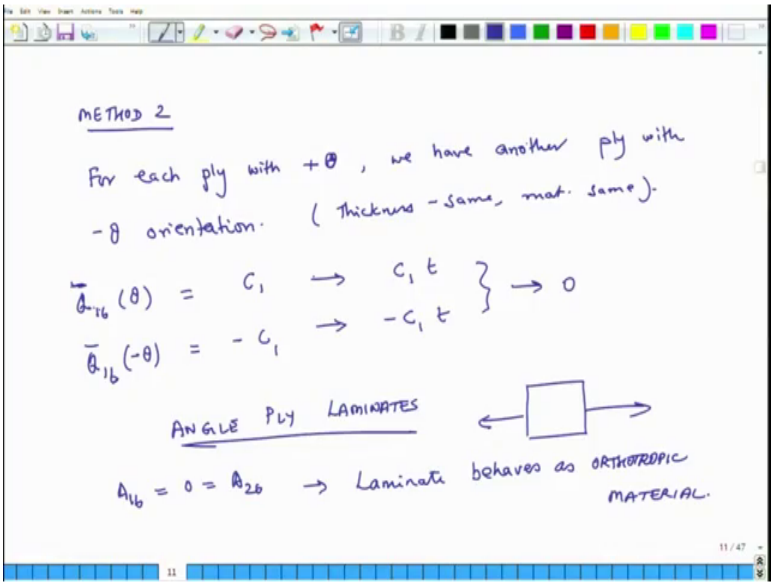

The second method, method 2: for each ply with positive theta orientation, we also have another ply with minus theta orientation ok. And here of course, we are assuming that thickness is same, material is same ok. So, in this case what happens; Q 1 6 for positive theta degrees will be let us say some number C 1 and Q 1 6 bar for the negatively oriented ply will be minus C 1 why? Because you look at this relation, if theta becomes negative theta, then it is an odd function in theta it is an odd function in theta. So, odd function means that if I change from theta 2 minus theta.

So, the contribution of positive theta would be C 1 times thickness of the layer and contribution of negative theta layer will be minus C 2 times thickness of the layer and when you add these two up it becomes 0; Q 1 6 bar here Q 1 6 bar. So, this is another approach through which you can ensure that A 1 6 and A 2 6 become 0. So, such types of laminates are known as angle ply laminates.

So, for every 30 degree ply there will be another ply it may or not may not be symmetrically place it does not matter, but there will be another ply which will be at an orientation of minus 30 and that will ensure that the overall value of A 1 6 and A 2 6 is 0; so, this is another. So, there are two methods through which we can ensure A 1 6 and A 2 6 to be 0 either we make sure that all the Laminae or layers are either 0 degrees or 90 degrees, and if we want to have non zero degree or 90 degrees layers also, then for each theta we should have another layer which has negative theta orientation and if that is the case then we are ok.

So, this is about how we ensure that the coupling between extensional stresses and shear stresses is eliminated. One more thing I would like to say in this context is that in this case; so if A 1 6 is equal to 0 and also A 2 6, then what does it mean? That if I have a flat plate and if I pull it will only exhibit extensional strains it will not exhibit shear strains. And similarly if I have flat plate and I apply shear strains on it shear stresses on it; it will only exhibit shear strains, it will not exhibit extensional strains.

Now, this is the behavior of orthotropic materials. So, if A 1 6 equals A 2 6 equals 0, then the entire laminate it behaves as orthotropic material. So, we are not talking about individual layers, but the at the laminate level at the laminate level; the whole laminate it behaves as a orthotropic material; layers may do their own thing, but at the laminate level the overall behavior is that of an orthotropic material.

So, this is another thing I wanted to mention. We will continue this discussion tomorrow and we will have; we will discuss the third coupling which is the coupling between bending and twisting in tomorrow's lecture and there we will see how we can eliminate or minimize the role of D 1 6 and D 2 6. So, that concludes our discussion for today and I look forward to seeing all of you tomorrow.

Thank you.