Noise Management & Its Control
Prof. Nachiketa Tiwari
Department of Mechanical Engineering
Indian Institute of Technology, Kanpur

Lecture – 04
Beats

So, very quickly we will try to see what is happening from a mathematical standpoint.

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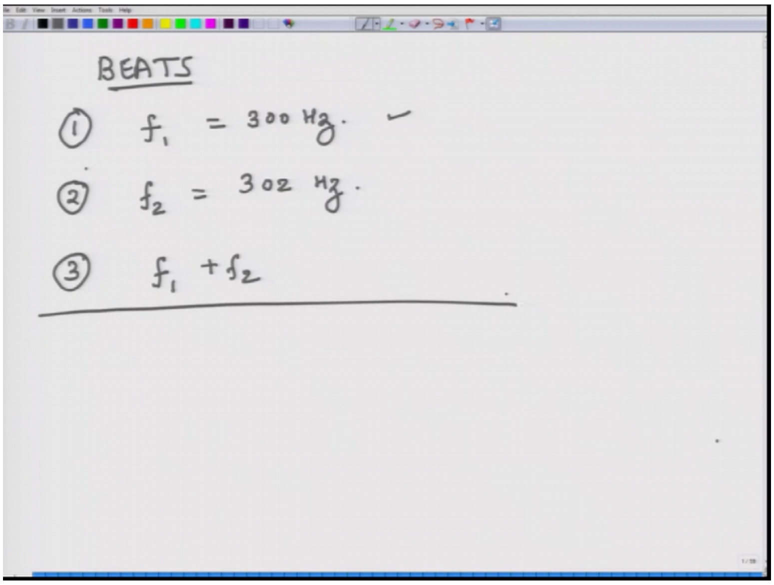

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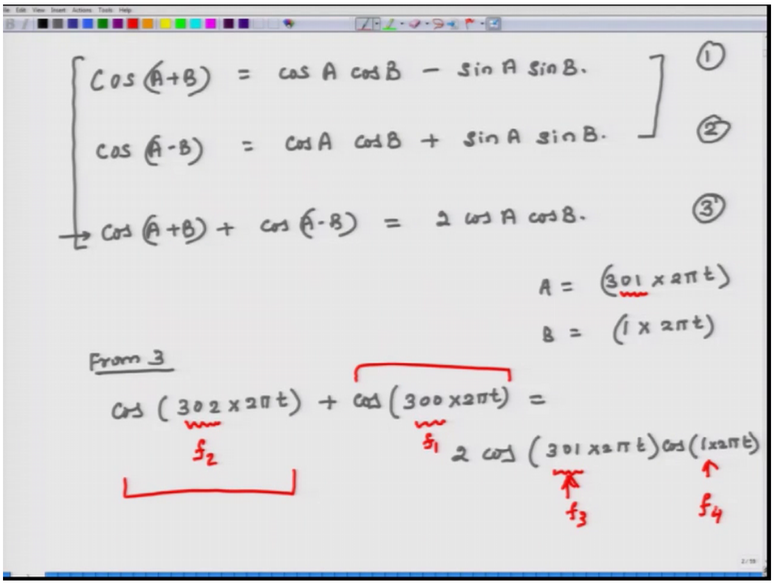

So, that you get an idea, why is this happening, first I will write 2 trigonometric expressions cosine A plus B is cosine A, cosine B minus sin A sin B and then cosine A minus B equals cosine A cosine B plus sin A sin B, these are standard trigonometric expressions and if I add these 2 guys these 2, what do I get cosine A plus B plus cosine A minus B equals 2 cosine A cosine B, right. So, this is equation 1, this is equation 2, this is equation 3 and how did I get equation 3, I just added one and 2 and I get this third equation.

So, these are 3 and the equation we have. So, we know how we get equation 3 basically from equation 1 and 2 and we will look at this third equation carefully and we will figure out what is happening with beats by understanding this equation in detail. So, what is A? A is the angle; A is the value of the angle. So, let us say that a is equal to 301 times 2 pi t and B is equal to is 1 times 2 pi t. So, what does that get us what is cosine A plus B. So, from 3 we get; what do we get from 3? We get cosine A plus B is 301 plus 1; 302; 302 times 2 pi t and what is A minus B cosine 300 times 2 pi t, this equals 2 times cosine A; A is what; 301 times 2 pi t cosine 2 pi t right actually I will make it I will write it slightly different cosine 1 times 2 pi t 1 times 2 pi t mathematically it is same, but just wanted to point out something.

Now, what is 302? 302 was the frequency of the second frequency f 2 which we played right this one was the second frequency f 2 and if I multiply by 2 pi that is the angular frequency 2 pi times f 2 is the angular frequency omega 2 and because it was a sinusoidal function. So, it is 2 pi f 2 times t. So, this represents the second wave; this represents the second tone which we played right whose frequency was 302 hertz; what is this guy f 1. So, this represents the first wave and when we played them together we basically mathematically how had we express it cosine 302 times 2 pi t plus cosine because when we are playing them together I have to add them up. So, this is what we are playing and mathematically we have through this mathematics we have shown that this left hand side is same as right hand side and what is this. So, this is one frequency and it is multiplied by another frequency. So, let us call this f 3 and let us call this f 4. So, this is f 3 and this is f 4.

Now, if you make a plot for the first function. So, when we played the in the third case essentially we played the second frequency and the first frequency simultaneously and mathematically it is equal to twice the multiple of a third frequency which is f 3 times a 4 frequency which is f one f 4 and what is the value of f 4 one and what is the value of f 3 it is 301; where does this 301 come from; it is basically this thing A right and B is 1; 1 hertz.

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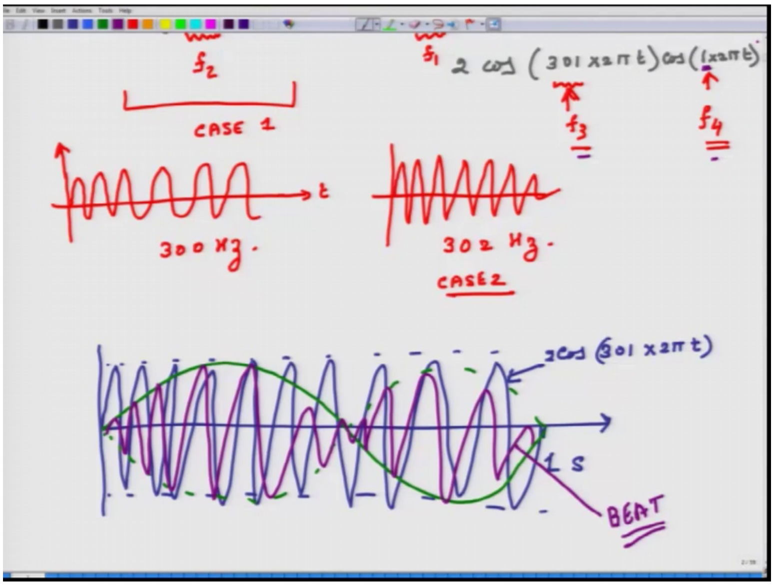

So, if n; so, in first case what did I play in case one; I just played a tone now this is not a good drawing or good graph, but this frequency. So, this is the time t and this is the amplitude some amplitude. So, what was the frequency in case one; it was how much 300 hertz. So, in one second this peak went up and down how many times 300 hertz; 300 times.

When I played the second sound file; in one second, it went up and down by 302 times, but it went up and down by the same amount because the amplitude is constant, right, amplitude was constant now, but when I play the 2 sound files together mathematically I can also show. So, this is case one, this is case 2 and this is because 300 and this is 302 for our ear it is not significantly different. So, we thought that it is the same sound, but when you play these slightly different sounds at the same level together what you are going to listen to is a multiple of f 3 and f 4 mathematically we have shown that right. So, what does that mean? So, what; that means, is. So, if I plot it; let us say this is one second. So, if I just plot f 3; how many times it will go up and down if I just plot f 3; it is frequency is 301. Now I will not be able to draw 301 times, but it is a large number of times.

So, this guy is cosine let us call this 2 cosine 301 times 2 t and the other thing is f 4; f 4 will go up and down how many times it will go only once up and down. So, I will plot it in a different color. So, actually I. So, I have started this from 0 actually cosine function starts from one. So, that some mistake, but anyway; so, this will go up and down once which means in one second it will do something like this if it was a sin wave its cosine maybe just shifted and what you are listening is what the product of green and the blue. So, the actual wave which you will listen to will be something; so, I will just take its mirror image also and the actual wave which I will listen to will be of sine wave which will go up and down how many times 300 times, but it will be bounded between the green curve. So, the actual wave it will be like this do you understand this because it is a multiple of f 3 and f 4 do you understand this.

So, just; so, too slightly different sounds if you play them individually you do not feel the difference, but if you play them together at some point of time they add up and they sound very loud and some points of time they cancel out each other and they sound very small you know at a much smaller level. So, this purple curve is the curve for the beat sound. So, this is the beat thing and what is the frequency of this beat; it is 1 hertz, it is essentially the defense and how do you do I you get this beat it is the difference between 302 minus 300 divided by 2 it is 302 minus 300 divided by 2. So, the sound will go up and down once each second if the difference is 2 hertz if the difference is 1 hertz, then it will go up and down only half times if the difference is 4 hertz then it will be 304 minus 304 divided by 2. So, each second it will go up and down to twice.

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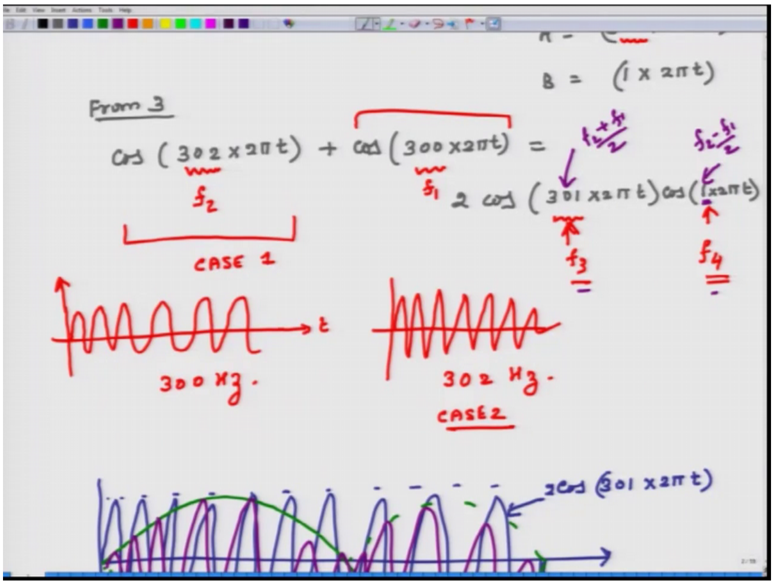

So, this is basically f 2 minus f 1 by 2 and this is what; f 2 plus f 1 by 2. So, this is the beat phenomena. So, let of times you may hear lot of sounds in a room and there may be in a big room there may be 2 machines and they may be running one may be running at I do not know maybe 500 hertz or 6000 rpm and another one may be running at slightly different speed and if you just run each of them they may sound similar, but when you run them together you may hear that the sound is going up and down up and down if the frequencies are identical you will not hear that difference, but if they are slightly difference that slight amount of difference you can sense because of this beat phenomena and that can also help you detect specific types of machines.

So, if you are; so, if ideally they should have the same frequency, but if one machine is slightly off because of some fault just by hearing this beat phenomena you can figure out that oh there is some problem with one of these 2 machines because the sound is not coming correctly it is going up and down up and down which means that the machines are at least one of the machine is not running at its desired rpm things like that.

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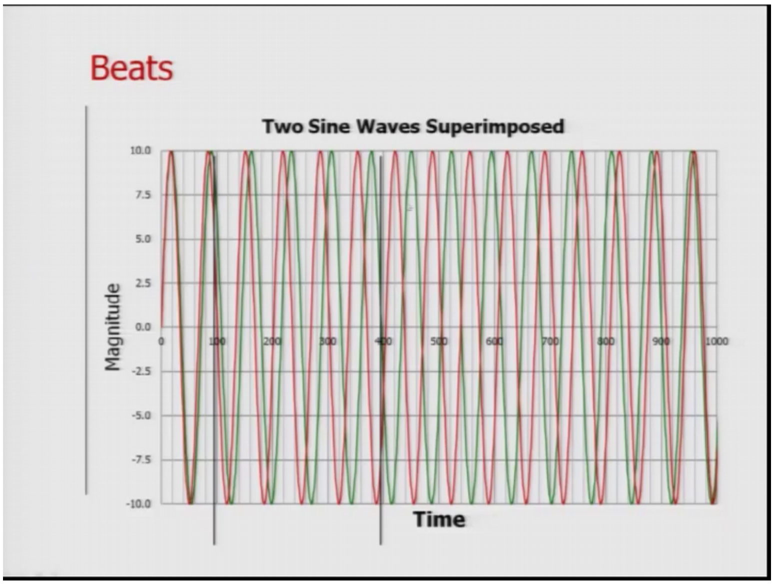

So, this is one small example; I wanted to share. So, this is again these are the 2 waves 300 and 301 and what I have done is the red one is maybe 300; green 1 is 302 hertz; they are individually plotted and when you add them up.

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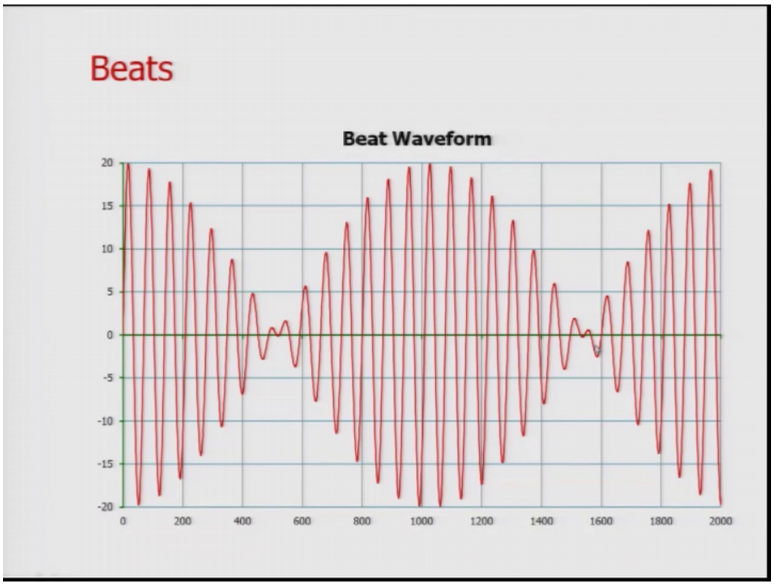

This is how the summation looks like and that is why sometimes you hear louder sounds sometimes you hear weaker sound and so on and so forth.

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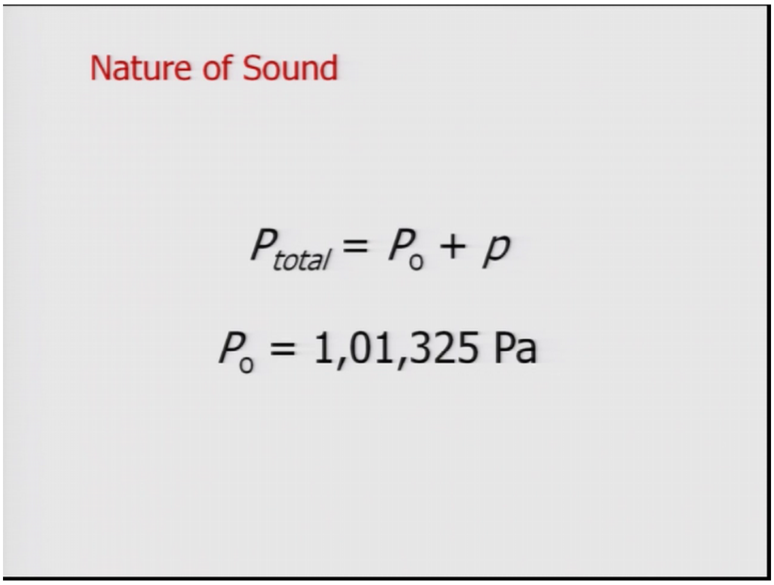

So, the last thing in today’s lecture which I wanted to show you is how or maybe 2 more slides is what are the typical sound pressures. So, remember we have said that this P is a very small number it is a very small number and I would like to give you a feel of how small these numbers are ok.

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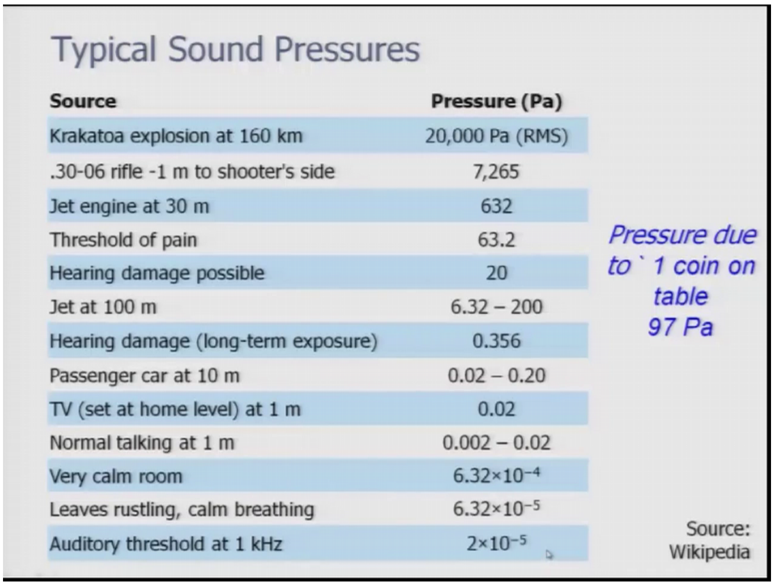

Source	Pressure (Pa)
Krakatoa explosion at 160 km	20,000 Pa (RMS)
.30-06 rifle -1 m to shooter's side	7,265
Jet engine at 30 m	632
Threshold of pain	63.2
Hearing damage possible	20
Jet at 100 m	6.32 – 200
Hearing damage (long-term exposure)	0.356
Passenger car at 10 m	0.02 – 0.20
TV (set at home level) at 1 m	0.02
Normal talking at 1 m	0.002 – 0.02
Very calm room	6.32×10^{-4}
Leaves rustling, calm breathing	6.32×10^{-5}
Auditory threshold at 1 kHz	2×10^{-5}

So, we will look at this chart and we will start looking at the lower you know the bottom most row to start with. So, we will look at the bottom most row initially and then we will go upwards.

So, it says auditory threshold at one kilohertz and the corresponding pressure is 2 times 10 to the power of minus 5 Pascals. So, you should understand what this means. So, suppose you have a room with no sound and I will give you an example maybe a mosquito a mosquito is about one meter away from you mosquito does not generate a whole lot of sound and in maybe your eyes are closed when it comes maybe one meter. So, it is approaching you if it is very far, you will not hear the sound being emitted by it at about one meter your ears barely start to listen perceive the sound generated by the mosquito and when it comes closer to you less than one meter then the sound becomes larger. So, when it is very far you do not hear and you about when it is 1 meter far away you start hearing the sound of the mosquito.

And it generates about 1000 hertz of frequency when it is coming and that is the minimum amount of sound you can your ears can perceive. So, the minimum amount of sound which your ears can perceive when the source is one meter away that corresponds to about 2 times 10 to the power of minus 5 Pascals. So, your ears can sense pressure fluctuations as small as 20 micro Pascals at 1000 hertz at 1000 hertz; they can perceive 20 micro Pascal pressure fluctuation. So, the value of t minimum value of P which our ears in general statistically some people may have very good ears they can perceive smaller values of P so, but in general the average value of P which our ears can perceive at a minimum level is 20 micro Pascals.

And that is known as auditory threshold because below that most of us cannot hear weaker sounds. So, that is why it known as auditory threshold and this is at 1000 hertz for different frequencies this threshold may change. So, at thousand hertz for thousand hertz tone this is the minimum sound most of us can perceive. So, it is a small number compared to 1.01 times 2 to the power of 5 Pascals; this is a very small number 10 to the power of 10 times less 10 orders of magnitude lesser another example is that suppose there is a empty room maybe the windows are open or windows are closed and there is no one, but you and there is no other sound then you become aware of your breathing; normally, when we are talking there is a lot of noise we do not hear our own breathing noise the pressure generated due to just because of our breathing which are ear sense is about 6.3 times 10 to the power of minus 5 Pascals 63 micro Pascals in a room if there is a person sitting next to you one meter away and this person is speaking typically the pressure fluctuations associated with his or her speaking lie between 2 milli Pascals to 20 milli Pascals. So, again these are small numbers very small numbers.

If you have a tv running at a normal level maybe one meter away it generates pressure fluctuations of 20 milli Pascals or 0.02 Pascals, again you should remember what is the value of P naught 1, 0, 1, 3, 2, 5 Pascals if you take in percent the terms of percentage, this is extremely small; if you are in a road then there is a passenger car just a regular car going away at normal speeds and it is 10 meters away from you it may be generating pressures in this range 0.02 to 0.2 Pascals if our ears are exposed to pressures more than 0.356 Pascals continuously on a regular basis they may get damaged. So, what this means is that our ears are very sensitive devices if the ear gets exposed to more than 20 Pascals of pressure then we may have immediate damage to the ear this damage when it is 0.356 Pascal this happens when there is long term exposure, but if the pressure exceeds 20 Pascals then our ear may get damaged right away.

So, most of the times, we hear sounds between 20 Pascals and 20 micro Pascals we do not we would not like to hear sounds which are corresponding to pressures in excess of 20 Pascals because that will create problems for our ears and the interesting and rather unfortunate thing is that at 20 Pascals when our ear gets damaged we do not feel the pain we will feel the pain only if the pressure is 60 Pascals or more. So, first the ear gets damaged and then you will hear the pain if the pressure is higher. So, you do not have to think that oh there was no pain. So, my ear was fine the ear maybe you maybe already def, but you will hear this damage pain only at higher pressures. So, these are very small sound pressure levels. So, this value of P is extremely small. So, our microphones or ears are designed in such a way that they can send very small pressures.

One final example to give you an idea what type of pressures we are talking about. So, if I take a small one rupee coin you know that weighs about 5 six grams and I place it on a table I can calculate how much pressure it generates on the table because I know the diameter of the coin. So, I can calculate its area I know its mass. So, I can multiply m times g divided by area the pressure which a small one rupee coin placed on a flat surface it generates it is about 97 Pascals and I place it on my hand and my hand does not feel a lot of pressure. So, the pressures we are talking about are much much smaller than this 97 Pascal number. So, when we are talking about sound acoustics noise we are talking about measuring managing controlling reducing extremely small pressure fluctuations extremely small pressure fluctuations.

So, that is what I wanted to discuss today tomorrow we will continue this journey and we will start talking about different scales on which we plot the results for acoustical measurements.

Thank you and have a great day, bye.