

Introduction to Composites
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Lecture – 37
Transverse Modulus of Unidirectional Composites

Hello, welcome to Introduction to Composites. This is the 7th week of this ongoing course and today is the start of this week or last several weeks we have discussed different aspects of composites, specifically in the last week we had started discussing how to predict different properties of unidirectional composite materials?

Specifically we had covered how to predict density of such composites, their longitudinal modulus and also their longitudinal strength. And in the last couple of lectures we had also covered the prediction of transverse modulus of such composites that is unidirectional composites.

Today we will continue that discussion and then subsequent in subsequent part of this week, we will touch upon different other properties of such unidirectional composites, such as transverse strength, shear modulus, Poisson's ratio, thermal conductivities, different transport properties and other important properties.

So, we will start by revisiting the transverse modulus of a unidirectional composite.

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The slide contains the following content:

$$\frac{E_T}{E_m} = \frac{1 + \xi \eta V_f}{1 - \eta V_f}$$

$$\eta = \frac{E_f/E_m - 1}{E_f/E_m + \xi}$$

$$\xi = 2 \left(\frac{r_f}{r_m}\right)$$

EXAMPLE

	GLASS	CARBON	} MATRIX Same
V_f	10%	10%	
V_f	50%	50%	

← $E_T \rightarrow ?$

$E_c = 7000 \text{ MPa}$
 $E_g = 1400 \text{ MPa}$
 $E_m = 70 \text{ MPa}$

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Now what we had stated was that the (Refer Time: 01:41) relation, gives us that the transverse modulus of a unidirectional composite can be defined as is defined through this relation, $1 + \frac{\eta}{\theta} \times \text{volume fraction}$ divided by $1 - \eta \times \text{volume fraction}$. And the results which we get from such a relation are fairly consistent with experimental data.

So, this particular relation gives us a fairly good estimate of the transverse modulus of a unidirectional composite. And here the parameter η is defined as $\frac{E_f}{E_m} - 1$ divided by $\frac{E_f}{E_m} + \zeta$ and we have this parameter ζ is defined as twice of $\frac{a}{b}$. Where a and b are the dimensions of the cross section of the fibers, which are used in the system.

So, I wanted to bring upon some important features of this relation and how each of these parameters volume fraction fiber modulus matrix modulus, they influence the transfer modulus. So, we will do that by looking at an example. So, consider the case that we have 2 types of fibers. So, we have glass fiber and we have carbon fiber. And both these composites have the same matrix material and volume fraction could also be or at 2 levels.

So, the first volume fraction would be 10 percent. So, V_f is 10 percent and the other volume fraction, which we could consider it, could be 50 percent. And the matrix material for both these composites is common. So, it is the same matrix material. So, we are going to make 4 different composites 1 with 10 percent glass fiber, another one with 50 percent glass fiber, third one with carbon fiber 10 percent and 4th 1 is 50 percent carbon fiber or graphite fiber.

And the representative properties of different materials we are going to state is young's modulus of graphite fiber, we say that it is 3500 MPa for glass it is 300 and 50 MPa and the matrix is the softest matrix and that is about 70 MPa.

So, for each of these composites let us see what is the value of E_t ? So, we have to find E_t for all these 4 different composites.

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$$E_f/E_m = 20 \quad (\text{glass}) \qquad E_f/E_m = 100 \quad (\text{graphite})$$

$$\eta = \frac{E_f/E_m - 1}{E_f/E_m + 2} = \frac{20 - 1}{20 + 2} = \frac{19}{22} \qquad \eta = \frac{100 - 1}{100 + 2} = \frac{99}{102}$$

$$V_f = 10\%$$

$$\frac{E_T}{E_m} = \frac{1 + \eta V_f}{1 - \eta V_f} = \frac{1 + 2 \times \frac{19}{22} \times 0.1}{1 - \frac{19}{22} \times 0.1} = 1.28 \quad (\text{glass})$$

$$= \frac{1 + 2 \times \frac{99}{102} \times 0.1}{1 - 2 \times \frac{99}{102} \times 0.1} = 3.28 \quad (\text{graphite})$$

So, we know that E_f/E_m is equal to 20, which is the ratio of fiber. So, this is the 20 for. So, we will address this problem, but before I address I wanted to say that there are some changes, I would like to change some of these numbers. So, this E_c is 7000 MPa and E_g is 1400 MPa ok.

So, for these values first we calculate the ratio of fiber modulus and matrix modulus. So, it is 20 for glass and E_f/E_m is 100 for graphite fibers. So, with that understanding next we compute this parameter η . So, this is equal to $E_f/E_m - 1$ divided by $E_f/E_m + 2$.

So, this is equal to 20 minus 1 divided by 20 plus 2, it gives us 19 over 22 and for the graphite system, this works out to be 100 minus 1 divided by 100 plus 2 that is 99 divided by 102.

So, now what we do is we compute the value of E_T/E_m this parameter and first we do it for volume fraction V_f equals 10 percent. So, for that case E_T/E_m equals $1 + \eta V_f$, divided by $1 - \eta V_f$ and if it is 10 percent for glass this is $1 + 2 \times \frac{19}{22} \times 0.1$ divided by $1 - \frac{19}{22} \times 0.1$ and that works out to be 1.28.

So, this is for glass and if we do the same thing for a graphite system so that is equal to $1 + 2 \times \frac{99}{102} \times 0.1$ divided by $1 - 2 \times \frac{99}{102} \times 0.1$

and that works out to be 3.28. So, this is for graphite this is for graphite. Next we do the same computation for a different volume fraction.

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The image shows a digital whiteboard with handwritten mathematical derivations. At the top, there is a toolbar with various drawing tools. Below the toolbar, the following equations are written:

$$V_f = 50\%$$

$$\frac{E_T}{E_m} = \frac{1 + 2 \times \frac{19}{22} \times 0.5}{1 - \frac{19}{22} \times 0.5} = 1.32 \text{ (glass)}$$

$$\frac{E_T}{E_m} = \frac{1 + 2 \times \frac{99}{102} \times 0.5}{1 - \frac{99}{102} \times 0.5} = 3.83 \text{ (graphite)}$$

The bottom right corner of the whiteboard shows the page number "3 / 47".

So, for V_f is equal to 50 percent E_T over E_m is equal to 1 plus 2 into 19 by 22 into 0.5 divided by 1 minus 19 over 22 into 0.5 and that comes out to be 1.32 for glass system. Finally, for graphite system when the volume fraction is 50 percent this ratio is 1 plus 2 times 99 by 102 times 0.5 divided by 1 minus 99 by 102 into 0.5 and that is equal to 3.83. So, this is graphite.

Now we had done similar calculations for same type of a system earlier also for longitudinal fiber longitudinal modulus. So, what we will do is we will put all these results in a small table and see what we make of it. So, we have 2 volume fractions V_f is equal to 10 percent.

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The image shows a handwritten table comparing the longitudinal modulus ratio (E_L/E_m) and transverse modulus ratio (E_T/E_m) for two fiber matrix systems (Glass and Graphite) at two different volume fractions ($V_f = 10\%$ and $V_f = 50\%$). The table is organized as follows:

	$V_f = 10\%$		$V_f = 50\%$	
	E_L/E_m	E_T/E_m	E_L/E_m	E_T/E_m
GLASS ($E_f/E_m = 20$)	2.9	1.28	10.5	3.28
GRAPHITE ($E_f/E_m = 100$)	10.9	1.32	50.5	3.83

Below the table, there are two handwritten observations:

- Transverse modulus changes marginally with increasing (E_f/E_m)
- Transverse modulus is MODERATELY to changes in V_f .

And the other volume fraction is V_f is equal to 50 percent and we have calculated today the parameter E_T/E_m earlier we had calculated the parameter E_L/E_m .

So, we will list down both these parameters. So, E_L/E_m and E_T/E_m and here again, and we had done it for 2 fiber matrix system 1 was glass and in this case E_f/E_m is 20 and the other 1 was graphite and here E_f/E_m equals 100. So, now, let us write down all the numbers. So, E_L/E_m is 2.9 this ratio is 1.28. Next 1 is 10.5, third 1 is 3 for last 3.28, then we have 10.9, 1.32 50.5 and 3.83.

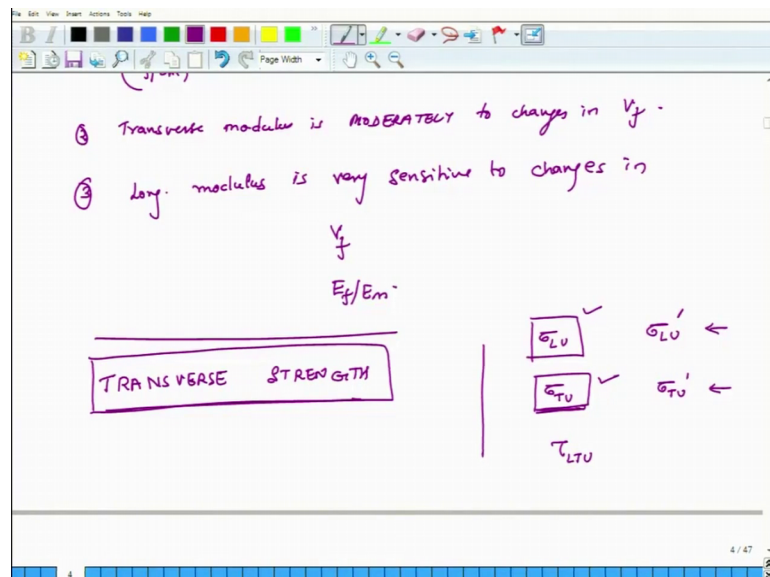
So, this is the overall summary. So, what do we make of it this table, when we look at this table, we observe several important things. First observation is that if the volume fraction is kept to be constant then if I increase my fiber modulus significantly, in somewhat similar terms the longitudinal modulus also goes up, but not much happens to the transverse modulus.

So, the first observation is transverse modulus changes marginally with increasing what do you call E_f/E_m parameter? I mean you have increased it from 20 to 100 and still the transverse modulus did not go up, but the longitudinal modulus unlike it goes up significantly, it goes from 2.9 to 10.9. So, roughly more than between 3 and 4 times it goes up by that much.

The other thing to note is that. So, the next 1 is that look at these numbers. So, this is the story is same here also from 3.2 8 to 3.8 3 here 1.2 8 to 1.3 2 not a significant change, but longitudinal modulus goes up significantly it goes from 2.9 to 10.9 and 10.5 to 50.5.

The second thing is if you look at the in the horizontal direction in this direction. In this direction volume fraction is increasing and as volume fraction increases from 10 percent to 50 percent E_L over E_m , again it goes up jumps from 2.9 to 10.5 transverse modulus also increases, but that increase is not that strong 1.2 to 3.2 8 is maybe something like 2 and a half times increase, but 2.9 to 10.5 is an increment of about factor of 3 to 4.

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So, we say that transverse modulus is moderately sensitive, moderately sensitive to changes in volume fraction of the fiber and then of course. So, these are 2 observations on transverse modulus and the observation on longitudinal modulus is that longitudinal modulus is very sensitive to changes in V_f and also in E_f over E_m ratio.

So, this is important to understand. In other words the transverse modulus is predominantly influenced by the matrix properties, unless the fiber volume fraction is really high 50 percent, 60 percent. It is predominantly influenced by matrix properties longitudinal modulus in on the other hand is strongly influenced by the properties of the fiber it is very strongly.

The contribution from fiber is really high on longitudinal modulus side, but it is not that great from fiber side for the transverse modulus. So, this is important to understand as you go around later in this course and as you try to work on composites and if you are interested in increasing the transverse modulus significantly. And if the composite is primarily unidirectional in nature, then you should not expect significant payoffs by if you choose a very stiff fiber what will really help you is that if you choose a very stiff matrix then that is going to influence your transverse modulus.

So, having said that we will next move to transverse strength so, in our earlier class in 1 of the earlier classes we had decided stated that a unidirectional laminate has to be characterized by 5 different types of strengths. The first strength, which we had stated was longitudinal strength in transition right longitudinal strength in transition and that we had designated by σ_L , meaning longitudinal ultimate strength U and then and this is in transition.

The other parameter which we had said was important is longitudinal strength σ_L U , but in compression and that is designated by this apostrophe. The third important property of unidirectional material related to strength is transverse strength σ_T U . When the material is subjected to tensile forces and the fourth one is σ_T U prime, which is the transverse strength in compression and the last one was shear strength σ_{LTU} .

So, what we will discuss in the next class is σ_T transverse strength and specifically we will be talking about this particular parameter. Later in the week we will also address σ_T U prime and σ_L U prime. Earlier in this course we have talked about σ_L U how to predict it? And now we are going to discuss how to get an estimate on σ_T U .

So, with this I close our discussion for today we will meet again tomorrow and we will discuss how does transverse strength of these unidirectional composites it can be estimated? And how what are the important parameters to which it is sensitive to. So, with that I close our discussion and I look forward to meeting you tomorrow.

Thank you.