Introduction to Composites

Prof. Nachiketa Tiwari
Department of Mechanical Engineering
Indian Institute of Technology, Kanpur

Lecture – 55
Relation Betweeen Engineering Constants and Elements of Stiffness and Compliance Matrices- Part I

Hello welcome to introduction to composites today is the tenth week of this course and the first day of this particular week. Over last week we had been discussing the hooks law for an isotropic materials and we had discussed and we had shown using concept of tensors. That in general an anisotropic solid would have 21 engineering constants, stiffness constants and if we are able to make the structure simpler then for a especially orthotropic lamina a unidirectional lamina this number of constants comes down to 5.

And then finally, we had also shown that if the I the solid if the material is having special or therapy and also if the state of stress is plane then the number of constant it comes down to 4 and for isotropic materials the number of elastic constants independent elastic constant it comes down to exactly 2. So, that is where we left in the last class and then we had also developed concepts like the Q matrix, the S matrix which is the compliance matrix and what we will do today and in the remaining portion of this week are 3 specific things.

One we will develop these relationships between Q S and engineering constants for a unidirectional laminar under plane stress, this is number one. The second thing is that we will also find out how do stresses and strains relate if a unidirectional lamina under plane stress is loaded in such a way that the material axis and the loading axis are not necessarily aligned. So, that is the second thing we will do.

And then the third thing we will try to accomplish over this week is we will have a detailed discussion of failure of unidirectional lamina under plane stress and we will also describe several failure theories which are relevant to composite materials. So, that is pretty much the agenda for this entire week and what we will do today is we will start by having a very quick recap of what we discussed in the last week and then we will extend that discussion further.

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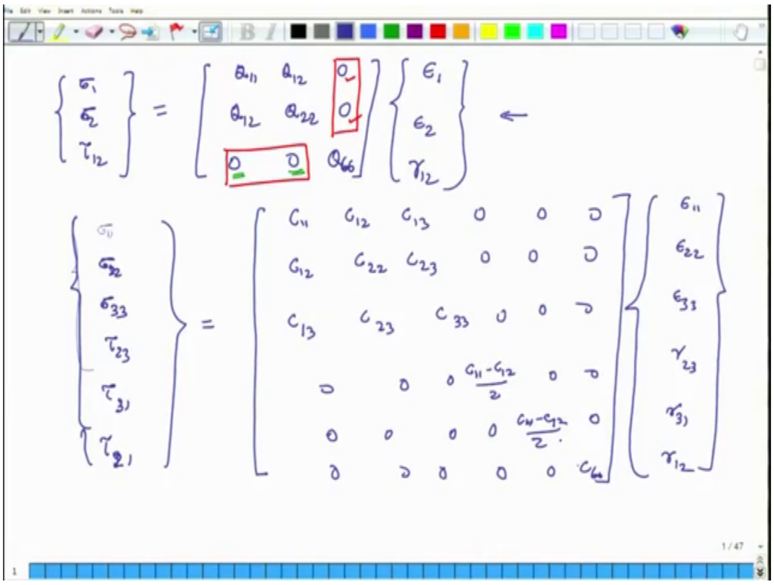

So, what we had shown was that if there is a unidirectional lamina under plane stress then the stresses sigma 1, sigma 2 and tau 1 2 they are related to the strains, the strains in the system through a Q matrix stiffness matrix. So, the stiffness matrix is something like this Q 1, 1 Q 1, 2 0, Q 1 2 Q 2 2, 0, Q 6 6, 0, 0 and on the strain side we have epsilon 1, epsilon 2 and gamma 1 2. So, these are 4 independent elastic constants and it should be noted that when there is especially orthotropic lamina under plane stress, a thin orthotropic lamina then if I have a shear stress or if I have a shear strain it will not generate any extensional stresses and if I have extensional stresses there they are not coupled to shear stresses.

So, extensional strains are not coupled to shear stress and shear strain is not coupled to extensional stress because these terms, these terms are 0. So, this term and this term ensure by the fact these 2 terms, these 2 terms ensure that there is no coupling between the extensional stress and the shear strain and these terms which are underlined in green since these are 0, they ensure that there is no coupling between the shear stress and the extensional strains.

So, that is there we had also shown the relationship between Q 1 1, Q 1 2, Q 2 tw2o and Q 6 6 and the elements of the c matrix which was defined earlier in the following way. So, the system which is under 3 d stress state the relation is something like this, sigma 1 1, sigma 2 2, sigma 3 3, tau 2 3, tau 3 1, tau 3 2, 2 2 1. So, this is my stress vector and this is coupled to the strain vector through a 6 by 6 square matrix the stiffness matrix. So, the strain terms are epsilon 1 1 epsilon 2, epsilon 3, 3 3, 2 2 epsilon excuse me.

So, this is gamma 2 3 gamma 3 1 and gamma 1 2 and the terms in the c are c 1 1, c 1 2, c 1 3 and then the other terms are 0, c 1 2, c 2 2, c 2 3, 0, 0, 0, c 1 3, c 2 3, c 3 3, 0 0 0 and then here we have c 1 1 minus c 1 2 divided by 2. Then c 0, 0, 0, 0, 0 and then we have the same term here and then we have c 6 6. So, that is how this matrix is constructed ok. So, just to clarify this is 0 and the fifth term, in the fifth row which is the diagonal term is again c 1 1 minus c 1 2 divided by 2 and just to be consistent I am going to make this a little clearer. So, because these equations are not tensor equations these are simple matrix equations.

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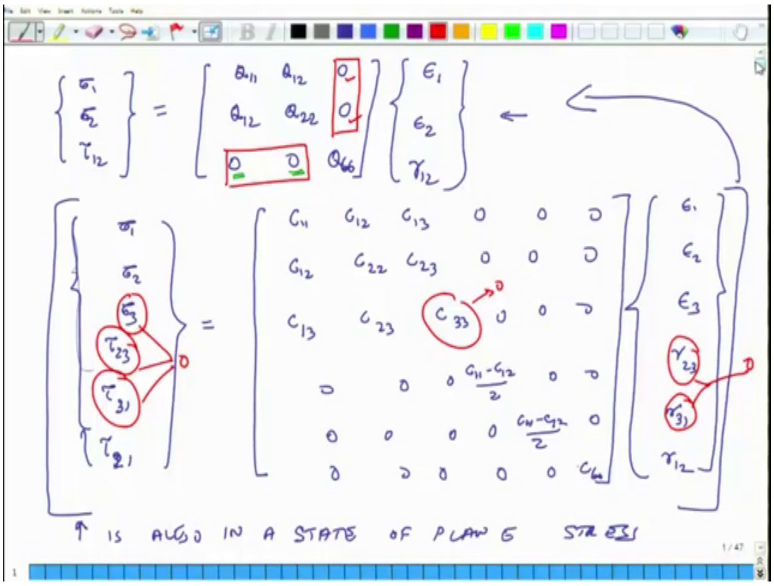

So, I will just call it sigma 1, sigma 2, sigma 3 and this is epsilon 1, epsilon 2, epsilon 3.

So, this is the situation, these 6 equations are applicable for especially orthotropic material which is along 2 planes of symmetry.

And if this material is also in a state in a state of plain stress then this equation, this set of equations this converges to the first set of equations because these terms sigma 3 3 is 0, in a plane stress situation tau 3 1 is 0 and tau 3 2 is 0. So, these are 0 in plane stress and because it is 0 in plane stress we can also say that gamma 3 1 and gamma 2 3 these guys are 0 and also c 3 3 0. So, when you do all this then you end up with 4 engineering concepts 4 stiffness constants which are Q 1 1, Q 1 2, Q 2 2 and Q 6 6.

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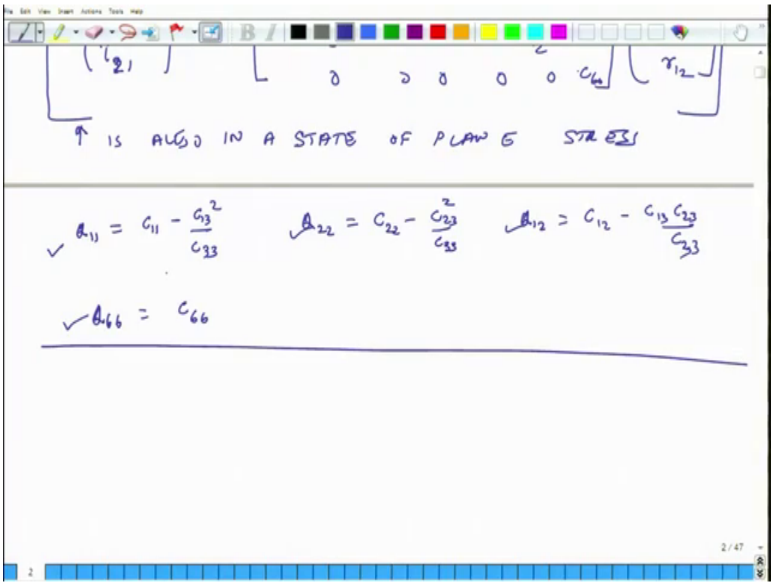

So, if we do all this math then we can show and we had shown earlier is that Q 1 1 equals c 1 1 minus c 1 3 square by c 3 3. So, actually I made a small. So, this is not 0, but epsilon 3 is 0. So, this is Q 1 and then Q 2 2 equals c 2 2 minus c 2 3 by c 3 3 the numerator square and Q 1 2 is equal to c 1 2 minus c 1 3 times c 2 3 divided by c 2 3 is c 3 3 and Q 6 6 is equal to c 6 6. So, these are the relations between the qs and the cs and what we see that for a plane stress what really matters are these 4 terms and they are. So, basically we have 4 fundamental elastic constants.

So, the next thing we look at is the compliance matrix.

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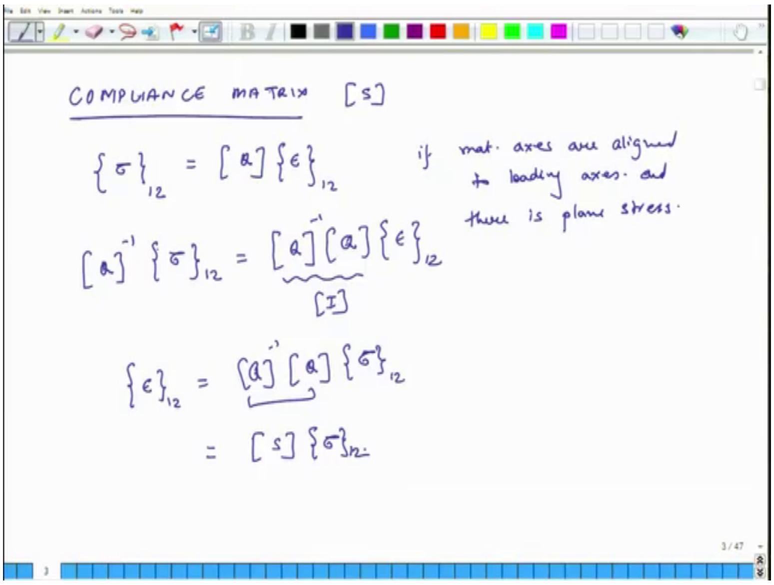

So, the stiffness matrix was designated as Q compliance matrix is designated as s. So, this is a little strange because S we one would think would stand for stiffness, but it is for Q. So, compliance matrix is designated by S and what is the relation between stress and strain if we use the compliance matrix. So, we know that sigma, if I use the you know sigma sigma vector if I use the 1 2 plane this is equal to Q matrix times the strain vector this is if material axes are aligned to loading axes. So, this is and there is plane stress ok. So, if material axes are aligned to loading axes and if there is plane stress then this is the relation which is there and this we have defined what is the nature of qs here we have shown this ok.

So, now if I take if we multiply both sides by inverse of Q then what I get is. So, what I have done is I have multiplied both sides by inverse of q, and Q inverse times Q is an identity matrix, unity matrix. So, this is a unity matrix and if I multiply this by epsilon 1 2 I just end up with epsilon 1 2. So, I get the strain I can express it as Q inverse times Q times sigma 1 2 ok. So, this I the product of these 2, I call it s, which is the compliance matrix times sigma 1 2 ok so, if I know the stresses in a unidirectional lamina which is extremely thin and if it is in a plane stress trace if I know the stresses.

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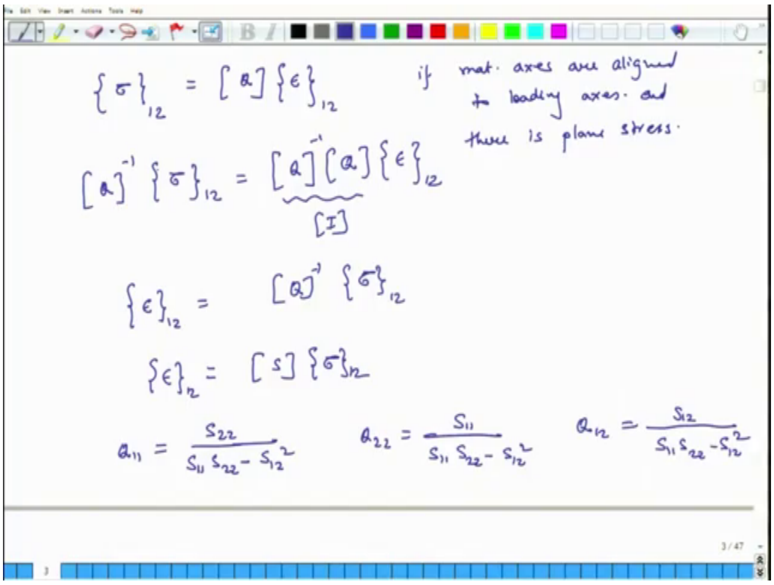

Then if I multiply the stiffness matrix by with the stress vector then I can compute the strains in all the 3 directions. So, what we will do is we will spell out the relations between the S matrix and the Q matrix. So, basically it is just Q inverse times Q and if I do all that mathematics what we find is that Q 1 1 is equal to S 2 2 divided by S 1 1, S 2 2 minus S 1 2 square and then Q 2 2 equals S 1 1 divided by S 1 1 S 2 2 minus S 1 2 square and Q 1 2 equals S 1 2 divided by S 1 one S 2 2 minus S 1 2 square.

Student: Sir you told Q inverse.

So, this should not be there right. So, I will just ye. So, there is an error. So, epsilon 1 2 is equal to Q inverse times sigma 1 2 and finally, we can say that Q 6 6 is equal to 1 over S 6 6.

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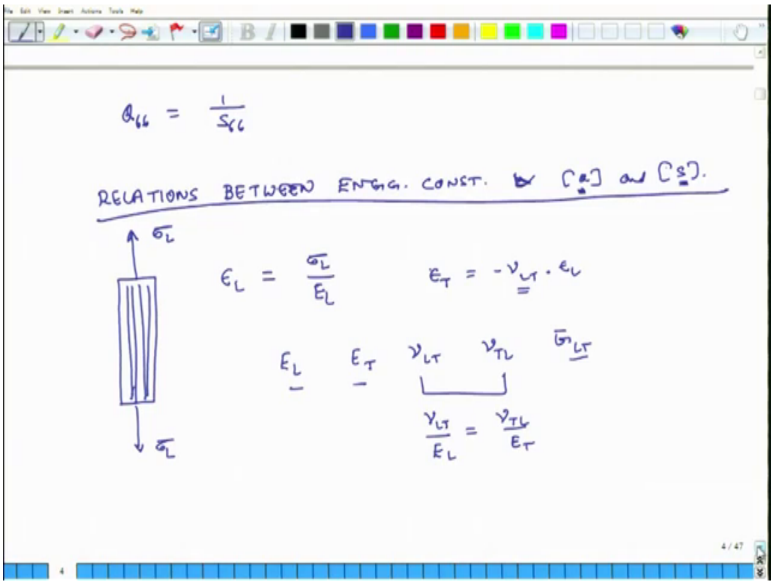

So, these are the relationships between Q and the S matrices Q is the stiffness matrix and S is the compliance matrix.

Lastly what we will do is we will look at the relations relations between engineering constants and Q and s. So, suppose I have a unidirectional specimen and the fibbers are in this direction and I just apply sigma L to it hm, if I just apply sigma L to it then what will happen it will also so it will have. So, in this case it will generate what, it will generate epsilon L and that epsilon L will be equal to sigma L divided by E L ok.

So, if I measure the strain in this material using some strain gauge and if I measure the stress and if I divide stress by strain I can calculate E L also there will be a transverse strain because of poisons effect. So, this is epsilon T and that will be only because of poisons effect. So, it will be minus of poison T L times epsilon L hm, it will be L T ok. So, you can calculate in this way the poisons ratio and also the E l, similarly if you test this specimen in the transverse direction you can calculate it E T and nu T L and so on and so forth.

And then if you test the specimen by applying only a pure shear load then you can calculate its g L t. So, the point is by making some simple engineering experiments we can experimentally find out E L, E T, nu L T, nu T L, and g L T and these 2 constants are related both of them are not independent of each other because we have shown that nu L T divided by E L equals nu T L divided by E t. So, ultimately there are only 4 independent constants. So, the question we are trying to figure out is what is the relationship between 4 independent constants in the Q matrix and these 4 engineering constants and what is the relationship between 4 independent engineering constants; in the S matrix which is the compliance matrix and these 4 engineering constants.

So, the basic thought process we have already explained. So, what I will do is I will just directly state the results.

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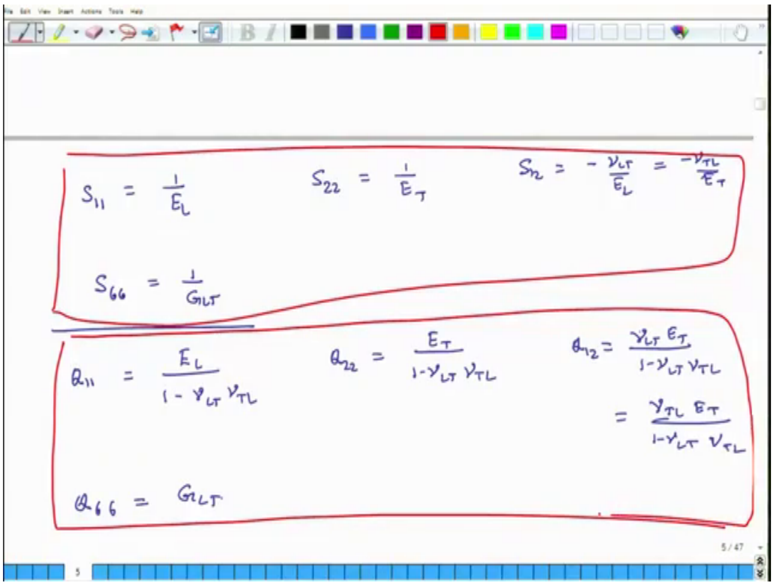

So, I will say that S 1 1 equals 1 over E L, S 2 2 equals one over E T and S 1 2 equals minus nu L T over E L or you can also state it as minus nu T L over E T and finally, S 6 6 is equal to 1 over g L t. So, this is how if you know the engineering constants we can very quickly find out the elements of the compliance matrix which is the S matrix using these relations.

Similarly, if you know the engineering constants then you can also directly calculate the members of Q matrix and this is Q 1 1 equals E L divided by 1 minus nu L T nu T L, Q 2 2 equals E T divided by nu L T, nu T L and Q 1 2 equals nu L T E T divided by 1 minus nu L T, nu T L. And you can also write it as nu T L E T divided by 1 minus nu L T, nu T L and finally, Q 6 6 equals g L t. So, so we have 3 sets of equations the first set of equations they relate q's to S s right. The second set of equations they relate S to E L you know engineering constants and the third set of equations they relate members of the stiffness matrix Q matrix to the engineering constants.

So, we have all sorts of relationships. So, if we have knowledge of even one set of constants then from that we can calculate all other constants by using these simple relations.

So, this concludes our discussion for today, what we will do in the next class is we will have a very brief continuation of the same discussion and then we will move on to transformation of these engineering constants if the loading direction is not aligned to the material axes of the system. So, with that conclusive remark I close our discussion for today and I am sure that you will have a wonderful evening and I look forward to seeing you tomorrow.

Thank you.