Noise Management & Its Control
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Lecture - 66
Helmholtz Resonator - I

Hello. Welcome to Noise Control and its Management. Today is the last day of the eleventh week of this course. And starting today we will introduce a new way to control and actual reduce noise and in this case we will be talking about resonators.

Now these resonators are devices which are used in several applications, but typically they are good in terms of reducing noise if there is a particular frequency, which dominates the overall noise spectrum. If the noise is pretty much broad band then these resonators will still work, but only around a particular frequency to which they are a tuned to, but for all other frequencies they will not be effective at all.

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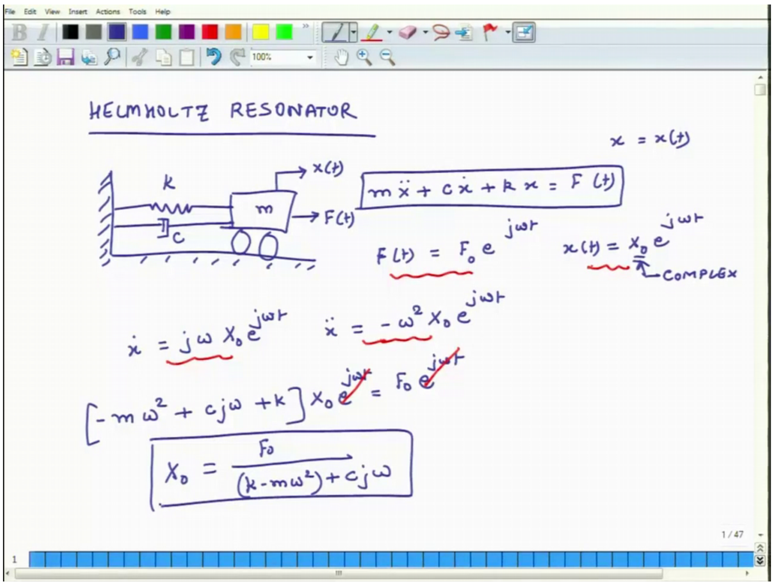

So, what we are going to discuss today is Helmholtz resonators. So, these resonators there these devices are known as resonators, because they have a single resonance point and they behave very effectively around that resonance point. But before we discuss that, I wanted to draw the analogy of a spring mass dash pot system. So, consider a mass and excuse me here I have to redraw it.

So, this is a mass and it is moving on a friction less surface and the mass is connected to a rigid frame, and it has a stiffness of k the spring stiffness and there is also a damping. Then the overall governing equation of this system, if I measure x in this positive direction and x is a function of time and suppose I apply a force F t then the overall governing system governing equation for this system is m x dot plus c x dot. So, its mass times acceleration m x double dot plus c x dot plus stiffness times is equal to F naught of t, where x is a function of time it changes with time.

Now if you solve this equation suppose this system is such that it is being excited by a sinusoidal force, then my F naught t in that case will be. So, this is not F naught t, it is just F t then my forcing function or external force is equal to F naught times e to the power of minus j omega t or you can have minus or negative it does not really matter; so e j omega t and because this is a linear system. So, I can also write x t and this linear system is being excited by a sinusoidal or a harmonic excitation. So, I will also have my solution of a similar form. So, that equals x naught e j omega t, where x naught can be a complex entity ok.

So, if I put this x naught in to my system then what I get is x dot equals j omega x naught e j omega t, and the second derivative of displacement with respect to time is minus omega square x naught e j omega t. Now, what I do is I can put this these all these 4 relations in my original governing equation and what I get is m omega square negative of that plus c times j of omega plus k x naught equals F naught excuse I have to multiply e j omega t on this side e j omega t equals F naught e j omega t and this exponent to the power of j omega t they cancel out.

So, my solution which is the amplitude complex amplitude of the displacement is nothing but F naught divided by k minus m omega square plus c j omega.

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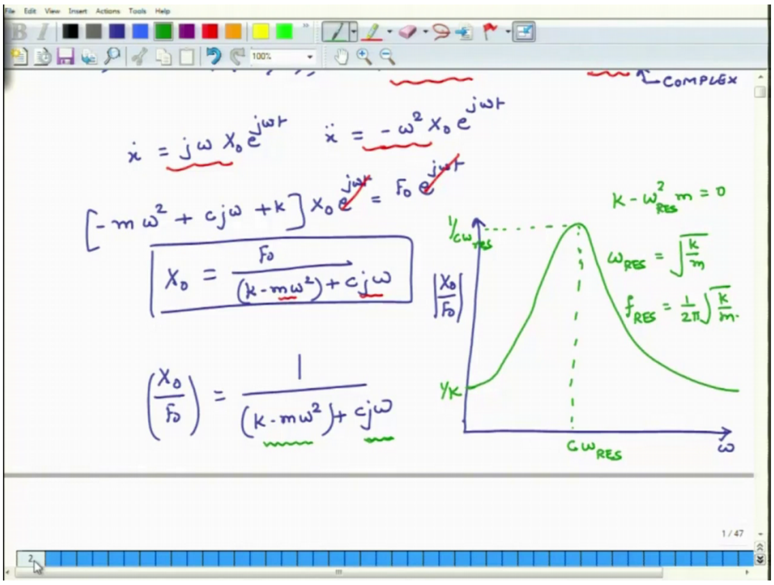

Now, if I plot x naught as a function of omega what happens. So, if on x axis I plot omega and on y axis let us say I plot x naught divided by F naught, let us say I am exciting it suppose by some Newton's times exponent j. So, x naught divided by F naught I am plotting on the y axis. So, at m at when there is d c or constant force then there is constant force then omega will be 0 when omega is 0 then these terms these terms will not exist.

So, at omega equals 0 my this thing. So, what is this my ratio? My ratio is x naught divided by F naught which is the transfer function is equal to 1 over k minus m omega square plus c j omega and when omega equals 0, then my ratio x naught over F naught will be 1 over k right and then what happens is that as I keep on increasing my omega this term, starts becoming less because initially omega is 0. So, in the brackets I have k, but as omega increases this term starts becoming less and this also, but this term starts becoming more, but anyway.

So, the solution if I plot with respect to omega it looks like this. So, what I am plotting here is I have to clarify, I am not plotting I am plotting x naught divided by F naught its ampli magnitude. So, it looks something like this. So, essential what this says is that it this thing becomes maximum when k is equal to m omega square. So, when k is equal to n m omega square then the ratio of x naught and F naught in terms of its absolute magnitude is 1 over c omega RES and I will call that resonance frequency, resonance angular frequency. So, this is resonance frequency c omega RES.

So, its maximum at c omega RES and what is the condition that omega is corresponds to resonance kind of thing when k minus omega RES square times m equals 0, which means that omega RES or angular frequencies resonance angular frequency is equal to k over m or if I want to find out the frequency not angular frequency, then it is equal to 1 over 2 pi k over m. Now one thing we should understand that when the resonance happens then we do not need a lot of force to excite the system, the system is very excitable at resonance frequency and this is what we see in the graph also that for any value of F naught this ratio of x naught over F naught becomes maximum at the resonance frequency at resonance frequency.

Now, what we will do is we will a have a similar understand develop a similar understanding in context of acoustics because the theme of today's discussion is Helmholtz resonators. So, this resonator which we have discussed is in the mechanical domain it is in the mechanical domain, but in acoustical domain also we have resonances. So, in acoustic domain one of one such resonator is known as Helmholtz resonator. So, how does it look like?

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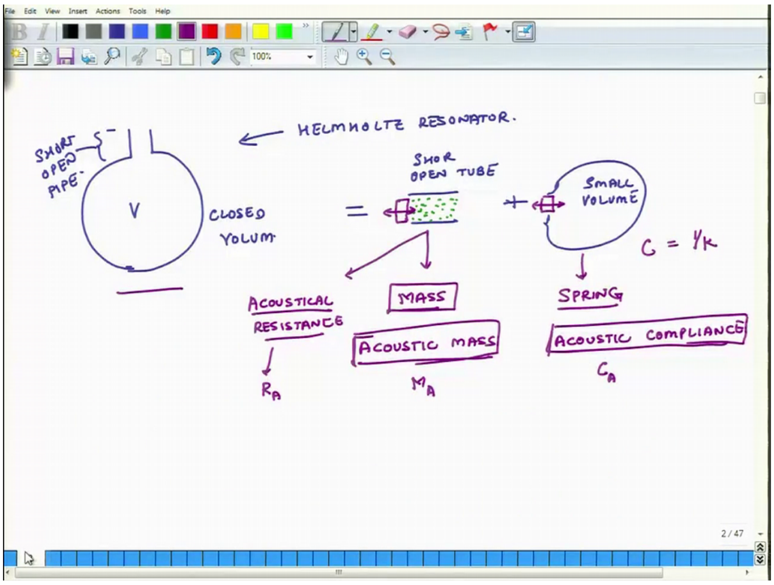

So, typically we can say that a Helmholtz resonator looks something like this. So, what this is a closed volume and this is a pipe short pipe short open pipe. So, this is a typical acoustical Helmholtz resonator. So, this entire thing is Helmholtz resonator and like mechanical resonators it also has its natural frequency, but before we discuss about that natural frequency, we should understand why does it have that natural frequency. So, think about. So, what does it what is it made up of. So, this is made up of a short opened tube and also a small volume it is made up of a short opened tube and a volume. So, this is volume closed volume and this is open tube and both of these are small in dimensions.

Now, what is the property of an open tube? Now if you go back to our one dimensional equations we have already discussed it, but we do not even have to go back this open tube is full of air and if I excite it with the piston like this then what will happen to the air the air will try to move in if the piston is going in, and it will try to come back if the piston is moving out. So, this thing, if the tube is long the amount of air which will be moving back and forth will be larger if the tube is short it will be smaller. So, this open tube acts as mass it acts as mass a short open tube acts.

So, I will say short this is important thing and this is small volume. So, a short open tube acts as rigid mass and now consider this small volume. So, here again consider a small piston which is placed at the opening of volume, and let us say this piston moves in and out now this air is sealed. So, when the piston moves what will happen a air will try to compress, it has nowhere to go. So, air tries to compress and when the piston moves in the back direction air will try to expand, because they will you will create some vacuum which will be filled by this small amount of air.

So, if you press the piston too deep, air gets compressed more and it tries to push back. So, this air in a small volume acts as a spring, in a small volume closed volume it acts as a spring. So, this resonator it has a mass and a spring connected in series. So, this looks similar if you think about it to this picture, you have a mass and a spring and the spring is fixed at the other end and you also have a spring which is fixed because the ends of the volume are closed. So, this acts as a combination of spring and mass in series.

So, it has its one resonance. Now this is in the area of acoustics. So, this mass like property of a short spring cube is called acoustic mass. So, we do not call it regular mass we call it acoustic mass because we use some other parameters to compute mass not just the actual mass of the air and the springiness of this small volume is known as acoustic compliance. So, what is compliance? Compliance of a spring is nothing but one over its stiffness.

So, in context of small volumes the you can have they have an acoustic stiffness or the inverse is called acoustic compliance the other thing which is here is that when. So, these are the 2 elements, acoustic mass and acoustic compliance. So, this is the ideal situation, but in real situations there is always damping also involved whether in mechanical system or in acoustical systems. So, this tube is not only mass, but it also has some acoustical damping. So, there is also an acoustical resistance associated with the tube.

So, predominantly it is mass, but it also has some resistive component. So, we label acoustic mass as M A acoustic compliance as C A, A being acoustical and acoustic resistance as R A and when you connect them then the overall system looks very similar to this spring mass dashpot system. So, then the next question is what are the values how can we calculate this acoustical compliance acoustical mass and acoustical resistance.

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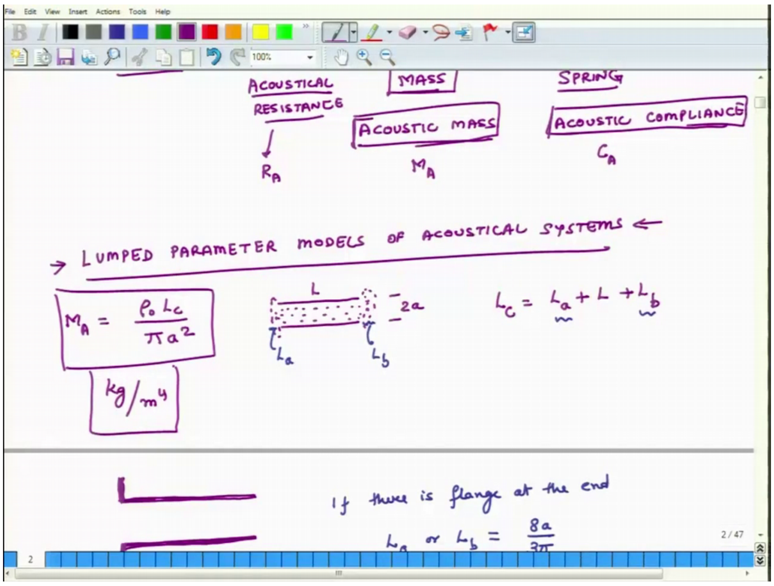

So, to understand this you have to understand the theory of a this lumped parameter models of acoustical systems. In one of few of my earlier courses on NPTEL actually I have covered this lump parameter modelling, but in this course we will not discuss this, but we will directly discuss talk about the results.

So, if we go and look at this approach what we will find is that the acoustical mass is nothing but rho naught Lc divided by pi a square. So, this is acoustic mass for a short tube and this tube has a length L, but here I am using L c and its diameter is 2 a. So, the equations says rho naught which is density of air inside the tube times L c, but this L c is different than the length of the tube divided by pi times a square where a is the radius of the tube a is the radius of the tube.

So, then the question is what is L c? So, it turns out that when you do experiments and you do also lot of analysis you find that it is not just the air which is inside the tube which moves in and out rigidly, but also because you have atmosphere outside also. So, some air outside at both ends also moves some air also moves which is on the outside. So, this additional length is called L a and this additional length of air on the other side is called L b. So, L c equals L a plus L plus L b and then how do you figure out what is the value of L a and L b.

So, once again you can have a tube like this and let us say in one case the end of the tube could have a flange like this it could be a flanged end. So, this is the thickness of the tube and. So, this end you can call it flanged end because there is a flange. So, this tube has one end which is having a flange and the other end is having no flange. So, if there is flange at the end then L a or L b depends you know L a or L b if the flange is in the beginning, then I will worry about L a if the flange you know. So, what we are doing is we are trying to find what are the incremental values of what are the values of L a and L b. So, L a or L b is equal to 8 a over 3 pi this people have figured out from analytical methods and finite element analysis, ok.

If there is no flange, then L a or L b equals 0.613 times a; so if the tube. So, we will compute.

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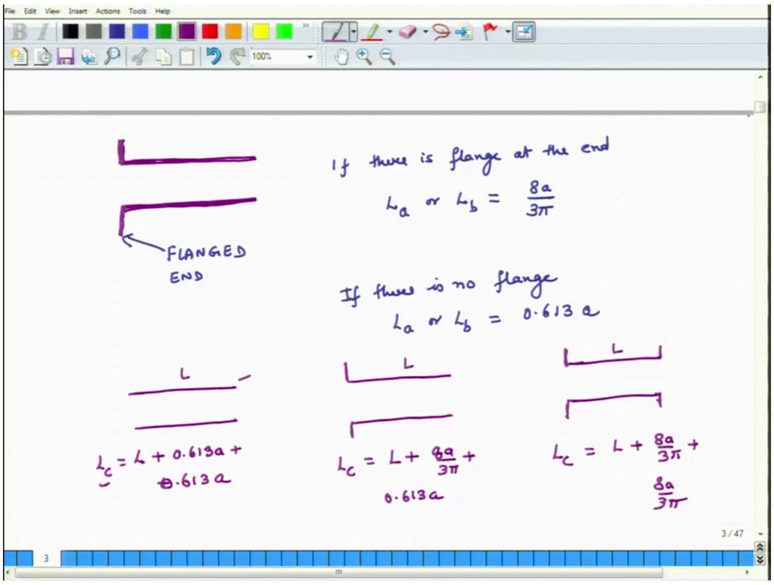

Suppose the tube is just like this both of its ends are having no flanges, then L c will be equal to L plus L a L a is what? 0.613 times a plus L b and there again at this end also there is no flange. So, it is 0.613 a if the tube looks like this now its starting has a flange, but it end point does not have a flange then L c equals L. So, L is the basic length plus 8 a over 3 pi plus 0.613 a and if the tube both the ends of the tube if they are having flanges then L c equals L plus 8 a over 3 pi plus 8 a over 3 pi.

So, this is how we compute. So, if we can calculate L c I can calculate the acoustical mass.

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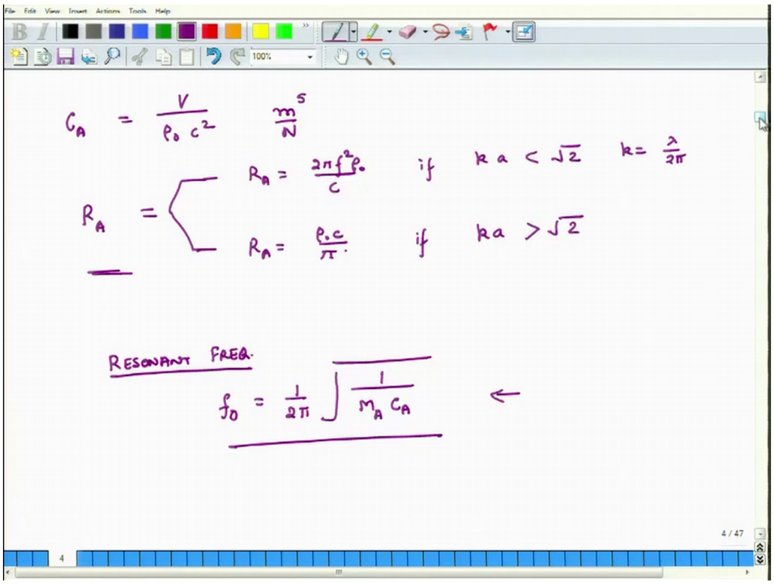

We will see acoustical compliance C A. So, the relation for C A is pretty much simple, volume divided by rho not c square where c is the speed of sound oh by the way the units of acoustical mass are what they are not kg’s because it is not exactly mass, but it is something like mass. So, it is kg per meters to the power of 4. So, these are the units of acoustical mass.

So, the next one is compliance. So, we said that whatever is the volume of the enclosure divided by rho naught which is the density of air times C square speed of sound and the units here are meters to the power of 5 divided by Newton’s and finally, acoustical resistance of the system is equal to and it will depend, you can use you have to select the right relation. So, the first relation is R A equals 2 pi f square rho divided by c rho naught if k times a is less than square root of 2 where k is the wavenumber and what is wavenumber lambda over 2 pi.

So, if the frequency of interest is such that k a divide is less than square root of 2 then we use this relation or else this is equal to rho naught c divided by pi. So, that is there. So, this is if k a is more than square root of 2. So, this is our acoustical resistance and the Helmholtz resonator. So, what is Helmholtz? It has a mass type of thing known as acoustic mass it has acoustical compliance it has acoustical resistance and the resonance or its resonant frequency resonance frequency of a resonator is we will call that f naught. So, F naught is what 1 over 2 pi divided by 1 over mass times compliance.

We know that for a spring mass system it is k over m 1 overs compliance is nothing but stiffness. So, it is mathematically equivalent. So, this is the resonant frequency of the system. So, if you want to use a Helmholtz resonator to absorb sound of a particular frequency, you have to design a resonator which has a resonance frequency which matches the frequency of your interest. So, we will do very quickly an example.

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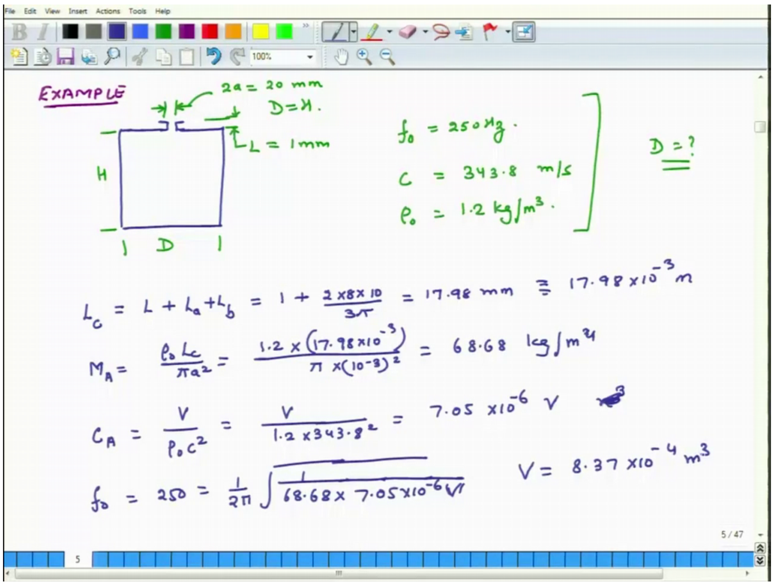

So, here the question is that we have to design a resonator and we are told that the resonator looks something like this. So, this is a cylinder and it has a short neck it is something like this.

So, this distance is diameter is D and the height is H and we are given we are told that D equals H and we have to design the resonator, the other thing we know in this problem is that this height which is L. So, L equals 1 millimeter. So, it is a very slender neck the diameter of this neck is 2 a and 2 a equals 20 millimeters and we are also told that this a flanged end the neck has a flange at the end. So, it is connected to the other side something else through a flange and we have to design such that the resonance frequency of the system is 250 hertz. We are told that c equals 343.8 meters per second and the density of air in the system is 1.2 kilograms per cubic meters.

So, with all this information we have to design. So, what does it mean? Basically we have to find D because if we know D then it is same as H, but we have to find the value of D such that the resonator gets its resonance at 250 hertz. So, what do we. So, we say first we will compute L c. So, L c equals L plus L a plus L b now what is L? L is one millimeters plus L b L a L a is what? Both the ends in this case are flanged. So, L a will be same as L b and that is equal to 2 times 8 times a is ten millimeters divided by 3 pi ok.

So, if we do the math you get 17.98 millimeters or in meters it is 17.98 times 10 to the power of minus 3 meters. So, now, I know L c. So, acoustic mass equals rho naught L c by pi a square and that equals rho naught is 1.2 times 17.98 into 10 to the power of minus 3 divided by pi into 10 to the power of minus 3 whole square. So, this comes out to be 68.68 kilograms per cubic per meter to the power of 4. Similarly acoustical compliance is v over rho naught c square and that equals v divided by 1.2 into 3, 43.8 whole square. So, this works out to be 7.05 into 10 to the power of minus 6 v. So, F naught is 250 hertz is equal to 1 over 2 pi, 1 divided by 68.68 times acoustical compliance 7.05 into 10 to the power of minus 6 v.

So, everything in this equation is known except for v. So, I compute V to be 8.37 into 10 to the power of minus 4 cubic meters.

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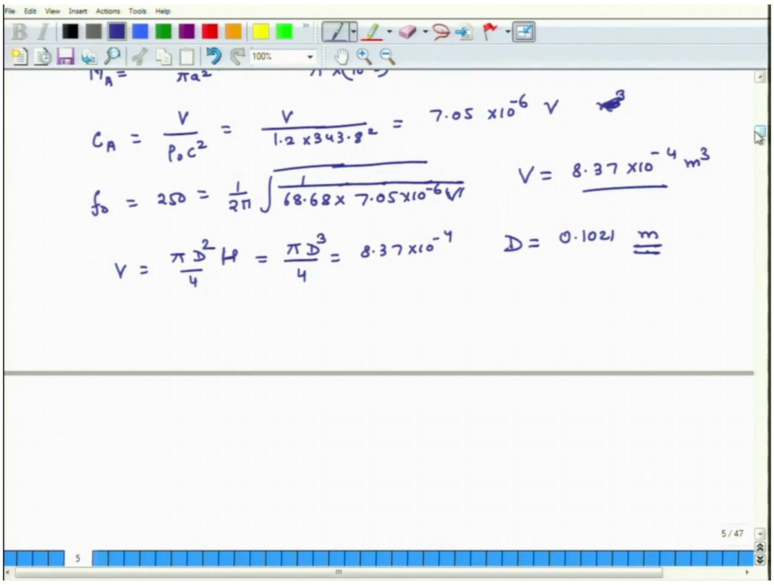

So, now I know v and I know that v equals pi times D square times height divided by 4 equals because D equals H. So, it is pi D cube divided by 4 and that equals 8.37 into 10 to the power of minus 4. So, solving for D, I get D equals 0.1021 meters ok.

So, this is how we can compute the a particular dimension for a resonator. So, now, we have learnt how to compute the resonance frequency of a Helmholtz resonator, but we do not now right now how to actually use this. What we has said till so far is, that if I have a Helmholtz resonator which looks something like this, then I can in some way use it to suck in up sound of particular frequency, but how will it suck in that particular sound how should it be connected to the area where sound is being generated all that stuff we will learn in our next class.

What we have learnt today is how to design a resonator which is tuned to a particular frequency, next we will learn how to tune a resonator or use a particular resonator to in a particular situation and how effective it is; because whatever what we have learnt does not tell us what is the transmission loss or the how much energy it is going to suck in all we have learnt is it will suck in acoustical energy at its resonant point. Will it suck in 100 percent of the energy, 90 percent, 80 percent we do not know.

So, that is something we will learn in our next class. So, with that we conclude our discussion for today and have a great week end and then I look forward to seeing all of you on the coming Monday.

Thank you very much. Bye.