Introduction to Composites
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Lecture – 44
Stress Transformation (Two Dimensional)

Hello. Welcome to introduction to composites course. Today is the second day of the ongoing week which is the 8h week of this course. Yesterday, we got a brief idea as to what are tensors and we had discussed 0 order tensors which we call them scalars we have talked about first order tensors which we call also which are also known as vectors and then general tensors which could be second order fourth order and so on and so forth.

What we will do today is we will look at equations which help us transform second order tensors from one coordinate system; one set of one Cartesian coordinate system to another coordinate system and then we will use this knowledge in context of 2 different types of tensors one is strains and the other one is stress.

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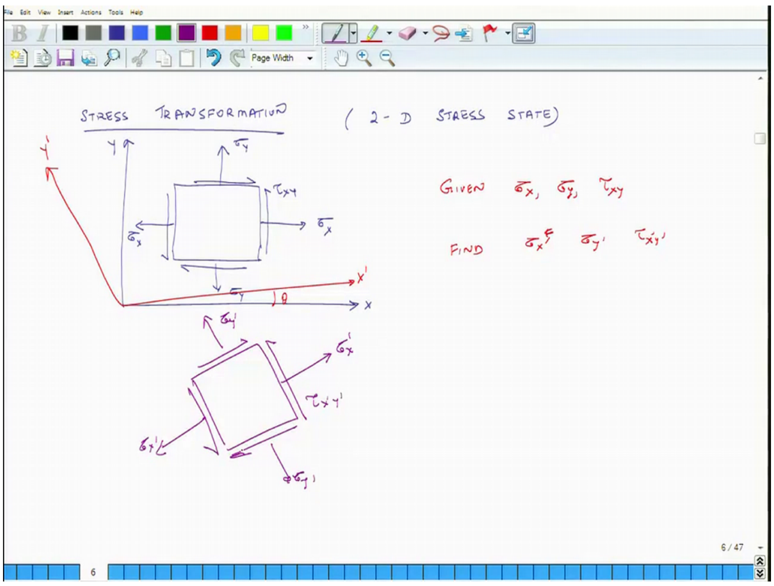

So, here is the problem. So, so first thing we will discuss is stress transformation. Now, you may have already seen this in your earlier classes, but for purposes of this course, it is important that we revisit these relations. So, that there is no confusion. So, suppose. So, I have a rectangular block or rectangular let us say plate. So, this is a rectangular plate. So, it is thin and it is seeing. So, this discussion will be for 2 D stress; stress state because in context of composite laminate individual layers the thickness is very small. So, we do not have to worry about the third direction.

So, 2 D stress state. So, this is a thin lamina or thin layer, it could be metal or composite unidirectional lamina or whatever it does not matter and it is having a 2 dimensional state of stresses. So, this is my x axis and this is the y axis. So, I am applying an external stress, here I call this sigma xx or for purposes of simplicity I just call it sigma x because that is self explanatory. So, I will just call it sigma x. So, it is the normal stress in the x direction is strictly speaking I should use sigma xx first index indicates the direction of the force and the second index indicates the direction of the surface on which it is being applied.

Then in the y direction it is being up exerted upon by another stress sigma y and then I am also applying a shear stress on this. So, I call this tau x y. So, this is tau x y. So, this. So, the value is sigma x sigma y and tau x y normal stress in x direction is sigma x normal stress in y direction is sigma y and the shear stress with respect to the x y coordinate system is sigma x y.

Now, suppose instead of using this coordinate system I want to use another coordinate system which is indicated in red and this coordinate system is nothing, but the same old coordinate system, but just rotated by an angle theta. So, the question is given sigma x sigma y tau x y find sigma x prime sigma y prime and tau x y prime x prime y prime tau x prime y prime this is what we are interested in meaning in figuring out.

So, what does that mean what; that means is that here the rectangular block the top surface and the bottom surface were aligned with the x axis and the vertical surfaces were aligned with the y axis, but now what I am interested in finding out is the stress in the system. So, suppose instead of this block I had another; the same block, but I cut it in a different direction right and this is my x direction. So, sigma this is sigma x this is sigma; sigma x prime sigma y prime and this is tau x prime y prime this is there.

So, in this case we can say. So, this is something we have already done in our solid the strength of materials courses. So, we will just recap those results. So, a sigma x prime sigma y prime and tau x prime y prime can be expressed in terms of sigma x sigma y sigma tau x y using this relation.

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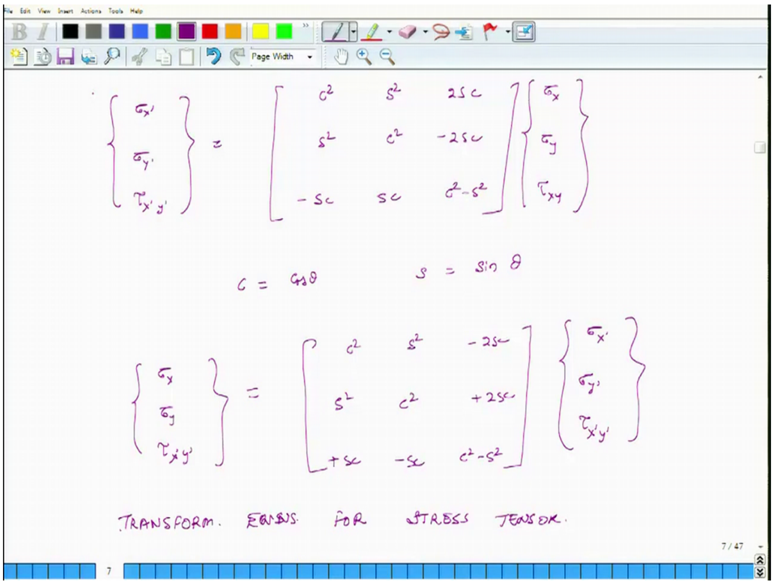

So, this is c square s square 2 times s c s square c square minus 2 times sc minus sc; sc and c square minus s square and I will explain what is cns in a moment.

But these are the transformation equations for stress tensor where c represents cosine of theta and s represents sin of theta. So, c represents sin of cosine theta and s is sin of theta. So, if I know sigma x sigma y tau x y and I know the value of theta then I can transform from one coordinate Cartesian coordinate system to another coordinate system which is rotated by an angle of theta in the anti clockwise direction.

So, positive theta is anti clockwise direction this is positive theta this is positive theta this is important to remember and vice versa if I have to transform sigma x prime sigma y prime tau x y prime into sigma x sigma y tau x y then what do I do; all I have to do is what theta becomes negative of theta right theta becomes negative of theta. So, the transformation matrix; so, this is sigma x prime sigma y prime tau x prime y prime and this is c square s square 2 s c becomes minus 2 s c because theta becomes minus of theta cosine theta cosine minus theta it does not change, but sin becomes negative and then this is minus plus 2 s c c square minus s square c square minus sc s square and plus sc ok.

So, these are the transformation equations for. So, these are transformation equations for stress tensor. So, these equations help us transform stress from one coordinate system to another coordinate system as long as these coordinate systems are Cartesian in nature.

So, next we will look at strains. So, in mathematics it does not matter when you transform one second order tensor from one coordinate system Cartesian system to another system the transformation matrix is the same.

So, the equations for transforming strains are also same.

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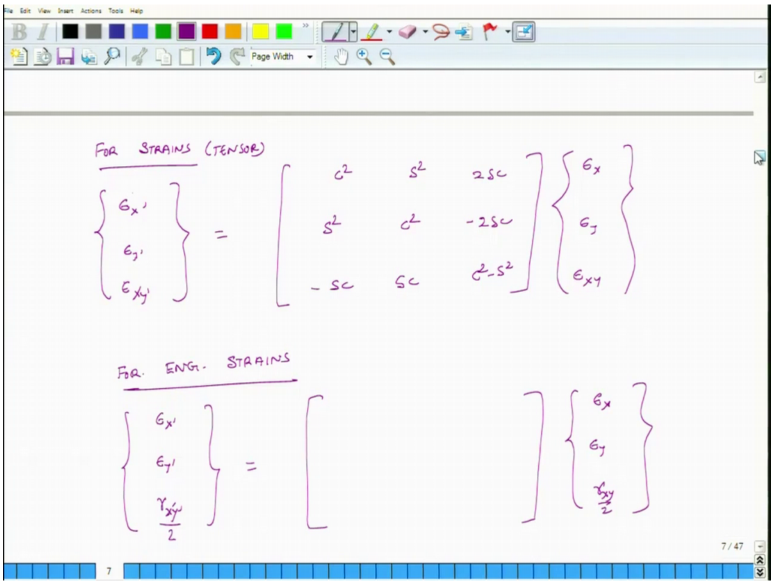

So, again; so, for strains, the transformation equations do not change. So, we can again write epsilon x epsilon y and epsilon x y. So, these are tensor strains. So, we are not talking about because the strains for the transformation for engineering strain may be different, but for tensor strain the equations are identical.

So, this is again c square s square 2 sc s square c square minus 2 sc minus sc sc c square minus s square epsilon x epsilon y epsilon x y now. So, this this is this this is further strain. So, far and the tensor version, but a lot of times we use engineering strain and engineering shear strains. So, if we have are for engineering shear strain the only thing we have to do is we have to bring in a factor of two. So, epsilon x prime epsilon y prime and gamma x y prime, but then gamma x y x prime y prime is not a tensor.

So, it to express it in tensor value I just divided by 2 and this matrix remains the same and here also it is epsilon x epsilon y gamma x y by 2 and this is what we get. So, this is how we transform the strain tensor or the engineering strain vector and this is how we transform the stress tensor and the transformation matrix remains the same and once again if I replace theta by minus theta then I get relations to transform x prime into x coordinate system and if theta is equal to positive theta then it helps us transform x to x prime coordinate system. So, this concludes our discussion for today tomorrow onwards we will start discussing orthotropic lamina and how if the load axis is not aligned to its material axis how do stress strain equations look like. So, with that we conclude for today and I will meet you once again tomorrow.

Thank you.