

**Introduction to Composites**  
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**Lecture – 27**  
**Modeling of Unidirectional Composites**

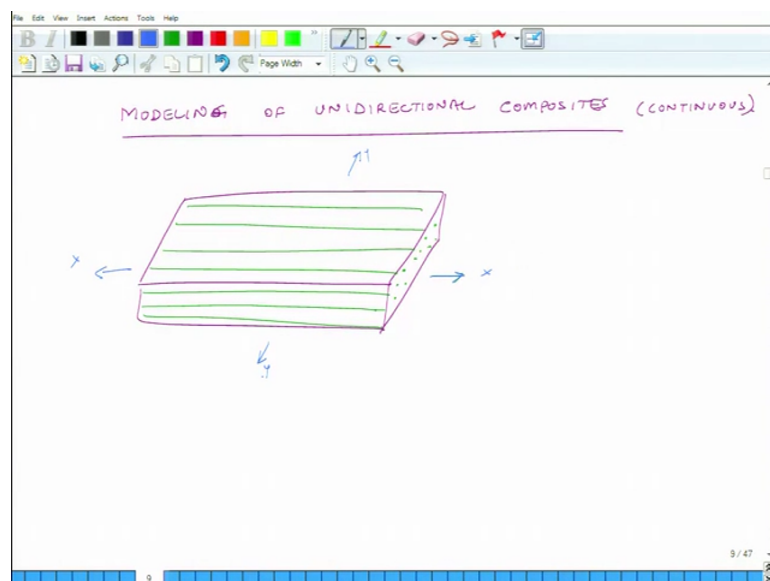
Hello, welcome to Introduction to Composites. Today is the 3rd day of the ongoing week. Over the last 2 days we have discussed some basic concepts and terminologies associated with composite materials and specifically we have discussed some the important attributes of isotropic materials, anisotropic materials and orthotropic materials.

Starting today we will start developing relations for different properties like density Young's modulus strain and load shared and so, on and so forth, for individual layers because layers in composite laminates are the building blocks of the overall system.

So, we learn as to how to calculate the properties some of the important mechanical and physical properties of individual layers which are having fibers only in one single direction.

So, what we will focus on is modeling?

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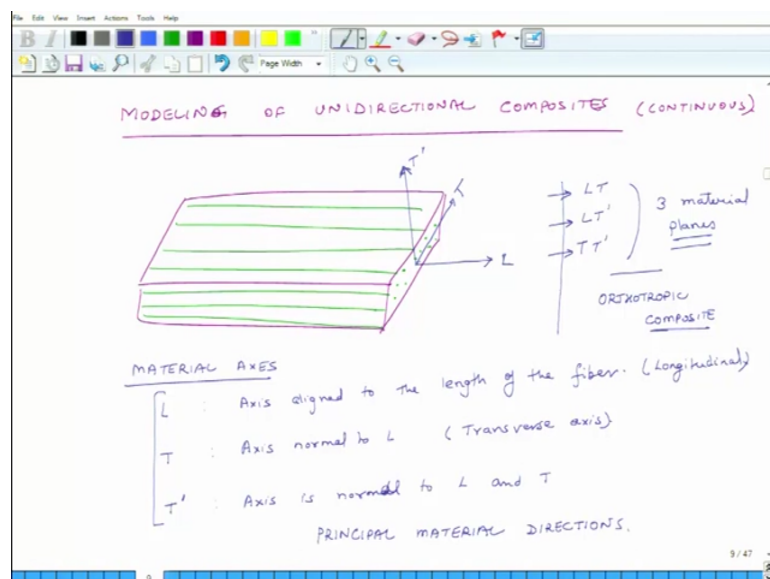
Modeling of unidirectional, unidirectional and unidirectional composites now here the fibers could be either continuous or short fibers. So, here we will focus on composites which have continuous fibers. We will worry about short fiber composites may be a little later.

Now unidirectional composites based on your intrusion itself they will have different properties in different directions for instance if you have a single layer and it has fibers which are running in the length direction. And the end view looks like something like this then if I pull this overall structure. In this direction let us say  $x$  it will have some stiffness if I pull it in  $y$ , it will have some other stiffness if I pull it in  $z$ , then it may have different stiffness. So, it will have different properties in different directions.

So, before we start wondering about the modeling the first thing we have to define is a consistent set of axis you know axis, which we can refer again and again. So, that when you are talking about a particular direction, I exactly understand what is that direction meant by, because  $x y z$  could vary and there could be infinite values of  $x y z$  because I can keep on rotating my axis system.

So, when we worry when we talk about unidirectional composites unidirectional composites or the composites or a single layer, which has fibers running only in one direction then the axis system which we refer to is always the Material axis of the system.

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So, these are our reference axis material axis. So, what are the material axis the first material axis is the axis is the length direction so or the  $l$  direct. So, let us look at this. So, this is called  $L$  direction  $\sigma_L$  and  $\sigma_L$ . So,  $L$  direction this is  $l$  axis  $L$  direction.

So, you have  $L$  and what does  $L$  corresponds to it is the axis aligned to the length of the fiber. Then the other direction we look at is the  $T$  direction. So, let us la this is  $T$  direction so  $T$ . So,  $L$  corresponds to longitudinal,  $L$  corresponds to longitudinal. Then the  $T$  direction so I did not draw it correctly. So,  $T$  direction is. So, this is the axis normal to  $l$  in plane of yeah. So, it is normal to  $l$  and it is shown as indicated ok.

So, axis is normal to  $l$  and this is called transverse axis and then there is a third direction because you always want to have a 3 sets of mutually orthogonal or axis, which are 90 degrees at ninety degrees to each other. So, the third axis we call it as  $T$  prime. So, this is also. So, this axis is normal to  $L$  and  $T$  yeah.

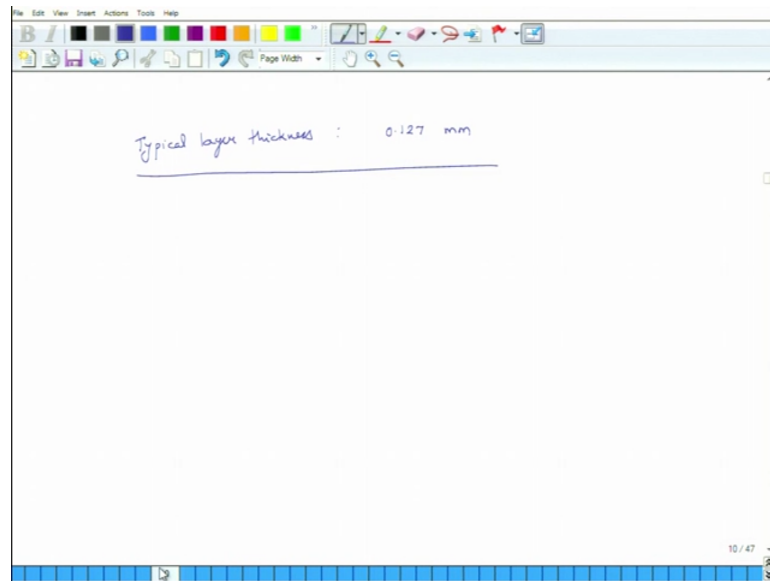
So, these are material axis and we also. So, these are material and we also call them as principle it is the same thing principle material directions. So, either you call it material axis or principle material directions. So, where would  $T$  be located? So,  $T$  would be located something like this  $T$  prime  $T$  prime is located something like this. So, these are the 3 principle material axis directions.

So, we calculate our young's modulus with reference to these axis stresses and so on and so forth. Now this material system if I have a unidirectional composite and in the and this is just a flat layer and in which layer all the fibers are running in the  $l$  direction. Then these 3 planes so for reasons explained earlier the plane  $L T$ , the plane  $L T$  prime and the plane  $T T$  prime, they are what they are 3 material planes? And in these planes within the plane itself the material properties will be isotropic for exactly the same reason which we had explained in the last class

So, a unidirectional composite which has ah unidirectional composite that is single layer, which has fibers running in the  $L$  direction continuously is an orthotropic composite. Because it has material symmetry in the  $L T$  plane it has material symmetry in the  $L T$  prime plane and it has material symmetry in the  $T T$  prime plane.

Typically each layer of composite, when you actually make is it is not a whole lot typical layer thickness.

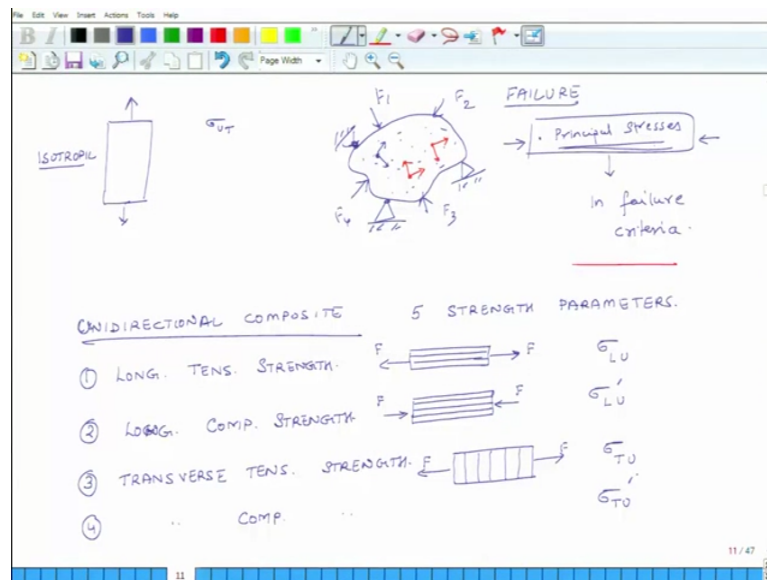
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Let us say we are making a glass fiber composite and we are making a laminate. So, we will stick several of these layers on top of each other. So, typical glass fiber refined fiber thickness layer would be about 0.127 millimeters. So, if you want to make up 6 millimeter thick composite plate, you have to stack up a lot of layers on top of each other. So, that is another thing to consider. So, you should have some idea that these thicknesses are not very large they are less than 0.25 millimeters.

So, that is one thing another contrast I wanted to share relative to isotropic materials. So, anisotropic materials suppose you have a piece of steel.

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So, now we are going to talk about failure and again we will discuss some basic information and then we will start looking at some of the details so failure. So, if it is an isotropic material, you pull it and it has a tensile strength right.

So, you can call it ultimate tensile strength, it has ultimate compressive strength, it may have a shear strength, it may have a yield point. And if you have a material structure something like this and you are putting force as so, this is all isotropic material and you are holding it at 3 points or several points. This structure will develop stresses it will develop stresses and the way you are. So, this is an exerting it some forces  $F_2, F_3, F_4$  and you want to figure out whether this structure is going to fail or not and what do you do?

The first step you do is you find out stresses at all the points in the system. And how do you do that you do that using your regular formula for beams effect the structure looks like a beam, then you use formulas for beam as displayed you use formulas for plates or if it is very complicated structure. Then you have methods like finite element solution and so on and so forth. But essentially the point is that at every point in the system you figure out what is the stress and then at every point in the system. So, at all these points you find out what are the principle stresses.

You find out principle stresses at all the points in the system. And then what do you do you want to know whether this structure is going to fail or not and then you use some of

those criteria, principle stress criteria, principle strain criteria, one misses stress criteria, distortion energy criteria, but the first step in understanding whether the stress is going to fail or not is first you figure out what is the principle stress at every point in the system. And then you use this information in failure criteria. To see whether it is failing or not and at every point you see whether there is a margin of safety or not, this is the overall way we figure out whether the structure is going to fail isotropic structure or not.

Now, we can use principle stresses in context of isotropic materials because of the material properties. So, principle stress here the principle stresses may be in these 3 directions. Principle stresses are always normal to each other. So, at every point you will have a set of principle stresses.

So, here they may be aligned like this, at another point they may be aligned like this, at a third point they may be aligned like this. And the reason we can compare these principle stresses with some failure criteria is because the material is isotropic. So, the strength of the material also does not change with direction. So, we can use the principle stress method.

But in case of orthotropic or anisotropic material this method just does not work because you can compute principle stresses at every point, but the strength of the material keeps on changing with respect to the direction. So, you do not know how what to compare to these principle stresses with.

So, typically we have 5 or 6 important parameters, which we use to characterize the failure of a composite of a single unidirectional composite. So, unlike isotropic materials, which have same strength in all the directions, composite materials have different strengths in different directions. So, composite may unidirectional Laminae, unidirectional lamina or I can call it a composite and this is again yeah unidirectional composite it has 5 strength parameters.

The first strength parameter is called longitudinal tensile strength. How do you calculate it you have a unidirectional lamina, you take a sample of it you pull it in tension in such a way that the fibers are aligned with the direction of your tensile stress? So, this is  $F$  and then you keep on pulling it till the thing fails and the stress at which it fails is called longitudinal tensile stress.

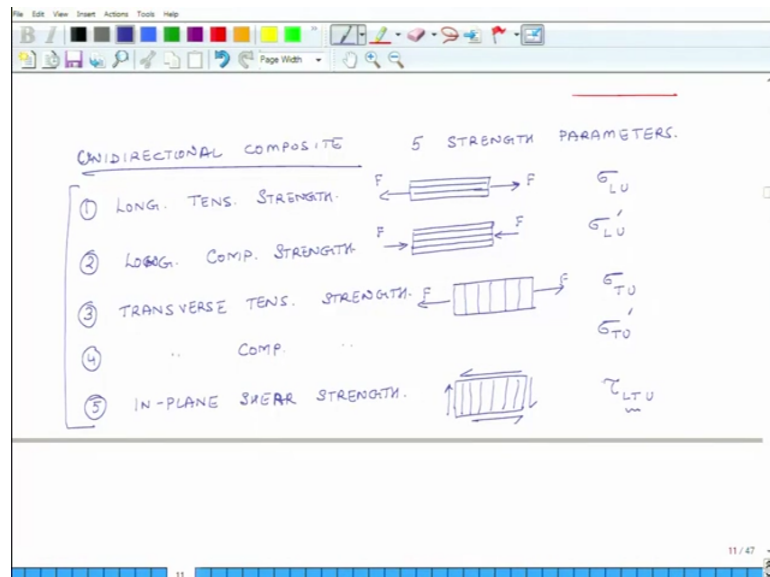
So, you can call it sigma because the stress is being applied in L direction here we call it sigma L and because it is in tension we call it sigma L T or it may create confusion because, I do not want T to V confused with the T direction. So, sigma L and U, U is the ultimate tensile strength.

Similarly, you have longitudinal compressive strength. So, what do we do in longitudinal compressive strength you take the same sample. And you subject it to compression and here also the fibers are aligned with the direction of the external stress. So, you are pulling it or compressing it in the compressive direction and this we call it sigma L U, but then this is compressive.

So, to differentiate between compression and tension we put an apostrophe here. So, that is sigma L U. The third one is transverse tensile strength transverse tensile strength. So, what do we do here? We have a sample and here, we pull it in tension and we pull it in tension in the transverse direction. So, this is these are the fiber force and the fibers are oriented like this and this strength is called sigma T U ultimate tensile strength.

And then the fourth is transverse compressive strength and this is sigma T U prime.

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And the last one is in plane shear strength. So, here you take a block of material and you apply some shear stresses to it like this. And how are the fibers aligned you can align fibers like this or they can be in the L direction does not matter because it is summative.

So, this is called. So, this is shear stress. So, it is  $\sigma_{LT}$  U is the ultimate shear in plane shear strength. So, what does LT mean LT specifies the plane in which the shear stress is being applied and U is the ultimate value of that.

So, these are 5 different properties of a unidirectional composite laminate associated with it is strength to predict failure in the unidirectional composite, we have to get a feel of, we have to know these 5 different values. And then we have to find different stresses in L and T directions and then using some methodology, we can see whether the thing is going to fail or not.

So, that is the overall scope of the process. So, the next thing we will discuss today is are some important parameters of unidirectional materials.

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The image shows a whiteboard with handwritten notes under the heading "VOLUME & MASS FRACTION".

- A tree diagram shows "Composite  $m_c$ " branching into "Fiber  $m_f$ " and "Matrix  $m_m$ ".
- A diagram of a unidirectional composite laminate is shown.
- Equation:  $m_c = m_f + m_m$
- Equation:  $1 = \frac{m_f}{m_c} + \frac{m_m}{m_c}$  with  $M_f$  and  $M_m$  written below the fractions.
- Equation:  $1 = M_f + M_m$  with "Mass fraction for fiber" and "Mass fraction for matrix" written below.
- Equation:  $\checkmark V_c = V_m + V_f$  (assuming no voids in system)
- Equation:  $1 = \frac{V_m}{V_c} + \frac{V_f}{V_c}$  with  $V_m$  and  $V_f$  written below the fractions.

So, we call them volume fraction volume and mass fraction volume and mass fraction. So, again all this discussion is happening in context of unidirectional composites, where fibers are just in one single direction.

So, typically you have a composite you have a composite and it is made up of 2 important ingredients fiber and matrix. So, it is a fiber and matrix. So, let us say you have a sample of composite. So, these are fibers and then you have matrix. So, let us say that the mass of the composite this sample is  $m_c$ .



So,  $c$  stands for composite. So, mass of the composite is  $m_c$ , now this  $m_c$  has 2 components mass of the fiber and mass of matrix so  $m_f$  and  $m_m$ .

So,  $m_c$  is equal to  $m_f$  plus  $m_m$  this is assume yeah. So, this is 2 this is assuming that you have only 2 materials, you do not have any additive failures and all those things this is assuming yet. So, if I divide this by  $m_c$ . So, I get 1 equals  $m_f$  over  $m_c$  plus  $m_m$  over  $m_c$ . And I re label these guys. So, this I called capital  $M_f$  and I call this capital  $M_m$ . So, I write this as one equals  $M_f$  plus  $M_m$ . This is mass fraction for fiber and this is mass fraction for matrix. So, that is our definition for mass fraction.

Similarly, if you have a sample of composite you can also have a similar relation for volume. So, volume of composite which is small  $v$  is equal to volume of matrix plus volume of fiber. Assuming no voids in system see in case of mass even if they were voids the weight of those voids what would be there in those voids here. So, it like almost 0. So, we can ignore that, but in volume we cannot ignore the volume of the voids.

So, if there are no voids in the system then this relation is if there are voids then I have to add also  $v_a$ . So, I will divide this equation by  $v_c$ . So, I get this equals  $v_m$  over  $v_c$  plus  $v_f$  over  $v_c$  and this is capital  $V_f$  this is capital  $V_m$  and what; that means is 1 equals  $V_f$  plus  $V_m$  and these are  $V_m$  and  $V_f$  are volume fractions for fiber and matrix.

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Handwritten notes on a whiteboard showing the derivation of volume fractions for a composite material. The notes include equations for volume of composite, mass fractions, and volume fractions, with annotations for assumptions and definitions.

Mass fraction for fiber:  $M_f$   
 Mass fraction for matrix:  $M_m$

$\checkmark v_c = v_m + v_f$  (assuming no voids in system)

$1 = \frac{v_m}{v_c} + \frac{v_f}{v_c}$

$1 = V_f + V_m$

$V_f$   
 $V_m$  ) → Volume fractions for fiber and matrix.

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If there is air:

$1 = V_f + V_m + V_a$

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So, there is a mass fraction and there is a volume fraction. If there is air in the system if there are some air pockets. Then if there is air then you have 1 equals volume fraction of fiber, plus volume fraction of matrix, plus volume fraction of air. So, that is there.

So, I think we will stop here we have covered lot of stuff in today's lecture, tomorrow we will extend this discussion and using all this information, we will develop relation between density and volume fraction densities and things like that.

So, that is what we plan to do tomorrow and when till the time we meet next I hope and I am sure that you will have a great time and we will meet once again tomorrow.

Thank you.