Introduction to Composites
Prof. Nachiketa Tiwari
Department of Mechanical Engineering
Indian Institute of Technology, Kanpur

Lecture – 57
Stress Strain Relations for A Lamina With Arbitrary Orientation Part I

Hello, welcome to introduction to composite MOOC course today is the third day of the ongoing week which is the tenth week of this course and yesterday we had introduced a new topic.

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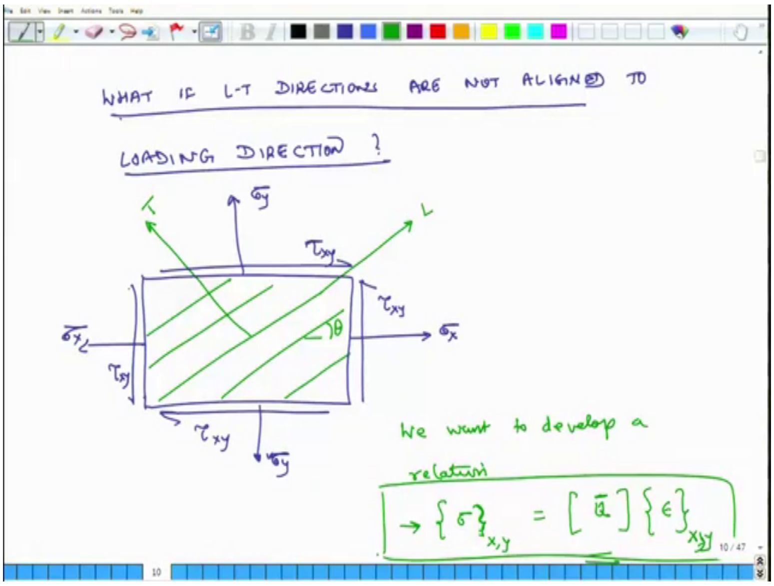

And the topic was that what kind of relationship exists between the stress vector which is sigma x sigma y and tau x y and the strain vector referred to x with respect to x and directions and what is the nature of the Q bar matrix. So, this is the relation we want to develop and we want to somehow relate this Q bar matrix to the Q matrix because we can very easily calculate the Q matrix Q 1 1 in terms of engineering constants.

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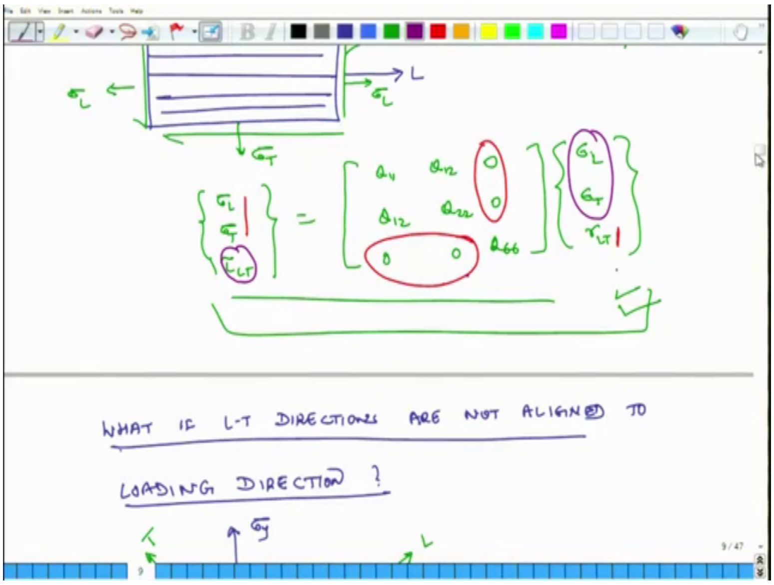

So, is there a way we can calculate Q bar in terms of Q because if we can do that calculation, then I can also calculate this Q bar matrix in terms of all the engineering constants. So, this is what we want to accomplish and to start with let us look at.

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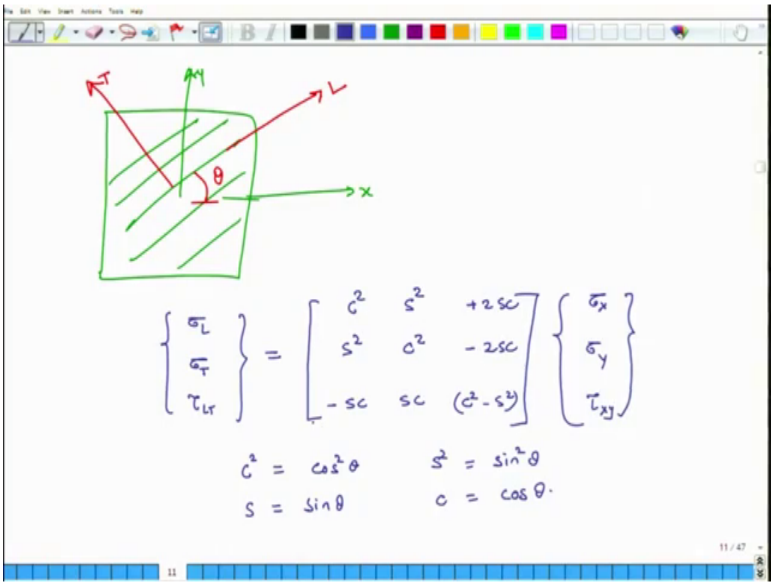

So, we know that suppose this is my x direction this is the y direction and suppose the fibers are like this, then this is my L direction and this is the T direction and the angle between L and x axis is theta degrees. Then based on principles of mechanics which you may have which you have studied in your previous classes I can transform sigma x, sigma y, sigma z sigma x, sigma y, sigma T or tau L T to sigma L in terms of tau L tau T and tau L T using this relation. So, I can write that sigma L, sigma T and the shear stress is nothing, but.

So, these relationships have been developed earlier, in your earlier classes. So, we will not capture that again. So, this is equal to a 3 by 3 matrix multiplied by this stress vector sigma x, sigma y and tau x y and the elements of this matrix are c square, s square my plus 2 s c s square c square minus 2 s c then I have s c s c and c square minus s square and what is c square. So, c square means cosine square theta, s square means sin square theta and same thing s means sin theta and c means cosine theta so if I.

So, the point is if I know sigma x, sigma y tau x y then I can calculate sigma L tau sigma L sigma T and tau L T if theta is known I can calculate these things.

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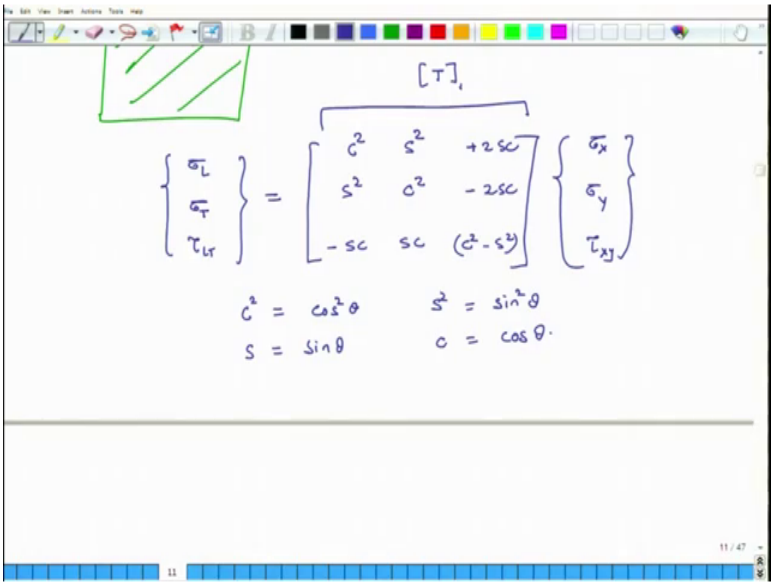

So, we will abbreviate this matrix and we will call it a matrix T 1 just a symbol. So, that we do not have to write these 3 by 3 relations again and again.

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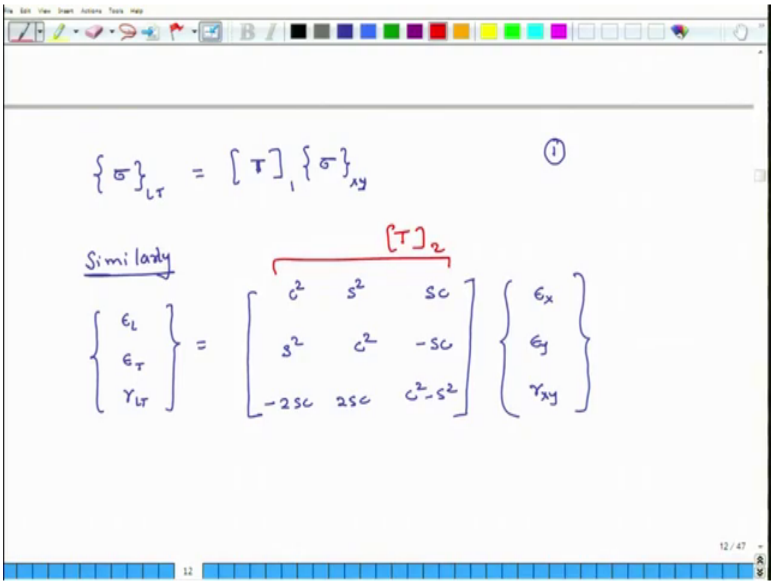

So, in brief I can write sigma when measured with respect to L T axis is equal to a transformation matrix T 1 times the sigma matrix when measured in x y coordinate system ok. So, this is equation 1, similarly there is a transformation matrix for strains also and the we will since we are if we used tensor strains then the transformation for from L T to x y is identical, but because in our regular practice we tend to use engineering strains and in engineering strain gamma x y is twice of the tensor strain.

So, in because of that reason the transformation matrix for strain is slightly different. So, we will write that relation also. So, the so if I am if I have a strains epsilon L epsilon T and gamma L T then these strains can be expressed in terms of a transformation matrix, times epsilon x, epsilon y and gamma x y ok. And what are the elements of this transformation matrix c square, s square, s c minus s c c square minus s square c square s square minus 2 s c, 2 s c. So, this is the and this I call it as T 2, T 2.

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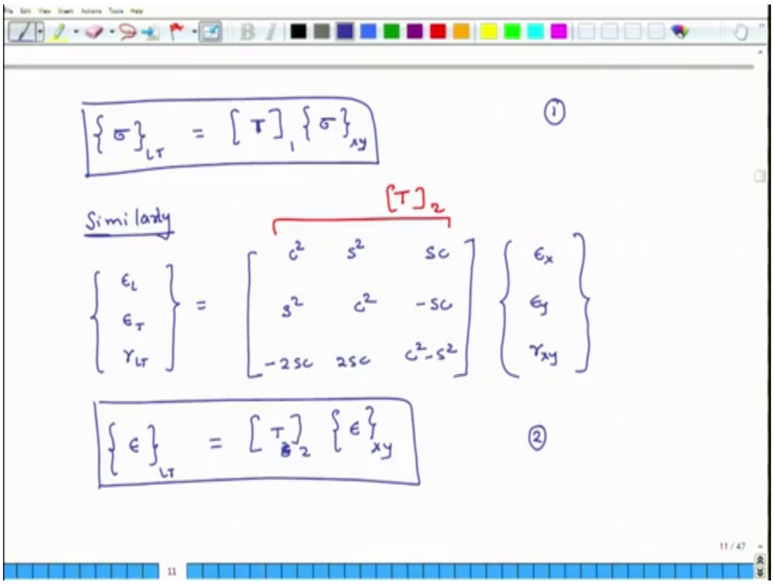

So, in brief I can write this as strain measured with respect to L T coordinate system equals a transformation matrix which is T 2, excuse me times a strain vector measured with respect to x y coordinate system. So, this is one equation and this is other equation ok, now we are going to merge these equations. So, this is equation number one and this is equation number 2 ok.

So, from one we can say if I multiply the first equation by inverse of T 1 on both sides then on the right hand side T inverse of T 1 minus T, inverse of T 1 times T 1 they just become an identity matrix, so it goes away.

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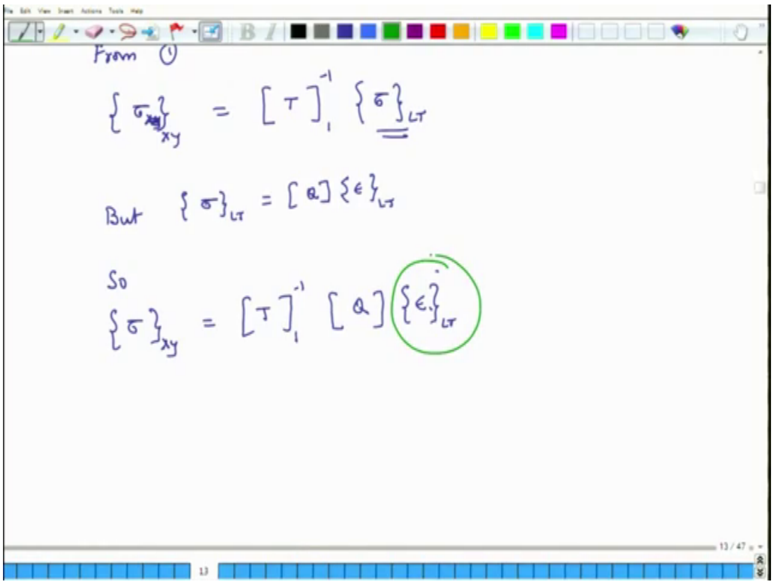

So, what I can say is that is from 1 sigma x y that is stresses as measured with respect to x and y axis are nothing, but inverse of T 1 times sigma L T ok, but we know that sigma L T is what we are shown several, I mean the several times now that sigma L T is equal to this Q matrix times epsilon L T right. So, we will do this substitution. So, we know that sigma when we are measuring with respect to L T coordinate system is what the Q matrix times the epsilon matrix measured in L T system. So, we can say sigma as measured with respect to x y frame equals T 1, inverse of T 1 times Q, times epsilon as measured in L T coordinate system ok.

Now, what we do is we replace this epsilon L T with equation 2. So, here I am expressing epsilon L T in terms of epsilon x y.

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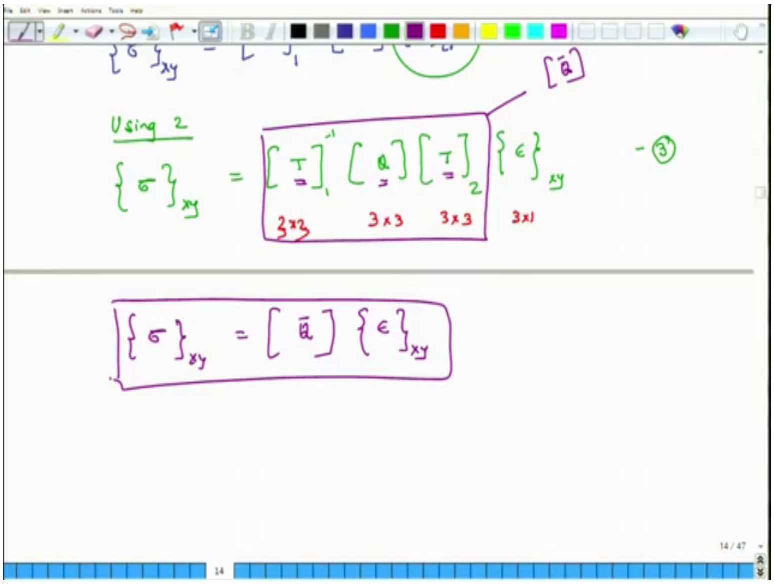

So, using 2, using 2 we get sigma x y equals T inverse 1 times Q, T inverse no T . So, epsilon L T is what T 2 times epsilon x y ok. So, this is there. So, this is equation 3 we finally, note that this is a 2 by 2 matrix oh I am sorry 3 by 3, this is a 3 by 3 this is a 3 by 3 and this is a 3 by 1. So, I can combine all these 3 3 by 3 matrices into 1 and I call this combo as Q bar ok, I know how to calculate T inverse of 1 I know how to calculate T 2 I already know how to calculate Q, from engineering constant. So, I so, if other material properties are known and if the direction of x axis with respect to L axis is known then I know all the elements of all these 3 matrixes and I can calculate the Q bar matrix.

So, I can say that sigma in the x y coordinate system is nothing but a Q bar matrix times strain as measured in x y system and what.

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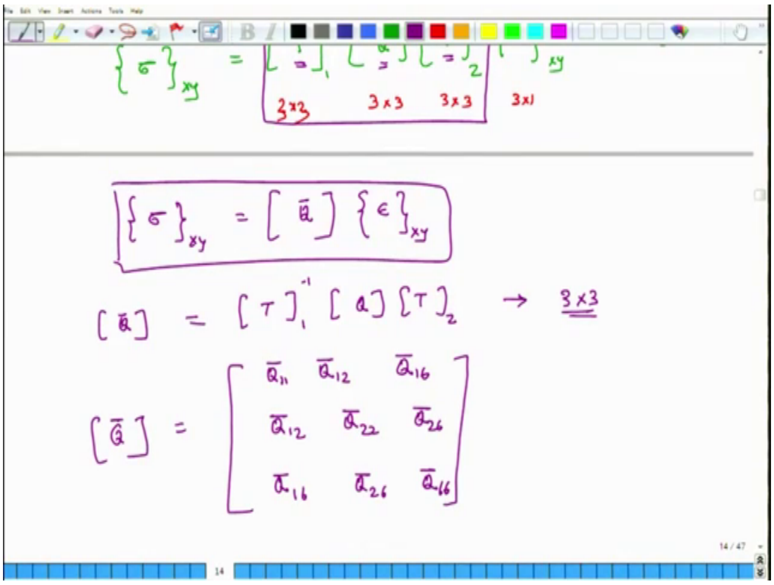

What is Q bar, to explicitly define Q bar matrix. So, Q bar and Q are not same, Q bar is for a generally orthotropic lamina Q is for a especially orthotropic lamina, but both are in a state of plane stress. So, Q bar is for a generally orthotropic thin lamina in a state of plane stress and this is equal to T inverse of T 1, times Q times T. So, inverse of T 1 times Q, times T 2.

So, this is again a 3 by 3 matrix, but it is a little different in terms of when we compare it to Q, Q matrix. So, this is a 3 by 3 matrix and if I want to express it in for using it all its elements it is a fully populated matrix. So, the Q matrix if you remember is not fully populated, why? Because these 2 elements are 0 and these 2 elements are 0 and what; that means, is that the extensional stresses and the shear strains are not coupled and similarly the shear stress is not coupled with extensional strains because these terms are 0.

The Q matrix is not fully populated, but the Q bar matrix in general is fully populated and we will write down their numbers. So, it is Q 1 1, Q 1 2 and the last element is called Q 1 6 it is just a convention. So, we do not have to worry too much about it last element is called Q 1 6, second row Q 1 2 Q 2 2 and Q 2 6 and Q 1 6, Q 2 6 and Q 6 6 and this is a Q bar matrix. So, every element has a bar on it. So, this is very important otherwise it will create confusion. So, it is a fully populated matrix, Q 1 1 bar, Q 1 2 bar, Q 1 6 bar, Q

1 2 bar, Q 2 2 bar, Q 2 6 bar, Q 1 6 bar, Q 2 6 bar and Q 6 6 bar. So, if we use this matrix to connect the stresses and strains then we have this relation.

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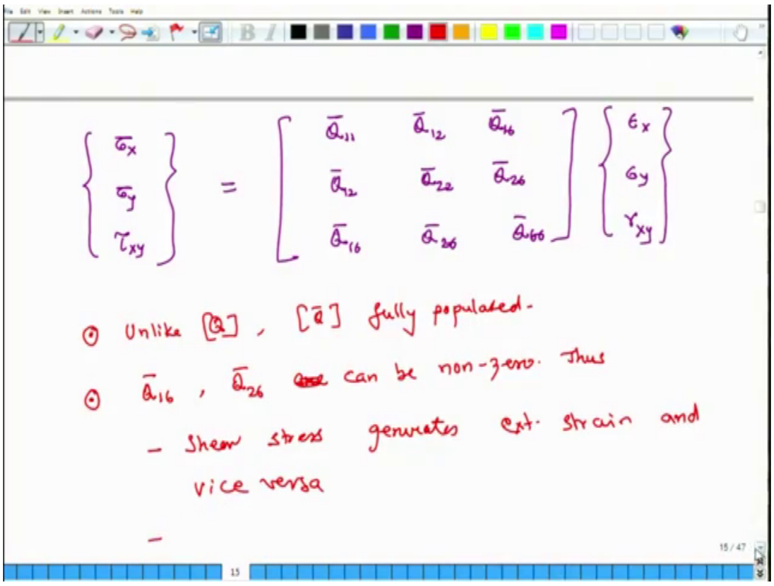

Sigma x, sigma y, tau x y equals Q 1 1 bar, Q 1 2 bar, Q 1 6 bar, Q 1 Q 2 6 bar, Q 2 2 bar and Q 2 6 bar oh I am sorry the first element is Q 1 2 bar. Because it is a symmetric matrix and that last line is Q 1 6 bar, Q 2 6 bar and Q 6 6 bar, times thus appropriate strains. So, appropriate strains are epsilon x, epsilon y and the shear strain gamma x y ok.

So, couple of observations unlike Q, Q bar fully populated, unlike you Q bar is fully populated ok, Q 1 6 and Q 2 6 are, I will not say always can be non 0 and because they are non 0 and. So, we can make 2 observations thus shear stress generates extensional strain and vice versa.

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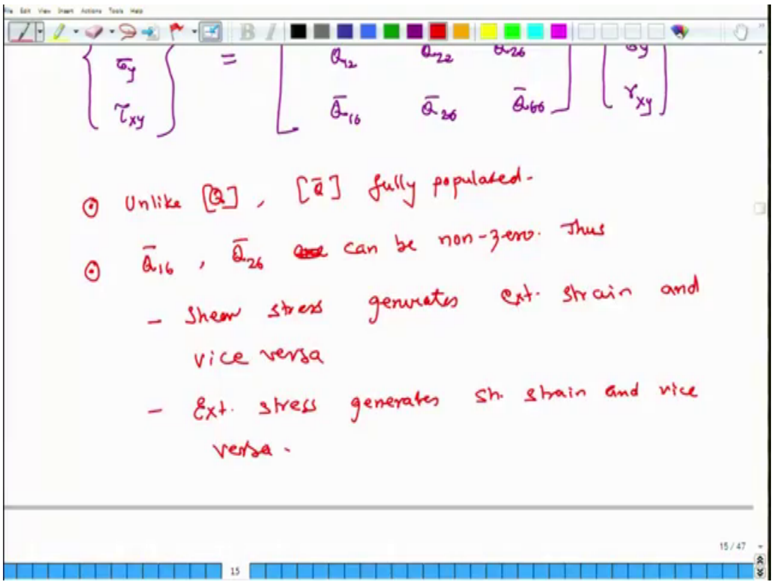

And further extensional stress generates shear strain and vice versa. So, this is the situation for generally orthotropic system, but even here the number of independent engineering constants is still 4 because Q 1 1 bar, Q 1 2, Q 1 6 bar only depend on what. Some sins and cosines which are not related to engineering, but they only depend on 4 independent values of Q bar matrix. So, the Q bar matrix has 6 constants, but at the fundamental they will they only depend on 4 independent engineering constants or stiffness elements. So, this is the overall theory. So, what we will do is we will just for purposes of completeness we will write down the exact relations between different values of Q bar and qs.

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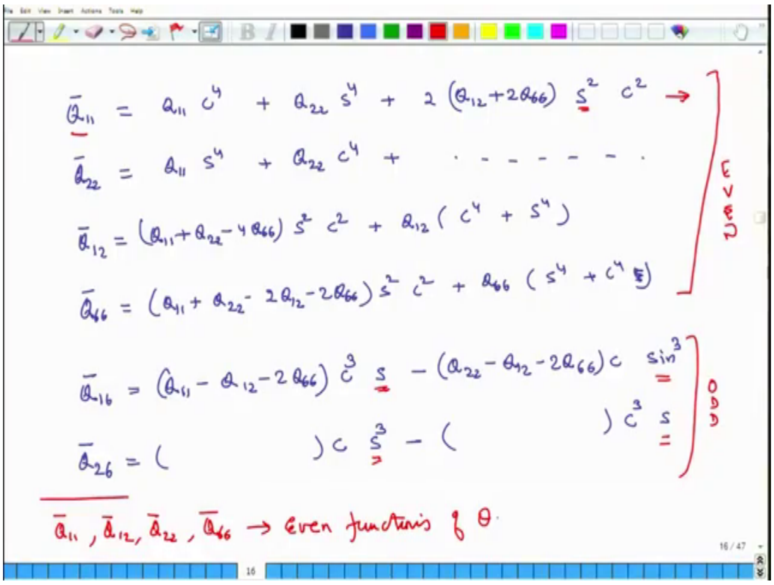

So, we will write those relations Q 1 1 bar equals Q 1 1 times cosine 4 theta plus Q 2 2 times sin 4 theta plus 2 Q 1 2 plus 2 Q 6 6 sin square theta cosine square theta ok. So, if you know Q 1 1 and Q 2 2 and Q 6 6 you can calculate Q 1 1 bar using this relation Q 2 2 bar is equal to Q 1 1 sin 4 theta plus Q 2 2 cosine 4 theta plus this term is the same. Then Q 1 2 bar equals Q 1 1 plus Q 2 2 minus 4 Q 6 6 sin square theta cosine square theta plus Q 1 2 times cosine 4 theta plus sin 4 theta Q 6 6 bar equals.

So, these are complicated relations, but if you know the mathematics then you can compute them and you can even create a small code which will compute all the qs for your composite. So, Q 1 1 plus Q 2 2 minus 2 Q 1 2 minus 2 Q 6 6 sin square theta cosine square theta plus Q 6 6 sine 4 theta plus cosine 4 theta ok. Q 1 6 bar equals Q 1 1 minus Q 1 2 minus 2 Q 6 6 and this is cosine cube theta times sin theta minus Q 2 2 minus Q 1 2 minus 2 Q 6 6 cosine theta times sin cube theta and Q 2 2 6 bar equals the same thing, but it is just the other way around.

So, it is cosine theta times sin cube minus this entire things times cosine cube times sin theta. So, we will make some more observations based on these relations, what we see here is Q 1 1 is an even function of theta. What is an even function, that if I make theta as negative theta cosine of negative theta and cosine of c theta is same, sin of positive theta is equal to negative sin of theta, but this is sin to the power of 4 and sin to the power of 2. So, Q 1 1, Q 1 2, Q 2 2 and Q 6 6 are even functions of theta that is if I make theta from let us say 30 degrees to minus 30 degrees the values of the first 4 Q bars will not change. However, Q 1 6 bar and Q 2 6 bar they are odd functions of theta.

So, if I make theta from 30 degrees to negative 30 degrees we should otherwise our calculations will be wrong see that the value of Q 1 6 becomes negative of its original value. So, these are bars ye. So, these are odd functions and these are even functions ok, these are even functions.

So, this is the conclusion of our discussion for Q matrix, similarly we can also calculate elements of the S matrix, but at least for today’s class this would suffice we will continue this discussion tomorrow also and till then you have a wonderful evening and we will meet once again tomorrow.

Thank you.