Introduction to Composites

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Lecture – 47
Transformation of Engineering Constants- Part I

Hello, welcome to Introduction to Composites. Today is the fifth day of this ongoing week and over this week we have started discussing the stress strain relationship for especially orthotropic lamina and in that context what we have seen is that there are four fundamental elastic constants mutually independent elastic constants and now, in the last class we discussed the stress strain relationships for a generally orthotropic lamina and there the number of elastic constants which we need to know are larger in number and today, what we will start discussing is the relationship between E L, E T, nu LT, G LT, these are the four constants for especially orthotropic lamina.

So, we will radiate these to the elastic constants for a generally orthotropic lamina. So, that is what we plan to do. So, first we will depth, so, so we will do this step by step.

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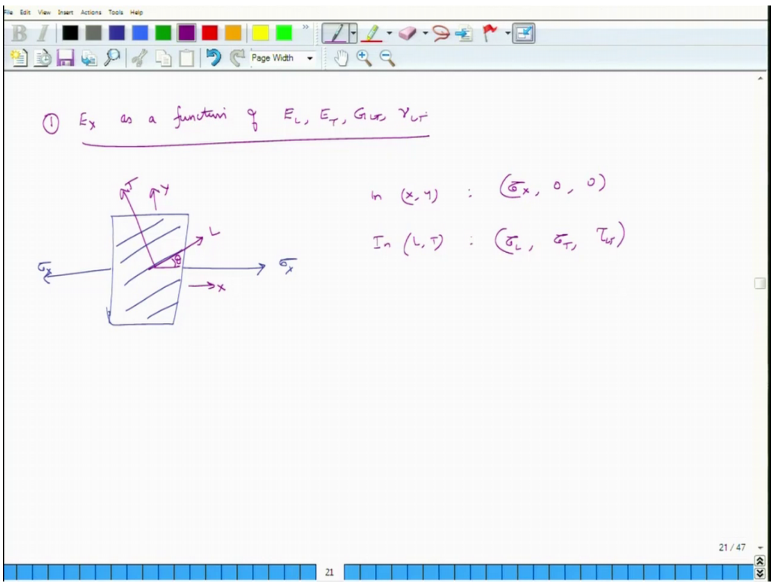

So, we will develop a relationship between E X as a function of E L, E T, G LT and nu LT. So, this is the first thing similarly we will develop a relationship for E Y, m X, m Y, G XY, nu XY, nu YX. So, we will do all this.

So, how do we do this? So, we will, what we will do is we will analyze lamina which is in a generally orthotropic case. So, here we are applying sigma x we are not applying any sigma y and neither we are applying any shear stress. So, this is my L direction this is my T direction and this is the value of theta. So, when I apply this stress sigma x, I can transform this sigma x in terms of in context of L, T Cartesian frame right.

So, in, so, this is my x axis system this is my y axis right this is my x and this is my y. So, in x, y frame, what are the stresses? Sigma x, 0 and 0 there is no stress in the y direction. In L, T frame, what are the stresses? They will be sigma L sigma T and tau LT. So, we have to transform sigma x in the L, T frame.

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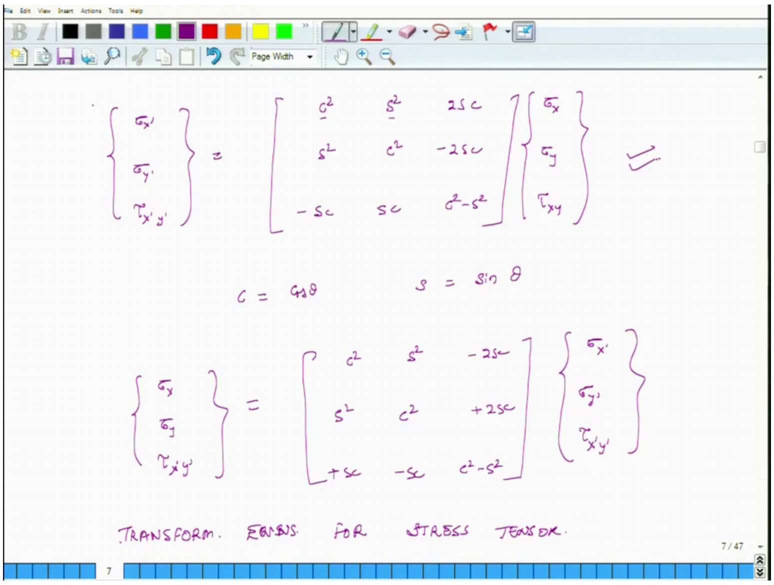

And, we will use the transformation equations which we had developed earlier I mean using these stress transformation relationships ok.

So, in this matrix sigma x is sigma x, sigma y is 0, tau xy is 0, sigma x prime means sigma L, sigma y prime means sigma T and tau x prime y prime means tau LT and this these numbers will be cosine square theta, sine square theta 2 sin cosine theta and so, on and so, forth. So, using this, these transformation relations we will express, we will find the relationship between sigma L, sigma T, tau LT and sigma x.

So, directly using those relations we get sigma L, sigma T and tau LT is equal to sigma x cosine square theta sine square theta and minus sine theta cosine theta, this is what we get all other terms come out to be 0, because sigma y is 0 and tau x y is 0. So, let us call these equations – A.

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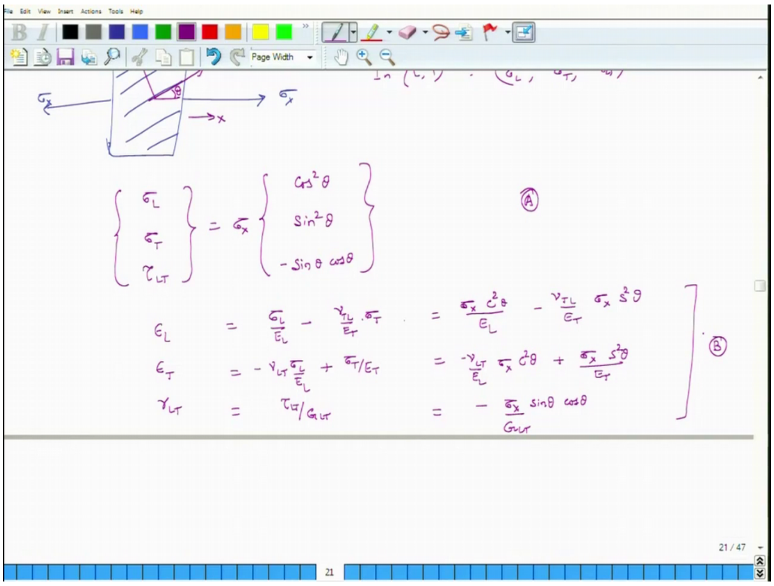

Now, we know the stress strain relationship between sigma L, sigma T and tau LT right. So, if I have to develop if I have to calculate epsilon L, epsilon T, and gamma LT, then what is this? It is sigma L over E L minus nu TL over E T times sigma T and this is equal to sigma T over E T and here we have the Poisson effect minus nu LT sigma L over E L and here we have, so, this is plus and here we have tau LT divided by G LT. So, these are the relations we have already developed and what is sigma L? sigma L is sigma x cosine square theta, ok.

So, this I can write these relations as sigma x cosine square theta divided by E L because sigma L is what sigma x times cosine square theta minus nu TL divided by E T and sigma T is sigma x sine square theta. So, sigma x sine square theta and then there is also the third component will be 0, because there is a no component of sigma tau LT the transverse strain epsilon T is what sigma x sin square this is minus nu LT divided by E L times sigma L and what is sigma L? sigma x cos square theta plus next term sigma T over E T and what is sigma T? sigma x sine square theta, so, it is sigma x sine square theta divided by E T.

And, finally, we have tau LT is now gamma LT is tau LT divided by G LT. So, what is tau LT? tau LT is as per this relation minus sigma x sine theta cos theta, so, it is minus sigma x sine theta cosine theta divided by G LT. So, this is another set of equations we call that B. So, what we have done now is we have expressed epsilon L, epsilon T, gamma LT in terms of sigma x which is the external stress applied on the laminate in x, y coordinate system, right.

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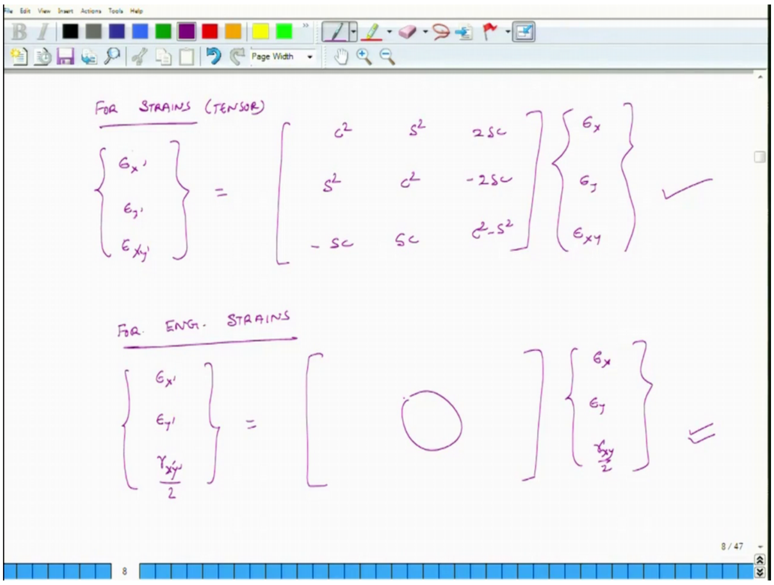

Now, so, this is equation B, we also know from our stress transformation equations these are the stress, I am sorry strain transformation equations or if we are talking about engineering strains, these are the transformation equations. From this set of equations, we can express epsilon L, epsilon T and gamma LT in terms of epsilon x, epsilon y and gamma xy in this case we are going from L, T coordinate system to x, y coordinate system, so, theta is minus theta is minus theta.

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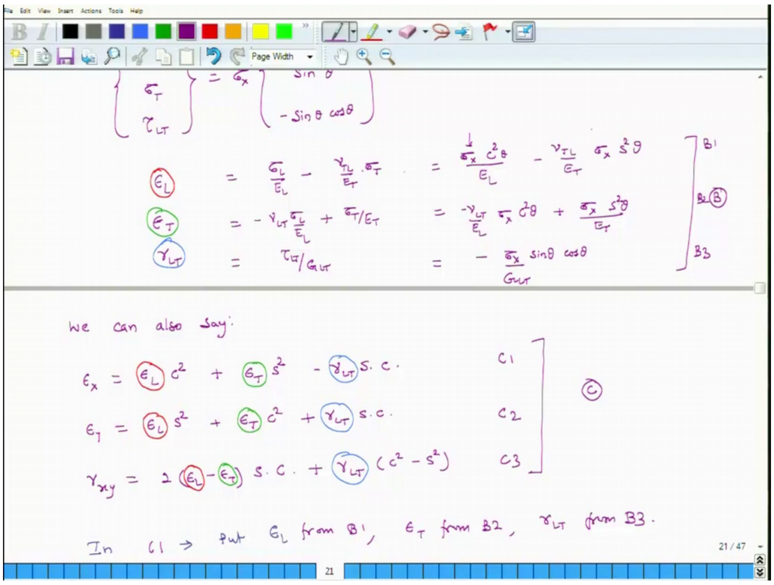

So, if we put the value of theta is minus theta in this matrix then I can say, so, I will write three more equations. So, so I can also, so, we can also say, what can we say? We can say that so, we are going from L, T coordinate system to x, y coordinate system. So, epsilon x is equal to epsilon L times cosine square theta plus epsilon T times sine square theta minus gamma LT times sin theta cosine theta and this comes directly from the strain transformation equations when we are dealing with engineering strains.

Similarly, epsilon y equals epsilon L sine square theta plus transverse strain times cosine square theta and plus gamma LT sine theta cosine theta and then the engineering shear strain gamma xy gamma xy is twice of epsilon L minus epsilon T times sine theta cosine theta plus gamma LT times cosine square theta minus sine square theta. So, this is equation another set of equations we call it equation – C.

Now, when we look at equation set B and set C on the right hand side of C we have epsilon L, epsilon L and epsilon L ok. Similarly, we have epsilon T, epsilon T and epsilon T and then we have the shear strain gamma LT and these guys also appear on the left side of set B. So, this was epsilon T, so, this I will put it in green and this one is light blue. So, what I can do here is I can put; so, let us say this is B 1, this is B 2 and this is B 3.

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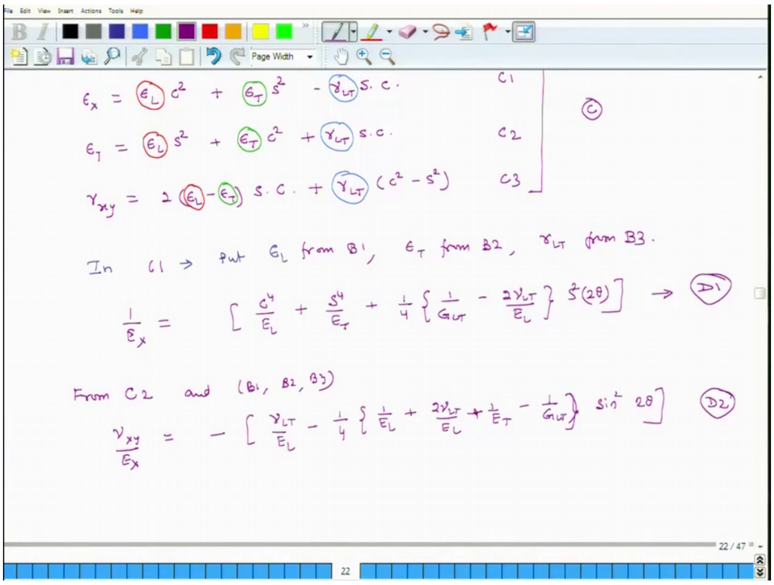

B 1, B 2, B 3. So, we will just work on B 1. So, in equation B 1, put epsilon L from C 1 no oh I am sorry this should not be B 1, B 2, B 3 it should be C 1, C 2, C 3. So, this is C 1 equation is C 2 and C 3 and this is equation B 1, B 2 and B 3. So, in equation C 1 put epsilon L from B 1, epsilon T from B 2 and gamma LT from what gamma LT from B 3.

So, once we do that what we will get is we will get a long expression which will involve only epsilon x, it will involve only epsilon x because it will involve on the, so, it will involve on the left side it will involve epsilon x and on the right side it will only have sigma x right, it will have only sigma x. So, we will get from that epsilon x is equal to sigma x times cosine 4 theta by E L plus sine 4 theta over E T plus 1 over 4 divided by 1 over G LT minus 2 nu LT divided by E L times sine square of 2 theta this long thing. So, this is from C 1.

From C 2 from C 2 we get. So, before we do C 2 and now what we do is we just divide both sides by sigma x. So, sigma x goes away from here and here I get epsilon x by sigma x and what is epsilon x by sigma x? This is nothing, but 1 over E x. So, this I will erase and I will call it 1 over E x. So, this is my definition for 1 over E x, D 1, ok.

Next, we have from C 2; from C 2 and of course, and B 1, B 2, B 3 we do the same thing you know in equation C 2 we plug the values of epsilon L, epsilon T and gamma LT and we get this thing epsilon y equals minus sigma x gamma LT over E L minus 1 over 4, 1 by E L plus 2 nu LT by E L minus plus 1 over E T minus 1 over G LT times sine square 2 theta ok.

So, this is the stress this is the stress in the system in y direction caused due to stress in the x axis on the x axis, left side tells us that it is the strain and y direction, right side is the source term which is sigma x. So, sigma x is causing some Poisson's contraction also and that is why we have a negative sign here right. So, if I divide both sides by E x essentially what I get is I can erase both these sides and what I get from here is nu xy divided by E x, if I divide both sides ha if I divide both sides by sigma x sigma x goes away from right side on the left side I am left with epsilon y divided by sigma x and that I can express it as this thing. So, this is equal to this. So, now I have a definition for Poisson's ratio x, y.

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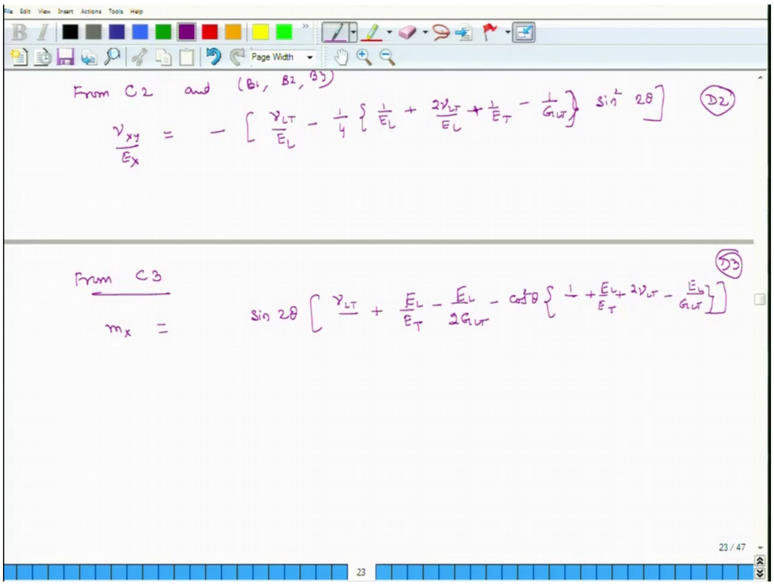

So, we do the same thing and from C 3, what do we get? We get gamma xy is equal to minus sigma x sine 2 theta time times this whole long term nu LT divided by E L plus 1 over E T minus 1 over 2 G LT minus cosine square theta, in brackets 1 over E L plus 1 over E T plus 2 nu LT divided by E L minus 1 over G LT, this is there, gamma xy.

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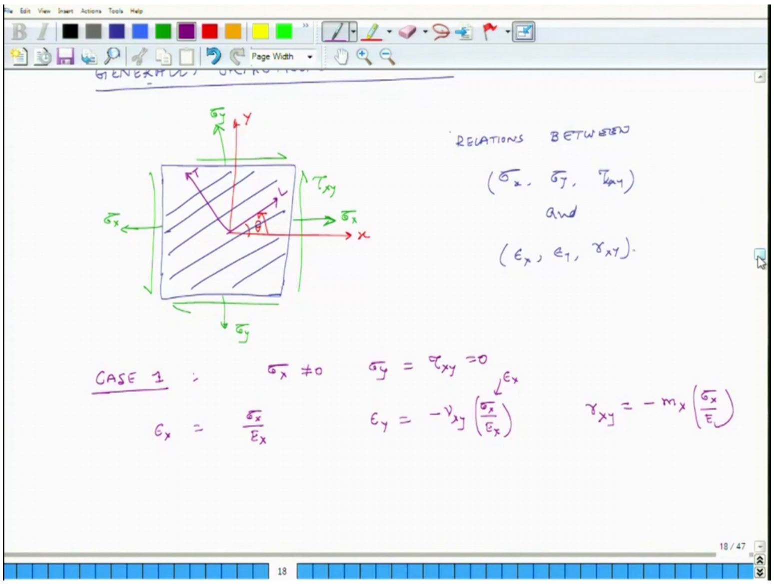

And, if you see the definition, so, if you apply a generally orthotropic and generally orthotropic case gamma xy is equal to minus m x times sigma x by E L. So, if you use this definition then from this C 3 you get m x. So, what is m x? So, you again divide both sides by sigma x and what you end up with is m x, m x is equal to sine theta and then E L is already here. So, E L goes away and E L comes, so, everywhere the whole thing gets multiplied by E L.

So, here it goes away and I have in the numerator, in the denominator it goes away and 2 E L. So, this is 1 over E L over G LT or this one is redundant just adds confusion. So, I will write it as E L over G LT. So, from C 1, C 2, C 3 we get E x relations for E x, nu xy and m x which is the cross coefficient, so, this is D 3.

We can do similar things and we will continue this discussion tomorrow and what we will show is that using similar approach we can also develop a relationship between for E y and m y. E y, m y and nu yx and then finally, we will develop an expression for the shear stress modulus or the shear modulus G xy. So, that concludes our discussion for today. We will continue this discussion tomorrow, until then have a great day. Bye.