Introduction to Composites
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Lecture – 29
Calculation of Longitudinal Modulus for Unidirectional Composites

Hello. Welcome to introduction to composites. Today is the fifth day of this ongoing week and what we will discuss today is how to calculate; computationally; how to figure out the longitudinal modulus of a unidirectional lamina, which has fibers, which are continuous in nature. So, our discussion will not be relevant if the fibers are broken and they are short in length.

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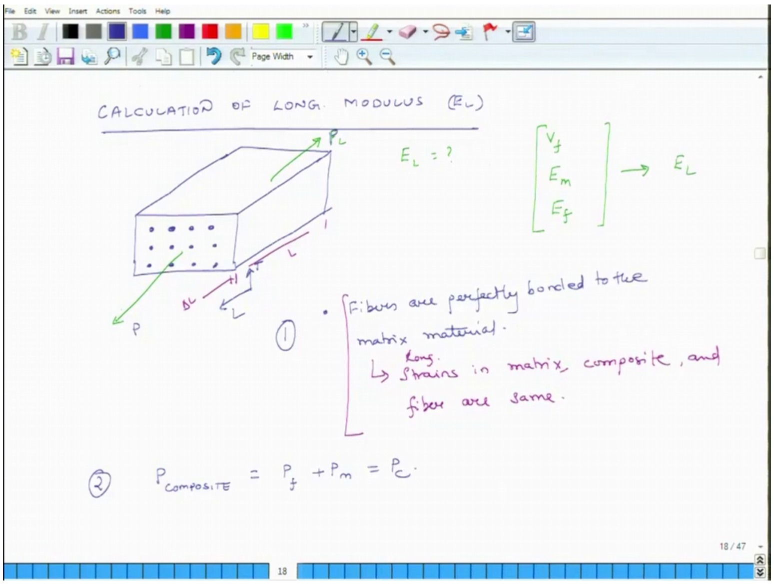

So calculation of longitudinal modulus. So, we will call it E L longitudinal modulus. Now before we start computing this, we have to understand; what is happening in the system. So, I will make a picture.

So, let us say this is what the sample looks like and the fibers. So, let us say that this is my L direction, this is my T direction and the direction in the third direction is T t prime. So, these are all the fibers and when I look at it from this end, I can only see their end points and what I am trying to figure out is that if I pull this composite in this direction and I apply a force f, L can I compute the value of Young’s modulus E L for this composite that is our goal that E L; how do I calculate this?

Now, what do I know about this problem? I know that what is the volume fraction of the fiber; without that I cannot figure it out I also know the Young's modulus of matrix and I also know the Young's modulus of fiber. So, these are the three parameters I know and using these three parameters my goal is to figure out E L. Now when I have; I am looking at this problem I make some assumptions and I assume and this is an important assumption.

So, I assume that fibers are perfectly bonded to the matrix material; this is the first thing we assume that fibers are perfectly bonded to the matrix material the moment I say that what; that means, is that when I apply the force in x direction, this body has an original length L, it will extend by some distance delta L, and if the fibers are perfectly bonded to the matrix material then the fibers the matrix and the whole composite will expand by the same amount.

If they are not bonded uniformly; a fiber will lose, then fiber will not expand, but the matrix will expand, but if they are perfectly bonded then the extension. So, the consequence of this is that strains in matrix composite and by strains I mean longitudinal strain that is in the L direction longitudinal strains and matrix composite and fiber are same this is very important to understand.

The second thing I would like to say. So, this is based on how I make the composite. So, if I make it nicely and everything is well bonded then this is what it means. So, the second thing I would like to state. So, this is the first thing first observation the second observation that f is the total force I apply f is the total force now some of these force is being taken by fibers and some of the force is being taken by the matrix, right and what is the relation between the force being taken by fibers matrix and total force it is that f total is equal to and actually I will not call it f, I will just make it f P. So, just to be consistent with my notes. So, P total. So, P total is what P composite, right.

The total force which is being experienced by the composite is equal to P f plus P m and this is equal to P c.

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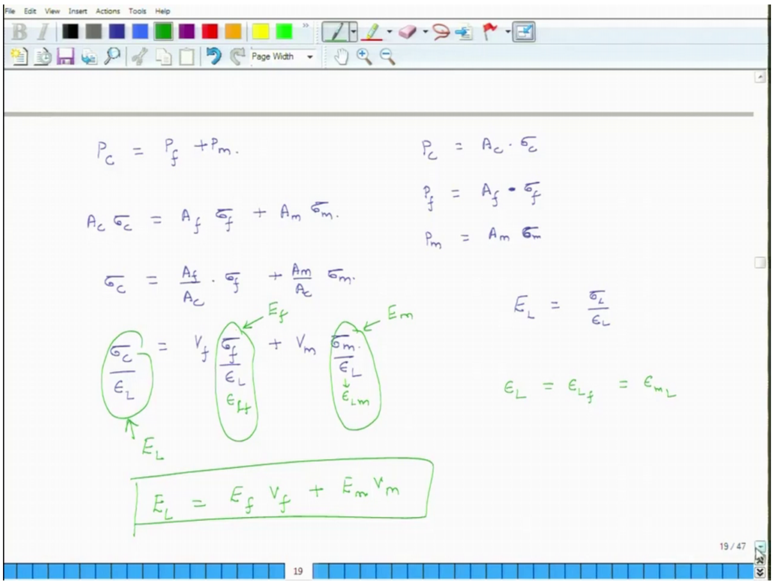

So, let us do some math. So, P c is equal to P f plus P m, I am just rewriting this relation now what is P c? P c is the total force which we are applying on the system. So, what is that? That is equal to area of the composite cross sectional area which is this entire area, right. This entire area of the cross section is A c. So, this is equal to A c times the stress in the composite similarly P f is equal to area of fiber times stress in the fiber and E m equals A m times stress in matrix. So, I can write A c sigma c is equal to A f sigma f plus A m sigma m, ok.

So, I divide this entire equation by A c and I get sigma c equals A f over A c times sigma f plus A m over A c times sigma m. So, what is again it is important to understand what is cross A c; A c is this entire area A, B, C, D.

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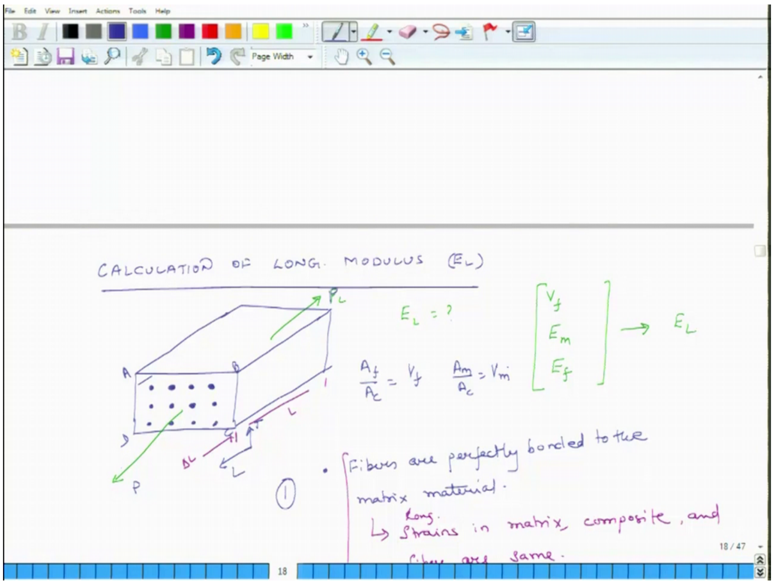

What is A f? A f is the sum total of all the cross sectional areas of all these fibers and A m; A m is the area of the matrix in the cross section A, B, C, D. So, if that is the case then A f over A c equals what.

Student: (Refer Time: 08:31).

Volume fraction of the fiber A m over A c will be volume fraction of matrix. So, I put these results in my equation back there. So, I get sigma c is equal to V f sigma f plus V m sigma m, again, I am not assuming; I am assuming that there is not a lot of air in the system ok.

So, what was our goal our goal was;

Student: (Refer Time: 09:05).

To find the Young's modulus; now Young's modulus of composite E L is equal to what stress in the composite in the length direction divided by;

Student: strain.

Strain in the length direction. So, I divide this entire equation by epsilon umm and what does this become this becomes E L. Now epsilon L is the stress in the composite, but we have said that the fibers and the matrix are bonded perfectly together. So, this is same as stress in the L direction in fiber and this is also same as stress in the matrix in the L direction.

So, I can replace this by epsilon L f and I can replace this by epsilon L m and then what is sigma f over epsilon f this thing is Young's modulus of the fiber in the length direction umm and this is Young's modulus of the matrix in the length direction. So, E L equals E f V f plus E m V m. So, this is the relation between volume fractions and moduli of the fiber and matrix and the overall modulus of the composite in the L direction.

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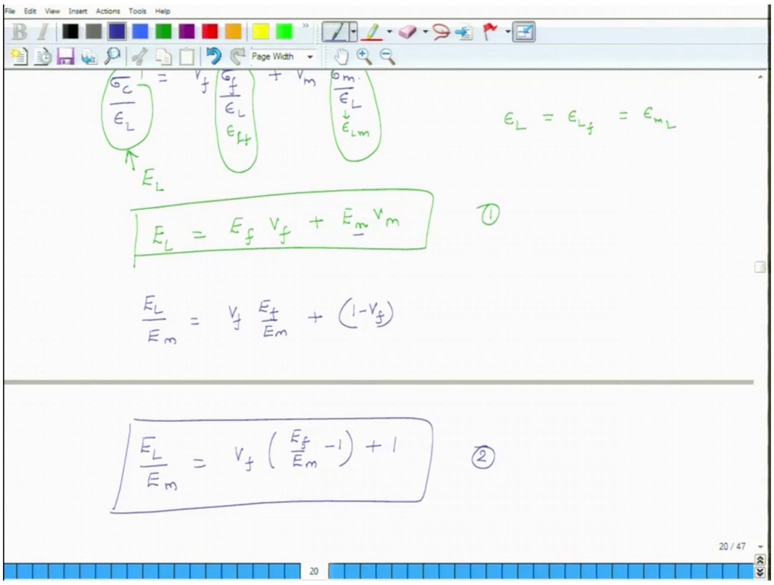

So, let us call this equation 1, I can reformulate this equation in slightly different form. So, what is do is that I divide this by E m. So, I get E L by E m equals V f; E f by E m plus and V m is what; 1 minus V f, right and I have divided the entire thing by E m. So, this E m is gone.

And this if I rearrange I can write it as E L over E m equals I take V f common E f over E m minus 1 plus 1 yeah. So, this is equation 2. So, these are the 2 forms and just wanted to give you some prospective of how strong is the influence of the fiber modulus on the overall modulus of the composite.

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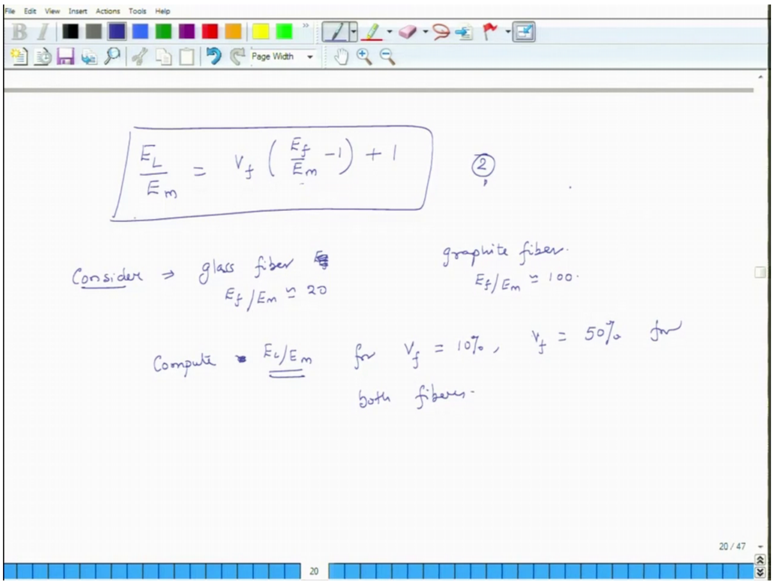

So, we will do an example very quickly. So, what we consider? We consider glass fiber and we consider graphite fiber and if you look at their modulus and the modulus of matrix typically E f or glass fiber is approximately equal to 20 and graphite is much more stiff. So, E f over E m is approximately equal to 100.

So, we want to compute this parameter E L over E m see if there is no fiber then the modulus will remain as same as E m, we want to see that if we had even a small amount of fiber; how much does the modulus of the composite change. So, compute E L over E m for V f equals 10 percent and V f equals 50 percent for both for both fibers. So, how do we actually just use this equation to directly we plug in E f over E m for glass as twenty percent point two and V f has 10 percent or 50 percent and we can compute E L over E m. So, we will just make a table.

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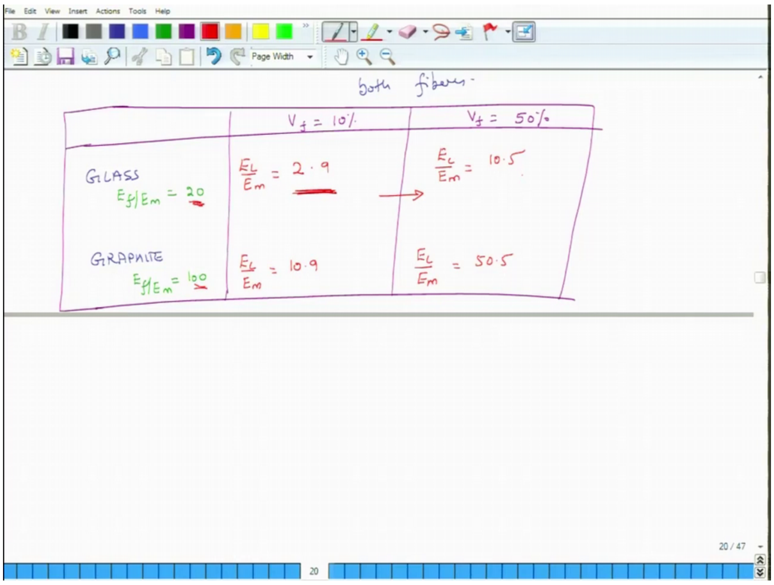

So, for glass and we have graphite for glass E f over E m is is how much E f over E m is about 20 and for graphite E f over E m is about 100.

And so, this is there yeah and the first case is V f is equal to 10 percent and V f is equal to 50 percent. So, let us write down these numbers. So, you can do the calculations, but I will just straight away write the numbers. So, what are these values these values are for E L over E m equals 2.9 E L over E m equals 10.5 E L over E m equals 10.9 E L over E m equals;

Student: 50.

50.5. So, that is there. So, couple of observations I mean these are simple calculations, but couple of important observations.

Even if I put a 10 percent glass fiber in the matrix material just 10 percent, it is not that we have to put a lot of glass fiber just a small amount which is 10 percent it increases the modulus by almost three times and glass is not that stiff compare to graphite glass is only 20 times stiff than matrix graphite is 100 times stiff. So, even small amount of fibers can rapidly increase the stiffness of the system and then if you increase the fiber content then if you increase the fiber content almost. So, you know. So, it almost linearly it increases the stiffness of the system. So, it goes from 2.9 to 10.5 and the same trend, we see for graphite fibers.

So, even if we put small amount of fibers we will start seeing the benefits in terms of higher stiffness for the system. So, this is what I wanted to say. So, remember we had originally the discussion on specific stiffness what this calculation sow shows is that even small amounts of fiber can rapidly increase the stiffness see stiffness went down went up by factor of 2.9 for the matrix and glass density is will be two point four something like that. So, the density of the overall composite has not gone up by that much, but may be it went up by 20 percent, 25 percent, we can calculate it using the formula for density, but the stiffness went up by 300 percent.

So, specific stiffness which is the ratio of modulus and density, it really went up significantly high and those pay offs or those benefits you see even to a much larger extent in case of graphite fibers because graphite fibers are light, they are actually lighter than water and they are extremely stiff.

So, you see start seeing benefits very significantly for such fibers. So, this is what I wanted to discuss in context of longitudinal modulus, next, what we will look at; in next 5-10 minutes is; how is the load shared between. So, remember we have this picture and we said that some load is shared by the fiber and some load is shared by the matrix.

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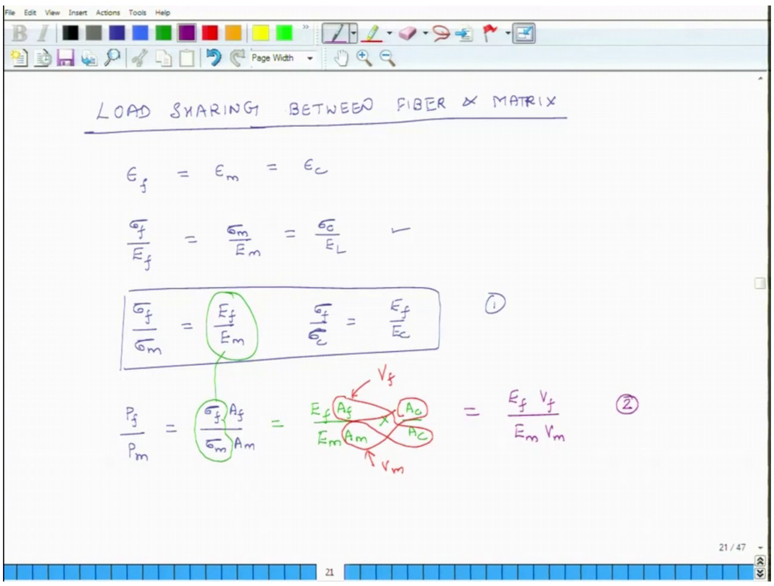

So, now we will figure out how much load is shared by fiber and how much load is shared by matrix in the system. So, our focus will be load sharing between fiber and matrix. So, we have to get a feel of this.

So, remember strain in for this system because it is perfectly bonded strain in the fiber is same as strain in matrix is same as a strain in composite what is the strain in fiber sigma f over E f strain in matrix sigma m over E m and strain in composite is sigma c over or sigma L over E L c means composite you can also write it sigma L now. So, this is based on the fact that fibers are perfectly bonded to the matrix and when you pull them the amount of extension of fiber experience is the same as the amount of extension in the matrix experiences. So, from this you can write sigma f over sigma m equals E f over E m.

So, how much load is being stress which is being handled by matrix and stress which is being handled by fiber is in the ratio of the Young's modulus this is there. So, if the ratio is 20 E f over E m is 20 then fiber takes 20 times more stress not load because load is also associated with area. So, if the fiber takes so, but the stress in the fiber will be proportionally higher.

Similarly, you can also say that sigma f over sigma c is equal to E f over E c. So, let us keep this as one equation. Now we want to find what is the ratio of loads because we are interested in loads how much load is being shared because ultimately we have see how much what we apply is load suppose I applied 20,000 Newton's of load I want to see how much is being taken by fiber and how much is being taken by matrix. So, P f is the load shared by fiber P m is the load shared by matrix. So, what is P m it is the stress in fiber times A f and what is P m it is the stress in matrix times A m ok.

Now, we just compute it that this thing is got same as this thing. So, I can write it as E f; E f over E m and what I can do is I can multiply it and divide it by what by A c and A c; A c is the area of the composite. So, what is A f over A c.

Student: V f.

This is V f and what is A c o m this thing V m. So, what I get is E f V f by E m V m. So, this is equation 2.

So, this equation two it gives us the ratio of how much load is being shared by fiber and how much load is being shared by matrix and what it tells is that if you want the fiber to take a lot of load of course, this E f over E m should be high and also make sure that E V f over V m is also as high as physically possible.

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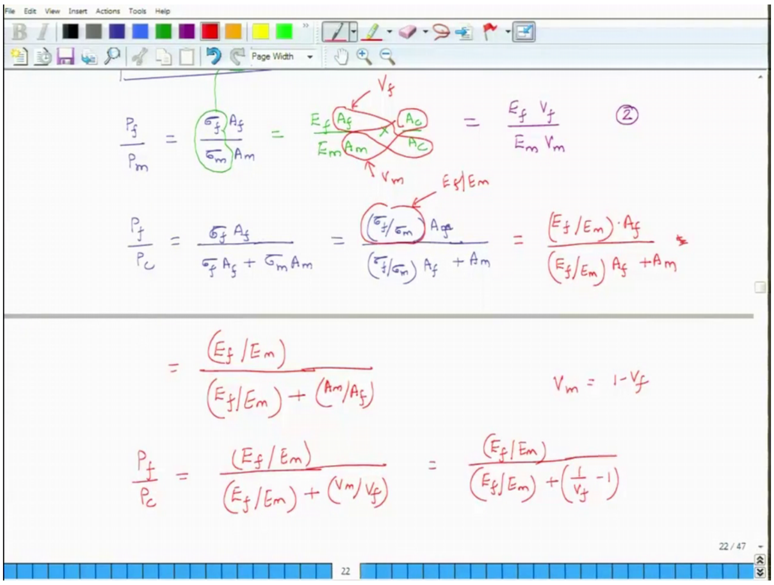

And then let us see; what is P f over P c. So, P f over P c is what it is equal to sigma f A f and P c if what load bond by the composite and that is what sigma f A f this is the load bond by composite plus load by matrix load by matrix ok.

Now, you divide the numerator and denominator by what sigma m. So, you get sigma f by sigma m A f by sigma f by sigma m A f plus A m; what is sigma f by sigma m; it is E f by E m, right. So, what you get is E f by E m times A f by E f by E m times A f plus A m, right and then you divide the numerator and denominator by A f. So, you divide this. So, you divide this numerator and denominator by a f. So, what do you get. So, you get E f by E m by E f by E m plus A m by A f and you can still this ratio of area is same as ratio of volume fractions.

So, load shared by fiber in relative to that by composite is E f by E m by E f by E m plus V m by V f and we know that V m is what 1 minus V f V m equals 1 minus V f, right, if there is no air, then I can write it as E f by E m by E f by E m plus 1 by V f minus 1; this is what I get. So, look at this.

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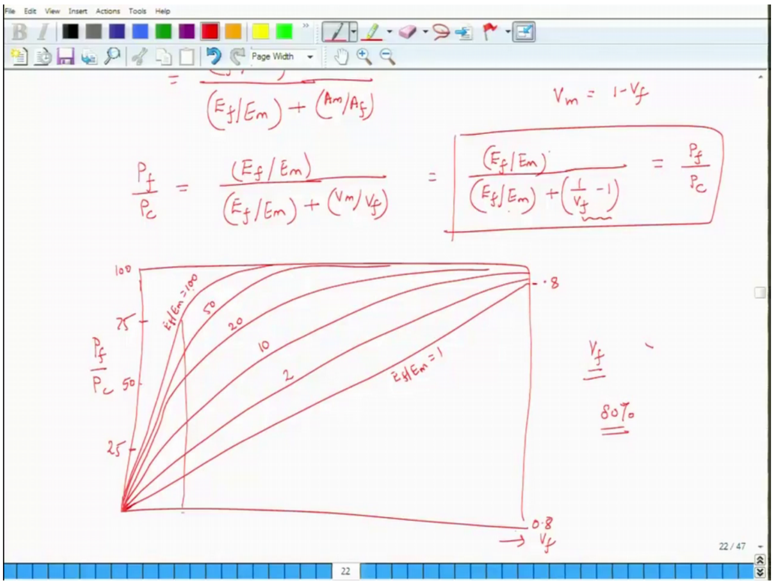

So, this is equal to load shared by fiber and load bond by composite. So, this is the third equation. So, what does it mean? Suppose, there is no (Refer Time: 28:25) in it; in that case volume fraction is of fiber is 0 if volume fraction of fiber is 0 I mean there is no fiber. Fiber will not share any load, what does this relation say if volume fraction is 0 of fiber, then 1 over V f will become very large infinity. So, denominator becomes infinity so; obviously, V f will be 0. So, that makes intuitive sense.

If fiber. So, the other thing is that if fiber volume fraction is even. So, so here is the thing. So, suppose E m over E f over E m is very large E f over E m is very large let us say it is 100. So, denominator numerator is 100 denominator is 100 plus some number.

Now, let us say V f is 0.1; suppose V f is 0.1; what does this become.

Student: 9.

This becomes 10 minus 1; 9. So, it is becomes basically 100 over 109; that means, that even for 10 volume fraction because E f over E m is very large 100 over 109 means what it is almost 1. So, that graph looks like this. So, let us say here I am plotting volume fraction and here I am plotting P f over P c. So, let us say this is 100 percent, this is 50, this is 25, this is 75 and here it is 0.8.

So, what we are going to plot is for different values of. So, if E m over E f over E m is one then it looks something like this. So, this is about 0.8. So, this is E f over E m equals

1 which means if the fiber and matrix are almost are same, then there is no difference between their behavior, right. So, whether you increase the fiber or you decrease the fiber. So, this is a straight line because the numerator this thing is 1, this is 1 plus 1 over V f minus 1 but as you increase the fiber content.

So, suppose you increase the fiber content to the modulus of fiber to 2. So, it looks like this. So, it goes like this. So, maybe this is E f over E m equals 100, this may be 50, this may be 20, this may be 10, this may be 2, this may be 1. So, what this picture shows is that if I increase the stiffness of my fiber relative to matrix, then I do not need a lot of fiber to bear most of the load, right because even get small volume for. So, so for when E f over E m is 100, I do not need a lot of fiber because. So, even at small volume fraction may be 10 to 15 volume fraction most of the load is taken by fiber.

Matrix the role of matrix is what just to bind everything together and keep everything in place. So, this is very important to understand. So, what naturally we would want is that we want to maximize V f as much as possible, but for practical reasons it is very difficult to increase V f over 80 percent because once you start having more fiber than 80 percent the matrix material finds it hard to flow between all the fibers and stick them together. So, there may be lot of areas in the system which may remain dry and the matrix may not be able to wet in such a case our original assumption that fibers and matrix and composites they have same strains that may not be true and in that case the actual modulus of the material may be a little less than what is being predicted by the relation which we have developed.

So, this is what I wanted to discuss for today and tomorrow we will discuss a new topic which is related to the failure of unidirectional composites in the length direction. So, with that we conclude our discussion for today and we will meet once again tomorrow with the new topic in hand.

Thank you.