Introduction to Composites
Prof. Nachiketa Tiwari
Department of Mechanical Engineering
Indian Institute of Technology, Kanpur

Lecture – 59
Strength of An Orthotropic Lamina

Hello welcome to introduction to composites. Today is the fifth day of the ongoing week. And today and tomorrow we will introduce and discuss a new topic in detail, and that is about how to predict whether a single layer of composite, which is loaded in a plane stress state, how can we predict it is failure.

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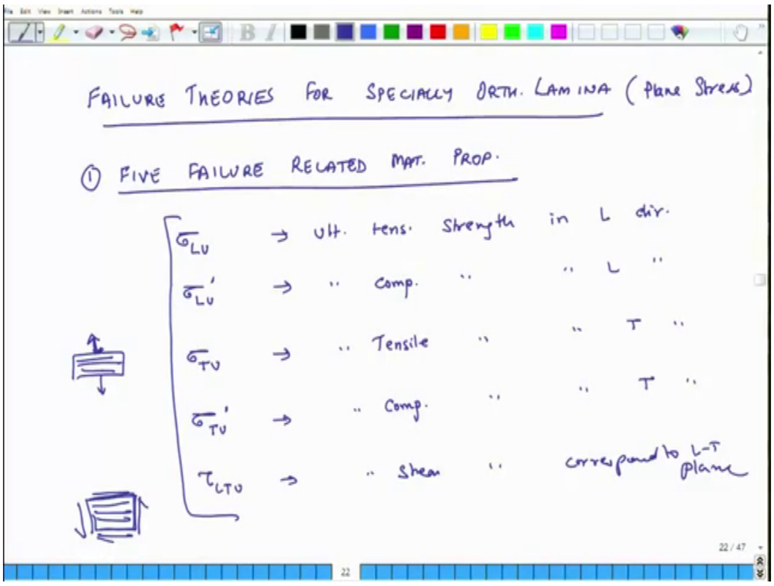

So, our theme of discussion is failure theories for especially orthotropic lamina. And since it is very thin, we assume that it is plane stress that is assumed. So, before we start discussing in detail we know based on our earlier studies that if we have to predict failure in case of metals which are isotropic materials. Then we have several theories of failure, for instance maximum stress theory, another one is maximum strain theory, another one is you know maximum strain energy theory, then there is another one known as von mises theory.

And in several of these theories we use the concept of principle stress, and we compare the value of principle stress. So, what we do is we compute the value of principle stress and we compare that value with the failure strength or with the strength of the material, and if the strength of the material is more, then we say the material is not failing, if the strength is less compared to principle stress or principle strain, then we say that the thing is failing. So, this is what is done a at least in context of isotropic materials.

And the reason we are able to do this in case of isotropic materials is, because the strength of the material is same in all directions. So, it does not matter when you whether you pull a rod a piece of if you take have a piece of metal, and you pull it in x direction or y direction or z direction, it is strength does not change with respect to the orientation of the force, it is strength remains the same, because it is an isotropic material.

So, when we compute the principle stress, the principle stress is associated with a particular direction. It could be anything, but in every direction because the strength is same, I can compare that principle stress with the strength, and because the stress is same I can compare and see whether the thing is going to fail or not.

The same cannot be said in case of composites, and the reason is I can still compute principle stress which will be the direction which will experience highest stress. But the strength of the material keeps on changing. So, for different directions it has different strengths. So, I cannot compare the principal stress with a particular number, which exists in case of isotropic materials.

So, because of this fundamental problem, we cannot use any failure theory which is used in isotropic materials as is and to predict the failure of composites or in this case especially orthotropic lamina; so, this is a background. So, we have to develop a different framework of predicting how do composites and specially orthotropic lamina loaded in the plane stress fail.

So, in that context, first thing what we will do is we will say that the 5 they especially orthotropic lamina under plane stress has 5 important failure properties. So, first thing is 5 failure related material properties, 5. So, these are 5 fundamental failure material related properties, what are these? The first one is sigma LU, and in this case this is the ultimate tensile strength.

So, U is ultimate, sigma LU tensile strength in L direction, this is the first fundamental. The second one is sigma LU prime. So, here it is if I have a lamina and if I instead of pulling it in tension, I compress it, it may fail also at some load. So, this is ultimate compressive strength in L direction. So, these are 2 properties.

Similarly, we have 2 properties for transverse direction, sigma TU is ultimate tensile strength in T direction transverse direction. And then we have so, what is sigma TU, it is fibres are like this, and I am pushing it I am pulling it in tensile direction. But in the transverse direction, and sigma TU prime here, the prime indicates the direction is compressive, it is ultimate compressive strength in T direction. And the last one is tau L TU it is ultimate shear strength, shear strength corresponding to L T plane corresponding to L T plane.

So, what; that means, is the lamina is loaded only under shear, and it may fail and the fibres are like this? So, these are 5 material related properties associated with failure of a thin lamina, especially, orthotropic lamina which is subjected to a plane stress situation. Now using these 5 properties, 3 theories will be discussed, which will help us understand when a composite is going to fail.

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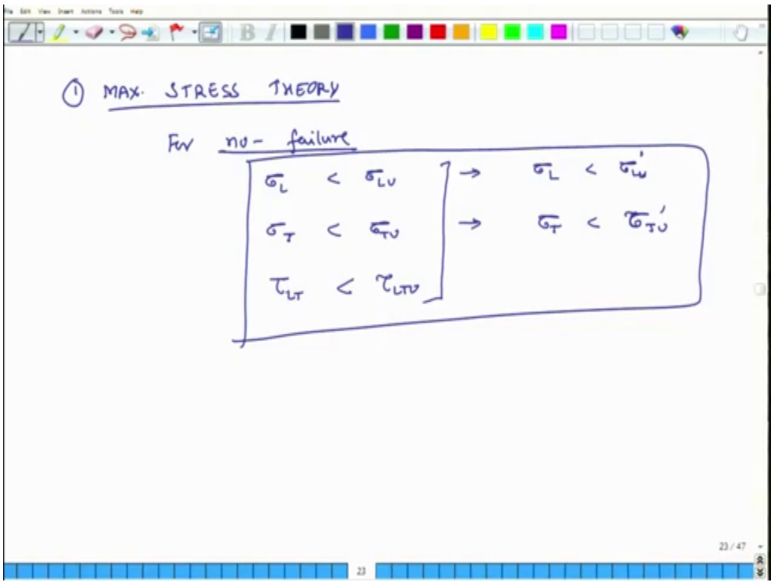

The first one is maximum stress theory, maximum stress theory.

So, what does this theory say? That if so the material so, a single lamina is not going to fail as long as the stresses in the L T, L T direction you know sigma L sigma T sigma tau L T, as long as they are less than their respective strengths, then the thing is not going to fail. If any one of those stresses exceeds one of these things, then the material is going to fail.

So, mathematically what would what does that mean? That for no failure the following conditions have to hold true. Sigma L should be less than sigma LU, sigma T should be less than sigma TU, and tau L T should be less than tau L T U. All of them have to hold simultaneously.

Now, there could be a case that sigma L is negative, then in that case the condition would be sigma L should be less than sigma LU prime. And if say tau sigma T is less than negative, then sigma T should be less than sigma T U prime. So, it depends if sigma is compressive, then you have to compare with it is compressive strength in the L or in the appropriate direction.

So, these 5 conditions are there, and as long as all these 5 conditions are true, even if one of the condition gets violated then the material is going to fail. So, this is the first theory for failure of composite materials. So, let us do an example.

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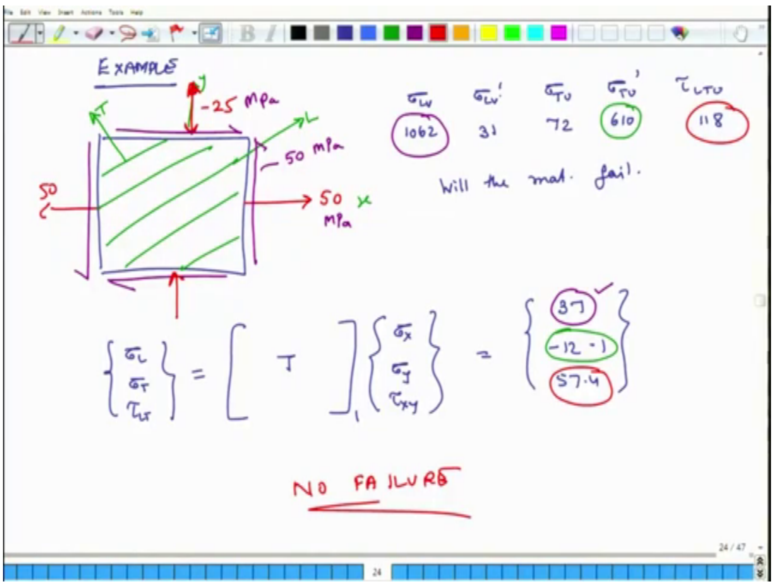

So, the example is we have a lamina, and I am applying some stresses on it. So, what are the stresses? 50 MPa in tension these are my direction of fibres. So, this is my L direction, this is my T direction, this is my x, this is my y. So, in the y direction the stress is negative. So, it is minus 25, and then I have also shear stress applied. And the value of shear stress is 50. So, all these are MPa. So, tensile stress in x direction 50 MPa compressive stress in y direction 25 MPa shear stress 50 MPa, and the material properties are also given.

So, let us see what the material properties are, sigma LU sigma LU prime, sigma TU sigma TU prime and tau L TU. So, this is all given. So, what are their values this is 1062, this is 31, this is 72, this is 610, and this is 118. So, these are all in MPa so, the question is will the material fail there is a question.

So, what do we do first? In we compute sigma L sigma T and tau L T.

So, we compute sigma L, sigma T tau L T, and what is this? This is nothing but that transformation matrix T 1 times sigma x, sigma y tau x y. And we know how to calculate this transformation matrix, we know it is. So, we do all these calculations and we find that these values are 37 minus 12 and 57.4. So, this is these are the values. So, sigma L is equal to 37 sigma T is minus 12.1 and the shear stress tau L T is 57. So now, we compare whether any of these values is exceeding it is limit.

So, first we see 37, and we compare it with 1062. So, it is not exceeding. So, this is the second number is 12.1, but it is negative it is 12.1, but it is negative. So, we compare minus 12.1 with 610, and again it is not failing. The third one is 57.4 and we compared with the shear strength of the material, and again it is not exceeding. So, all the conditions are met so, no failure, there would not be any failure. So, this is the way to do this. So, this is how the maximum stress theory works.

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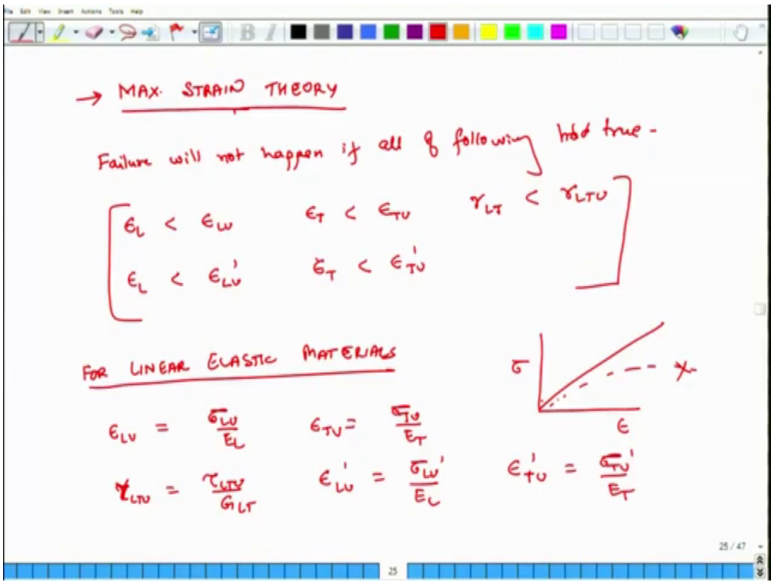

The next theory we are going to look at is maximum strain theory. So, in the stress theory we had 5 fundamental properties related to the, these are the 5 properties which are important. And here we are saying that the material will fail at a certain stress state. In strain theories we replace these 5 material properties by similar material properties, but strain related. So, ultimate tensile strain in the L direction will be epsilon LU ultimate compressive strain in L direction will be epsilon LU prime and so on and so forth. The maximum strain theory says, that the failure will not happen, if all of following hold true.

What are the following conditions? Epsilon should less be epsilon LU, epsilon T should be less than epsilon TU, and gamma L T should be less than gamma L TU. And suppose instead of tensile stress a tensile strain I have a compressive strain, then we look at epsilon L should be less than epsilon LU prime, and epsilon T should be less than epsilon TU prime. So, these are the 5 conditions ok. So, these conditions as long as they are valid the material does not fail if any one of them gets violated they fail right away.

Now, we will connect this theory with the maximum stress theory for linear elastic materials. For linear elastic material what does that mean? What that means is that, if I keep on pulling the material, if my I double the stress the strain also doubles. So, in that case if I have a stress strain response.

So, here I am having a strain, and here I am having a stress, that it is a straight line, it is not a line something like this, this is not the response. If it is this kind of a line, then I can say that epsilon LU ha epsilon LU is what it is equal to sigma LU by EL, only if the line is straight, if it is not straight, then I cannot have this. Similarly, epsilon TU is equal to sigma TU divided by ET, right? Tau L TU is oh gamma L TU is equal to tau L TU divided by g L T epsilon LU prime is equal to sigma LU prime divided by EL, and epsilon TU prime is equal to sigma TU prime divided by ET.

So, for linear elastic materials, the prediction from maximum stress theory and maximum strain theory are very close to each other. There is slight difference because of effects of poison ratios, but otherwise they are fairly close to each other, maximum stress theory and maximum strain theory give you the more or less the same results. But if the material is not linearly elastic, for instance, the matrix is plastic and it just deforms heavily before failing suppose. or if the fibres initially they may be elastic, but later say suppose they turn plastic before failing. Then, the material will not be linearly elastic, in that case, we cannot use these types of relations, we can still use these relations.

But these relations second set of relations cannot be used. So, for linear elastic materials, the predictions from maximum stress theory and maximum strain theory they come pretty close to each other. So, this is the second theory which has been talked about. And the final theory is known as which we will discuss today. And this is the most popular one is known as maximum work theory.

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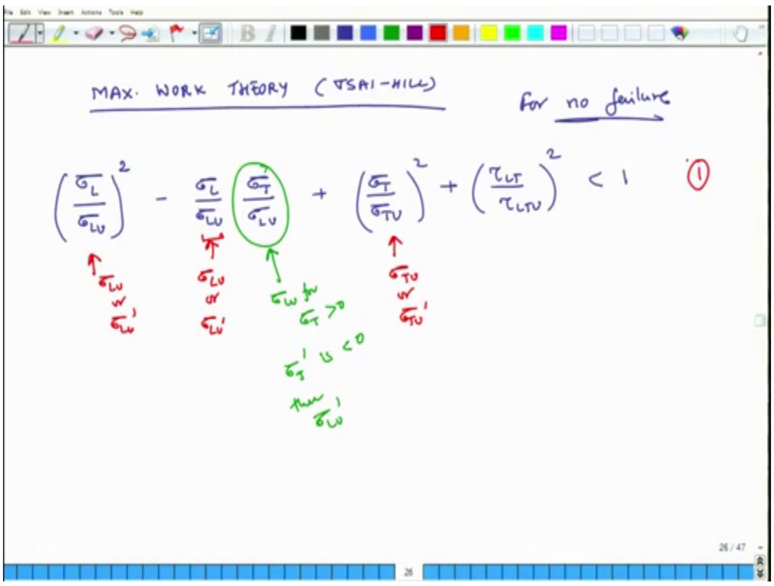

And it is also known as Sai Hill theory.

So, here it is related to the work done by the system strain energy and things like that, and we are just directly going to share you with the, tell you the results. So, what are what does the result say? It says that the system is going to fail, if the right-hand side of this equation exceeds one, if it does not exceed one then it will not fail. So, what is this sigma L divided by sigma LU square minus, sigma L divided by sigma LU times sigma T.

And remember here the denominator is still sigma LU, no TU I am sorry. Sigma LU plus sigma T divided by sigma TU whole square, plus tau L T divided by tau L T U, whole square should be less than 1 so, for no failure. So, here a good thing about this is that you do not have to have 5 different conditions. One equation will tell you whether the thing is going to fail or not, right? In other maximum stress and maximum theories, you have 5 different equations, and you to make sure that each of them is not being violated.

Now, here I wanted to make some things clear. Here if sigma L is negative then we have to consider if sigma L is compressive then we have to consider sigma LU prime. So, we have to modify this thing. Similarly, if sigma T is compressive then we have to consider sigma TU prime. So, this would be sigma LU or sigma LU prime. This could be sigma TU or sigma TU prime.

And same thing here, sigma LU or sigma LU prime. And there is a special situation in this case. What is that? That if sigma T is positive, then we consider sigma LU, and if sigma T is negative, then we use sigma LU prime this is important to understand, otherwise we may get confused and we may use wrong numbers.

So, based on this we can calculate whether the right hand or the left-hand side of the system is less than 1 or more than one. If it equals one or if it is more than one then there will be failure. If it is less than 1 then we are.

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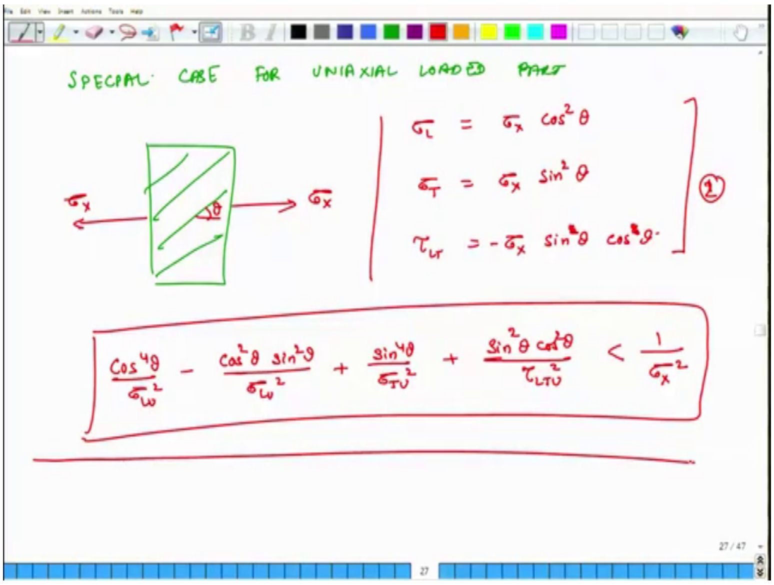

Now a special case could be for uniaxial specimen, what does that mean? That it could be something like this. So, here I am having loading just in the x direction, sigma x sigma y tau x y suppose they are all 0.

So, this is theta. Then if this is the case, then I can say for this kind of a situation, I can say that sigma L again using our matrix of T 1 sigma L is equal to sigma x, cosine square theta, sigma T is sigma x, sine square theta and tau L T is equal to sigma x sine square theta cosine square theta. I am sorry, it is just sine theta and cosine theta, I am sorry.

So, if if we are applying only sigma x, then these are the values of sigma L sigma T L tau L T. And if that is the case, then I can put let us say this is equation number 2, and let us call this equation number 1. Then I can put 2 in 1, and what I get is cosine 4 theta by sigma LU prime sigma LU square minus cosine square theta sine square theta by sigma LU square, plus sin 4 theta divided by sigma TU square, plus sine square theta cosine square theta by tau L TU square is less than equal to 1 over sigma x square. So, this is the equation for a uniaxially loaded specimen.

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So, sigma x sigma y tau x y are 0, right?

Student: Yes.

So, it is minus of that thing, yes. So, this is negative of it, but then we it gets squared. So, it gets corrected, ok. So, this is the maximum ah, I mean the sai hill criteria for predicting failure. So, what we will do is, we will again check on this example you know this example and see whether the sai hill criteria tells whether this thing is going to fail or not. So, we will again redo this example.

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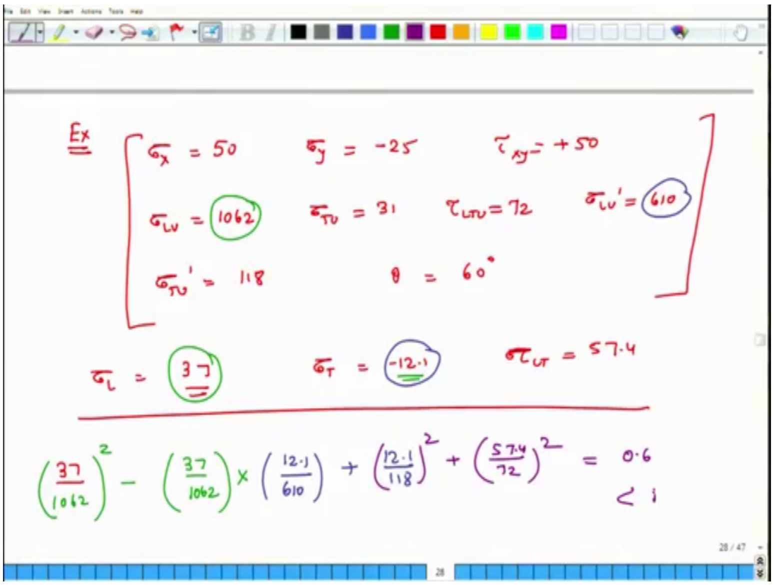

So, sigma x in this case is 50 sigma y in this case is minus 25 tau x y in this case is positive, and it is important to remember the signs of stresses and that is something we will discuss later also. So, minus plus 50, and then finally, sigma LU equals 1062 sigma TU equals 31. Tau L TU equals 72, sigma LU prime equals 610 and sigma TU prime equals 118.

So, we have already calculated all the values. So, sigma L we calculated earlier, and theta we had defined was 60 degrees. So, for this sigma L we calculated as 37, sigma T as minus 12.1 and tau L T is 57.4. Now with this condition let us apply the sai hill criteria.

So, here we have 37 divided by this 37. So, this is and this has to be divided by LU 1062, the whole thing square minus 37 divided by 1062 times the second thing. So, what is the second term? Sigma T divided by sigma LU, now here so, sigma T is minus 12, sigma T is minus 12.

So, of course, we just take the number we do not use the so it is 12.1. And we divide it by because this is compressive so, we consider 610. So, this is 610, and then of course, the third term is 12.1 divided by 118. Because the third term says sigma T divided by sigma TU. So, sigma T is negative 12.1.

So, we consider sigma TU prime which is 118. So, this is square and then finally, we have the shear thing 57.4 divided by 72. And this works out to be 0.6 which is less than 1 so; the material is safe and will not fail.

So, this concludes our discussion for today. We will close this discussion tomorrow by discussing some more finer aspects of this theory, and specifically we will discuss the direction of shear stresses in this context. And that will close the discussion for this week and also the theme of this week's topics. So, with that wish you the best of evening, look forward to seeing you tomorrow, bye.