

Design Practice
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Lecture - 23
Average quality loss

Hello and welcome to this design practice module, 23. We were discussing about the Average Quality Loss A Q L and we calculated the Average Quality Loss.

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Average Quality Loss

$$AQL = K \left[\frac{(n-1)\sigma^2}{n} + (\mu - T)^2 \right]$$

$\hookrightarrow K[\sigma^2 + (\mu - T)^2]$ in case $n \gg 1$

AgL ① The loss due to deviation of the average value of y from the target $\frac{1}{n}(\mu - T)$

② The loss due to loss due to the mean square deviation of y around its own mean.

Tab on the μ

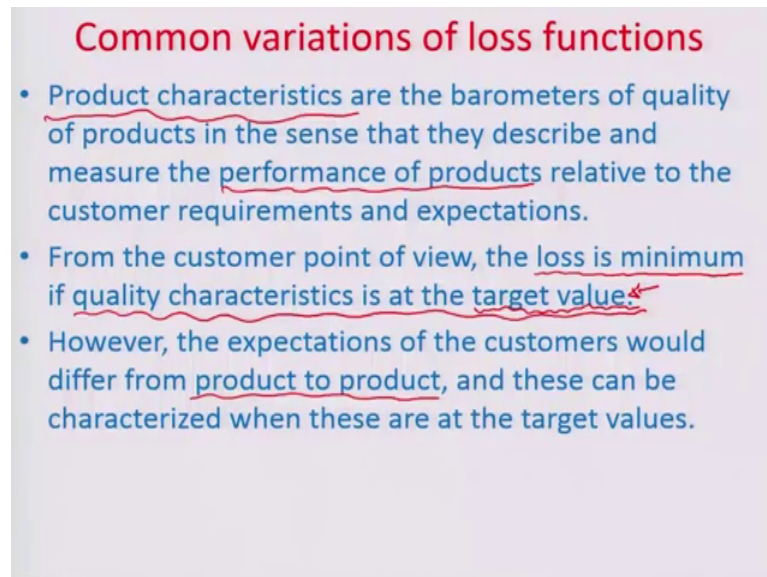
Tab on σ

A Q L to be equal to K times of n minus 1 by n square of σ plus μ minus T whole square, which converged into K times of square of σ plus μ minus T square in case n were very large. So, the average quality loss really, now is you know as you can see made up of two components. So, the A Q L has one component, which is the loss due to deviation of the average value of observations or according or measurements from the target.

And then there is yet another part of the A Q L. So, this is that K times of μ minus T square loss and then there is a component of the loss due to the K σ square part of the component. This is, because of the mean square deviation of y around its own mean. So, what we are referring here is that from a standpoint of prevention of any non compliance, it is important to not only have a tab on the μ or the central, you know tendency of the distribution, but also a tab on what is the overall deviation from μ .

Okay. So, both these need to be in control, for you to have a lower equal value and that is exactly the concept, what has been built in to it, to what is known as you know introducing robustness in, in the whole design so; obviously.

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Common variations of loss functions

- Product characteristics are the barometers of quality of products in the sense that they describe and measure the performance of products relative to the customer requirements and expectations.
- From the customer point of view, the loss is minimum if quality characteristics is at the target value.
- However, the expectations of the customers would differ from product to product, and these can be characterized when these are at the target values.

The common variations of loss functions are, because of product characteristics. They are like barometers, of the quality of the products and they sort of describe and measure the performance, the product performance relative to the requirements and expectations coming out of the product. So, the customer point of view is really about somehow minimization of this loss.

So, that most of the quality characteristics, which are set in as are more or less at the target values and; obviously, the target values as I have told you in case of many, as many kind of quality parameters are many different kinds for example, customer would have a different opinion from product to product and this can be characterized, at different target values. Let us look at some, some, some quality indices or quality characteristics from a customer point of view, which are very common place in normal products available.

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Common variations of Loss functions

- Nominal is the best when 'y' is at target. Examples include dimension, viscosity and clearance.
is the
$$L(y) = K(y - T)^2$$
- Smaller is better; that is, y tends to zero, where target is zero. Examples of quality characteristics include wear, shrinkage, deterioration, friction loss, and micro-finish of a machined surface among others.
Smaller the better $T=0$ $L(y) = K y^2$
- Larger is better; y tends to infinity when the target is at infinity. Examples include fuel efficiency, ultimate strength, and life.
Larger the better $L(y) = \frac{K}{y^2}$

Ah. So, there are for example, some products, which have dimensional specifications; length, breadth, height, you know diameter so on. So, forth some viscosity values, clearance values, these are definitive quantitative values, which are associated with the specification, which the customer wants and in such cases, a one can easily estimate that the loss would be minimalistic, if the deviation is lesser and lesser. So, why should we add the target value that is how the loss function is defined.

So, we call these kind of characteristics, you know the quality dimensions, where nominal is the best strategy. So, the lesser it is away from the target, the better it is in terms of inducted, you know minimalistic inducted losses; however, that is not the only kind of characteristics, which product may have, there may be characteristics, where they are essential, they are quite undesirable yet, they are there, because of a natural processes. For example, there are characteristics related to wear shrinkage deterioration of life of products friction losses, particularly moving machinery.

Let us say micro finishing aspects of machined surface ok. So, these are some of the quality characteristics, where one wants that, the smaller it is, the better it is ok. So, we call this smaller the better and here the type of loss; obviously, is set at a target value T equal to 0 ok, where we talk about L y simply as K y square. So, it is only how much, you know, the y is away from 0 target, which defines what is the loss.

So, there again other loss functions, which are related to some of the desirable qualities in different cases different situations. So, in these cases you; obviously, expect the value to be as large as possible, because they are desirable values for a system to run, in high quality and so, we call this larger the better and here the loss equation of course, will have to be different here. The last equation should have an inverse term K by y square.

Where y going to infinity would mean the loss is going to go to 0 and therefore,, because the loss function is varying in all the three cases; obviously, the average quality loss also should vary based on, whatever you know statistical significance, is there and one can calculate really the value of K for all these three different situations, and from there a lot of things related to what should be the target specification, etcetera could come out as consequences, which would help us to add the robustness at minimalistic loss, which is realistic for any system to operate.

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The value of K for nominal the best case is given by

$$L(y) = K(y-T)^2$$

Functional limit of characteristic $y = T \pm d$ is A

The unit cost because of such failure is A

$$A = K(T+d-T)^2 \Rightarrow K = \frac{A}{d^2}$$

Smaller the better case $\rightarrow$ Larger the better case

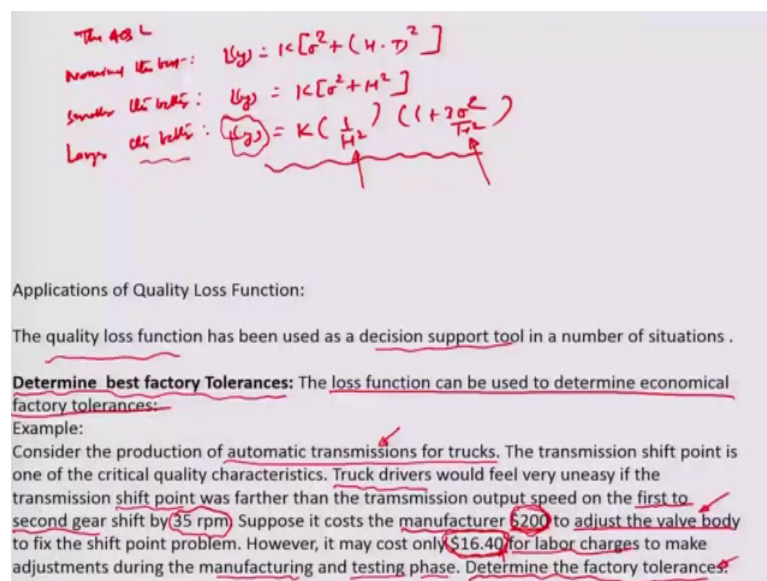
$K = \frac{A}{d^2}$ $\rightarrow$ $K = \frac{A}{d^2}$

So, the value of K for nominal, the best case is given by as you know $L(y)$ equals K times of y minus T whole square and, if we suppose, suppose that a tolerance of plus minus d is the operational the functional limit for the quality characteristic y . So, for example, functional limit of characteristic y is provided by y equal to T plus minus d the product is supposed to have failed or supposed to be scrapped, beyond these particular limits typically, if it does not meet T plus d standard or T minus d standard and let us say, if such a case occurs the unit cost, because of such failure not meeting y equal to T plus

minus d is some value A ok. So, we can easily calculate, what is the K? So, A becomes equal to K times of T plus d minus T square in other words K becomes A by d square. So, once the d is known to you.

And the A is known to you, you can find out what is going to be the lost constant and if we just, use the smaller, the better value, just as we use the nominal, the best case here, the lost constants would typically be K equal to A by d square or K equal to A T square, if it changes to larger the better value so. In fact, what you are seeing is that based on the loss equations modified to different aspects of quality parameters. You are having different loss constants in all these different cases, which would be somehow used to determine, some of the design specifications at the outset, based on certain cost decisions, which will come in as I will go along the way. So, the average quality losses for all these functions are completely different.

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The slide contains handwritten notes in red ink at the top and a printed section titled "Applications of Quality Loss Function:" below.

Handwritten notes:

- The best L
- Nominal the best: $Ly = 1[\sigma^2 + (\mu - T)^2]$
- Smaller the better: $Ly = 1[\sigma^2 + \mu^2]$
- Larger the better: $Ly = K\left(\frac{1}{\mu^2}\right)\left(1 + \frac{\sigma^2}{\mu^2}\right)$

Applications of Quality Loss Function:

The quality loss function has been used as a decision support tool in a number of situations.

Determine best factory Tolerances: The loss function can be used to determine economical factory tolerances.

Example:

Consider the production of automatic transmissions for trucks. The transmission shift point is one of the critical quality characteristics. Truck drivers would feel very uneasy if the transmission shift point was farther than the transmission output speed on the first to second gear shift by 35 rpm. Suppose it costs the manufacturer \$200 to adjust the valve body to fix the shift point problem. However, it may cost only \$16.40 for labor charges to make adjustments during the manufacturing and testing phase. Determine the factory tolerances.

So, the average quality loss for, let us say the nominal the best, you have already seen this value is going to be K times of sigma square plus mu minus T whole square assuming a large value of n for smaller, the better. This loss Ly goes at K times of sigma square plus mu square at T equal to 0, you want the target to be 0 in undesirable qualities or undesirable parameters and for larger the better case, although I will not derive this out, but the Ly typically, you can, you can do it for yourself in the same manner as I did

for the average quality loss for the other two cases, but here this is K times of 1 by square of μ times of 1 plus 3 sigma square by μ square.

So, that is how the loss is calculated in all these different cases, average quality loss is calculated ah. So, if you have a distribution of measurements associated with maintaining a system, you can always try to leave a cost and based on it you can take a decision, whether a lower at a lower cost, if the specifications could be made right or set right, will there be a definite advantage to do that. So, a tighter specification for example, might help to generate as less defects as possible and something, which can be easily done at the line side, can prevent any kind of failures or non compliance as to go into the market out, in the market and that is called robustness, adding robustness to the design.

So, let us look at a problem example here. For example, let us say we will, we will see how quality loss function can be used as a decision support tool, whenever we are designing a number of situations well let us talk about this particular situation, where we are determining what is going to be the best factory tolerance, the loss function here can be used to determine what is the most economical affected tolerance. Now, the example is that of.

So, for the automatic transmissions, for trucks you know there is a uncomfortable zone, that almost all truck drivers would, feel, when the shift point is shifted the gearshift point from let us say the first to the second gear is shifted by about 35 r p m ok. So, supposing, it was supposed to be changing gears at let us say 50 r p m instead of 50 . Now, it changes at 85 r p m . So, ah; obviously, there is a certain amount of time, which is spend to accelerate hard.

So, that the engine does not stall and it is really a quality problem, where the user is directly affected, the customer is directly affected and what was studied by this company, which produces the automatic transmissions and fits in the car in the tracks is that suppose, it costs the manufacturer about 200 dollars to adjust the valve body; obviously, when you are trying to assemble or disassemble and change the parametrics within the transmission ah. So, that. So, that you know the shift point can be again shifted back ah.

So, that cost is quite high, because of the disassembly involved and instead of that, if supposing you were to invest about 16 and a half dollars for just labour charges with the person who will make adjustments during the manufacturing stage itself.

So, that this problem does not go ahead, you see that in that case the, because of manufacturing and testing introduced to the line at a lower cost, you are able to do the very same. So, for that we need to determine what is going to be this level of tolerance that we want to set. So, that this cost option is better than the 200 dollars cost option. So, let us look at this problem and try to see, if the specifications can be designed ah. So, the loss caused by the product variation here from the target value.

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The loss caused by product deviation from the target value

$$L(y) = K(y - T)^2 \quad \text{where } y = T \pm d$$

$A = \$200$

$$\rightarrow K = \frac{A}{d^2} \Rightarrow K = \frac{200}{(35)^2} = 0.164 \text{ \$/rpm}^2$$

The factory tolerance should be determined based on the cost incurred

$\$16.40$ because of adjustments within this factory

$$16.40 = Kd^2 = 0.164 \times d^2$$

$d = \pm 10 \text{ rpm}$

This means that if the transmission shift point is further than $\pm 10 \text{ rpm}$ than nominal, it is cheaper to adjust the shift point at this factory than to wait for a correction in the order to make an adjustment.

Let us say $L(y)$ is K times of y minus T square, where y happens to be equal to T plus minus d and accordingly the K can be defined as a ratio between A by d square. So, this is the case, where we are talking about nominal the best and, because it has to be at the target point, target shift point and it is a way by about 35 r p m right now. So, a total amount of loss 200 dollars is incurred, if it goes out and it has to be adjusted in the market. So, let us say A equal to 200 dollars so; obviously, the K that would get built up is 200 divided by 35 square, because d . So, y is set at T plus minus d , where d is now 35 r p m ah. The transmission shift point is away by 35 r p m, while training from first to second gear. So, this happens to be.

Now, a value of K , which is equal to 0.164 dollar per r p m square and the factory tolerance needs to be now, determined, based on the cost incurred, because of an additional labour that you are putting on the line, who will do the adjustments and repair. So, that cost is about 16 and a half dollars, this is, because of adjustments within the

factory. Let us say of adjustments, so; obviously, in this particular case and the new value of d that will come out is going to be based on the K value, in the case of transmission shafts, which is 0.164. So, the d happens to be again equal to plus minus 10 r p m.

So, if one can set within the factory tolerances or what one can set within the factory? The r p m shift point to be within plus minus 10 r p m, then you do not have a problem of any, vehicle, going out in the market and it is cheaper to adjust the, vehicle in the factory itself, rather than wait for a complaint and make adjustments in warranty; obviously, it is a lost repetition as well when such failures start happening in the market. So, just write this down here that this means that if the transmission shift point is farther than 10 r p m in the nominal. It is cheaper to adjust the shift point at the factory, then to wait for a complaint in order to make an adjustment. So, you can see here, that we are essentially determining the specification, the final specification of the product based on an impost quality loss on us.

So, this is how you going to add robustness to all the, designs or design specifications. So, that you can avoid all criticalities, which come as a result of different phases of the product lifecycle, levied on to the way that the propagates through the lifecycle. So, I think, I could give you an understanding of what I mean by, adding robustness to a particular design. So, I think you know, there are many other example problems, where similar kind of situations would be arrived and, where you could actually make decisions related to even sometimes, which products should be selected, to be assembled into a design. for example, this other, problem which is there is about such a product selection we have.

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• **Product Selection :**
The loss function can be used to select products as illustrated by the following example.
High-Tech Rotor Dynamics is planning to buy a couple of thousand bolts to be used in their systems. The system requires highly reliable bolts. In case of bolt failure the system repair cost is estimated to be \$15.00. Two companies that offer different kinds of alloys in their products bid to supply the bolts. High-tech decides to go for destructive testing using 20 specimens. The criterion used for testing is the ultimate tensile strength measured in Kgf/mm². The lower specification limit is 11Kgf/mm². The purchase quantity is 20,000. The unit costs of products A and B are \$.14 and \$.13, respectively. Advise high tech rotor dynamics for its purchase decision.

Ah you know some kind of a design, where, we have a part, a screw or a bolt, which has to be bought in order to do the assembly and this is exactly an example of what would happen, when there is a characteristic, which is desirable and we have these characteristics desirable to be operating at level infinity, for the last to be as low as possible.

So, here we talked about assembling bolts and this company here buys about thousands of bolts to be used in its design; obviously, the designer has given options, related to you know some tested systems of two or three different companies across which, such bolts, can be received, they are highly reliable. In fact, and they are one of the critical components, for the rotor to successfully operate.

So, in of bolt failures, the system repair cost is estimated to be about 15 dollars and you know the two companies, that are being sourced out for these bolts, which would be able to meet the quality requirements of the high tech rotor dynamics, they offer different kind of alloys in their products and be to supply the bolts to the company ah; obviously, the high tech decision is based on destructive testing of 20 specimens to see, if the ultimate yield strength measured in kg force per millimetre square is as per the specification or not.

So, the lower specification limit for these bolts is about at least 11 kg force per millimetre square and one needs to have bolts above this specification limit and the and

much above 11 is going to be better. In fact, 11 is the selection criteria for the bolts, the quantity that is being purchased is about 20000 and; obviously, there are unit costs of the products. So, the different unit costs are represented here and one has to help to select, which bolt would be the better bolt in this particular case to add to the design.

So, I like to, solve this example in the next module, in the interest of time, but as of now, you kind of understand, what we mean by adding or building robustness or in the in the overall design through situations, which come along the various phases of product life cycle.

Thank you very much again.