Introduction to Composites
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Lecture – 32
Minimum Volume Fraction and Critical Volume Fraction

Hello. Welcome to introduction to composites. Today is the second day of the sixth week of this course. Yesterday, we had defined two different strengths of unidirectional composites when they are loaded in tension in the direction of the fibers. The first scenario was when fibers are in extremely small fractional proportion and when that is the case, then the breakage of the fiber causes transfer of this additional load to matrix this increase in the load in the matrix or stress in matrix is not sufficiently high to cause immediate breakage of matrix. So, I can still load my composites further and eventually the matrix also fails.

So, in this case, the breakage happens first fiber fails then at a higher load the matrix fails and that is when the whole composite fails the other scenario which we had explained was when fibers are in reasonably large amount and when that is the case initially, the fibers break because of the breakage of fibers the load gets transferred to matrix and this transferred load is significantly large and as a result matrix gets over stressed beyond its bearing capacity and that also fails at the same load level. So, these are the 2 scenarios. So, which we had discussed and we had developed 2 relations.

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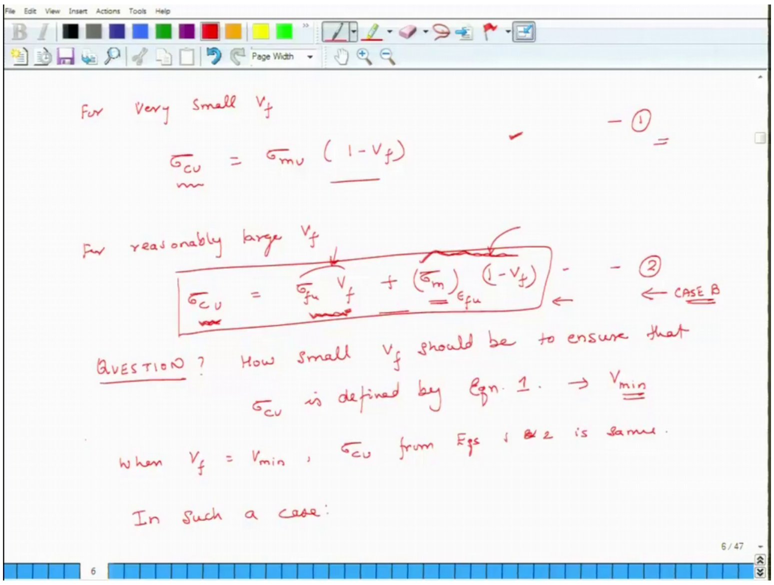

So, for very small V f, strength of the composite, we had defined or we had developed this expression is strength of matrix materials times 1 minus V f.

For reasonably large V f strength of the composite, material was found out to be strength of fiber times the volume fraction of fiber plus stress in the material matrix material at breaking point of the fiber times 1 minus V f which is the volume fraction of the matrix material.

Now, so, the question is how small V f should be. So, let us call it equation 1 and let us call it equation 2. So, the question is how small V f should be to ensure that sigma c u is defined by equation 1 that is what we are interested in understanding. So, the way to find out the answer to this question is so as I for at 0 percent fiber volume fraction equation 1 holds, as I increase it, equation 1 holds, right.

As I increase it by very small amount in the equation 1 holds and then there will be a threshold. There will be a cross over threshold when equation 1 will not be valid and equation 2 will be valid. So, the fiber volume fraction has to be at least a certain percent and that can be found if we equate sigma c u from equation 1 and sigma c u from equation 2 that is the critical threshold level.

So, that is that is what we call V min. So, at this cross over level the volume fraction of fiber we call it V min. So, when V is equal to V min the prediction by equation 1 and prediction by equation 2 will be the same, it will be the same. So, if that is the case. So, when V f is equal to V min, then sigma c u from equations 1 and 2 is same. So, when that is the case, I can equate. So, in such a case; so, what is the value of sigma c u from the first equation?

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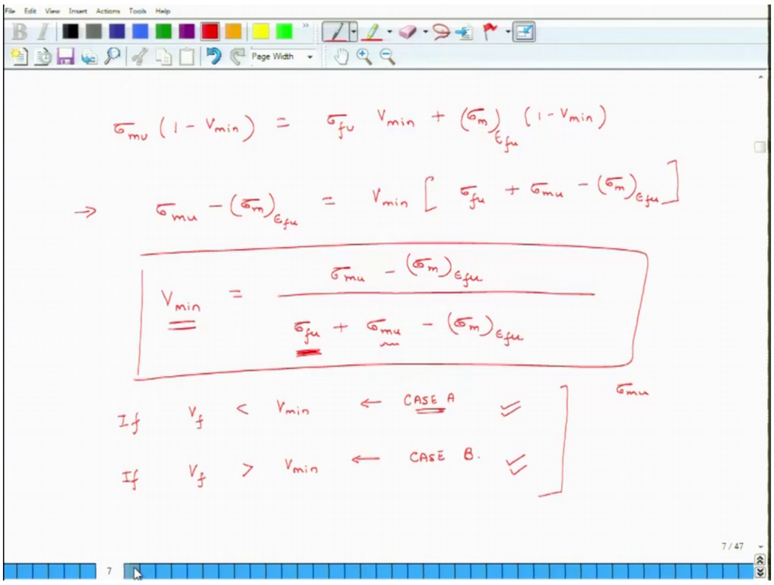

It is sigma m u 1 minus volume fraction of fiber is V. Now volume fraction in this case is V min. So, this equals sigma f u V min plus sigma m epsilon f u 1 minus V min. So, if we process this further we get sigma m u minus sigma m evaluated at breaking strain of the fiber and that equals V min times sigma f u plus sigma m u minus sigma m evaluated at breaking strain of fiber. So, this is the thing.

So, V min is calculated out to be sigma m u minus sigma m evaluated at breaking strain of fiber divided by sigma f u plus sigma m u minus sigma m elevated at breaking strain of fiber. So, that is the expression for V min ok.

So, if V f volume fraction of the fiber is less than this value V min if it is less than this value of V min then which formula which relation we use we use, then that is case a then that is case a and in that case, fiber breaks load gets transferred to matrix at that point is not breaking. So, I can load the composite further and at A; some future point the matrix base fails and if V f is more than V min then we have case B.

And in this case, the failure of the fiber the failure of the matrix and the failure of the composite it happens at the same point at the same strain level at the same strain level. So, this is the thing now it will be worthwhile to spend may be a minute or 2 to understand how small this value is how small this value is this V min.

If you look at the denominator you have the ultimate strength of the fiber and in the numerator you have ultimate strength of the matrix material and then of course, you are subtracting here you are adding sigma m u and subtracting sigma m epsilon f u.

But typically sigma m u is about one hundred times is about one hundred times less than sigma f u it is 50 to 100 times or sometimes even more. So, the numerator is very small compared to the denominator. So, this V min is extremely small when we will later do some calculations and you will find that the value of this V min is barely a percent or 2 in most of the cases. So, even at extremely small volumes fractions of fiber chances are that this formula will work at really small levels once the fiber volume fraction becomes very small less than 1 percent, 2 percent and you have to actually calculate it only, then this case a is valid.

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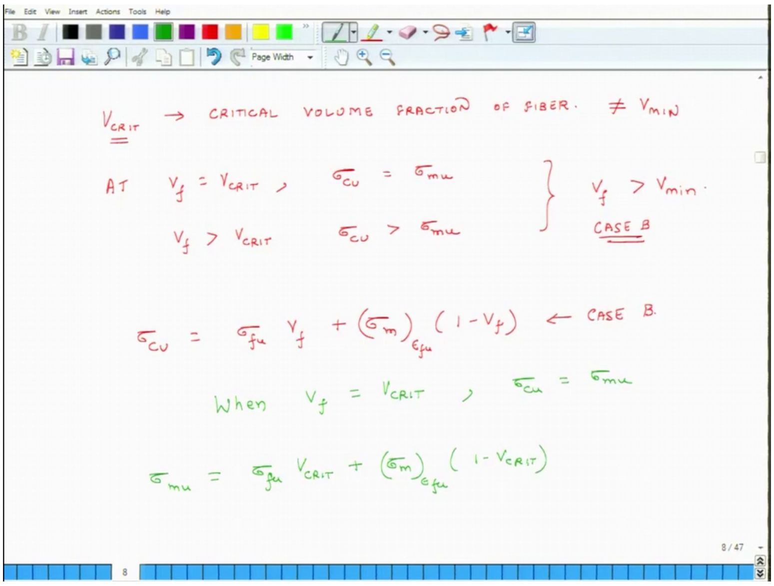

So, this is very important to understand. So, this is the discussion on V min now we will also define another expression called V CR IT and this is called critical volume fraction of fiber critical volume fraction of the fiber this V CR IT is not same as V min, we will define what V CR IT is we have already understood what is V min if volume fraction is less than a fiber is less than V min, then we have to use the first expression for case a to find out the failure of the composite, if it is more than V min, then we have to use case B to predict the failure of the composite.

So, now let us look at; what is let us look at this figure this expression. So, this is for case b and now we are trying to get a physical understanding is; what is V CR IT consider a case when volume fraction is very small; it is small, but not so small that case a holds, but it is. So, it suddenly we are not having a volume fraction.

So, there could be a situation when volume fraction is more than V min, but it is still very small let us say three or four percent typically V min is about one percent sometimes even 0.1 percent. So, let us say V me volume fraction is still very small, but more than V min. So, in that case we have to use this relation to predict the strength of the composite now in that case if volume fraction is very small the strength of the composite will have 2 terms one is this term and the other one is this term, right.

If volume fraction is very small the contribution of this term where the strength of the fiber is involved will be very small most of the strength of the composite will come from this term which is sigma m epsilon f sigma m at breaking strain of the fiber times 1 minus V f ok.

Now, if V f is very small then the contribution of this term will be very small because contribution of this term is going to be very small. So, in that case the chances are that the strength of the composite will be very small and it may be actually less than that of the matrix itself because this not the strength of the matrix, right.

So, we, but what do we want when we make a composite we want matrix is very weak in general we have seen earlier through material properties fibers are very strong now we cannot have hundred percent fiber in a composite, but from a functional standpoint we want that our composite should be stronger than the at least it should be stronger than the matrix if it is weaker than the matrix then there is no point in having a composite ok.

So, we want a situation where the composite strength which is sigma c u should be more than the strength of the matrix which is sigma m u, but if V f is very small this contribution is going to be very small contribution because of sigma f u. So, the chances are that the composite strength will be less than that of the matrix itself there is the concept of V crit. So, at V crit; so, at V f is equal to V CR IT which is critical volume fraction strength of composite equals the strength of the matrix and if V f exceeds this critical volume fraction then composite strength is more than that of the matrix material

But in both these cases here V f is anyway more than V min that is a given. So, we have to calculate; what is the value of V CR IT or critical volume fraction now because in both these cases volume fraction is more than the minimum volume fraction required. So, we use the relation as developed in case b right. So, what is the expression for case b that sigma c u equals sigma f u V f plus sigma m times 1 minus V f; this is the expression for case B ok.

Now, when V f is equal to critical volume fraction what happens at critical volume fraction sigma c u is same as the strength of the matrix material. So, we can say that sigma mu equals sigma f u times V CR IT plus sigma m at breaking strength of the fiber times 1 minus V CR IT.

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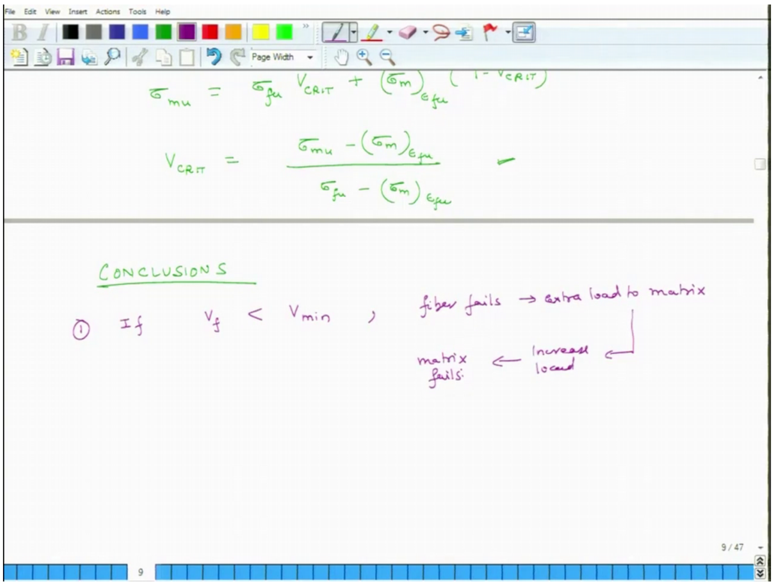

So, from there we can find V CR IT is equal to sigma m u minus sigma m evaluated at breaking strength of the fiber divided by this is sigma f u minus sigma m evaluated breaking strength of the fiber. So, please check I think this expression is correct. So, this is my critical volume fraction.

So, conclusions; so, the first conclusion is if volume fraction in the composite unidirectional composite is less than V min then fiber fails extra load to matrix, then you increase load and then matrix fails, right.

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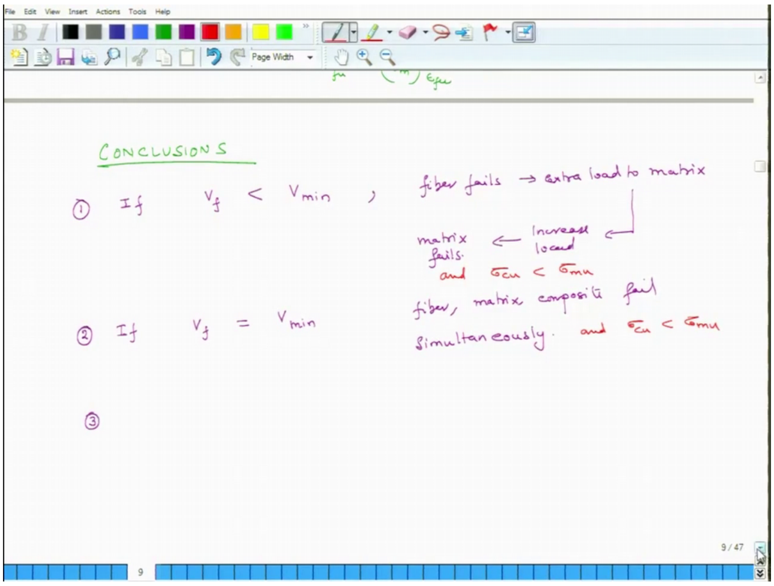

So, this is the first scenario second case if V f equals V min, then fiber matrix composite fail simultaneously third oh not only that we also have to make a comment on and. So, this is there and sigma c u is less than sigma m u right because we have not reached the critical volume fraction. So, here also and sigma c u is less than sigma m u. So, the third situation is V f is less than V CR IT and it is less than.

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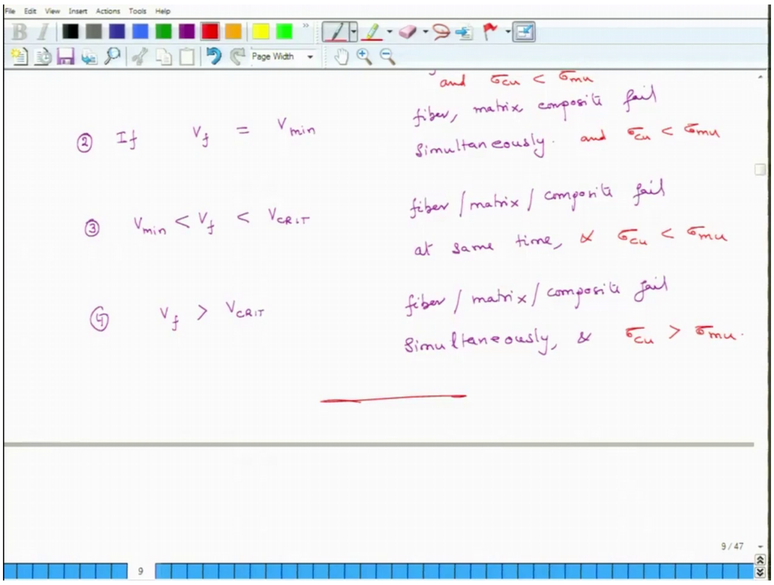

But more than V min then again fiber matrix composite fail at same time and sigma c u is less than sigma m u strength of the matrix.

Even at that stage the overall composite is weaker than the matrix itself and finally, when V f is more than V CR IT, then what happens fiber matrix composite fail simultaneously and sigma c u is more than sigma m u, this is the case. So, as we design a composite laminate especially when we are talking about unidirectional laminates we have to make sure that the volume fraction of the fiber should always exceed the critical volume fraction otherwise we will not get the benefits of higher strength as we would like the composite to be ok.

And the other moral of the story is that we should try to maximize the fiber content in the matrix. So, not only its strength will be more than that of matrix, but it will be significantly larger the ma the than that of matrix I mean the higher is the difference the better it is for us because then are specific weight and right specific strength of the system becomes less. So, this is the overview for unidirectional composites tensile strength and the longitudinal direction.

Tomorrow, we will may be we will do a couple of examples and then we will move on to some other properties of unidirectional laminates. So, that is all for today and we will once again meet tomorrow with some more knowledge about unidirectional composites.

Thank you.