

Noise Management & Its Control
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Lecture – 32
Interference of 1-D Spherically Propagating Sound Waves – II

Hello, welcome to noise management and its control. Today is the second day of this week and yesterday, we were discussing the phenomenon of interference in context of sound waves specifically sound waves as they are coming from 2 point sources or monopoles and they meet at a distance r away from the midpoint of these 2 sources and what we are trying to figure out is that what is the pressure level at this point P .

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The image shows a handwritten derivation of the complex pressure $P(r, \theta, t)$ for two point sources. The derivation is as follows:

$$P(r, \theta, t) = \frac{j\omega\rho_0}{4\pi r} e^{j\omega t} \left[\frac{V_{v1}}{c} e^{-j\frac{\omega}{c}(r - \frac{d}{2}\sin\theta)} + \frac{V_{v2}}{c} e^{-j\frac{\omega}{c}(r + \frac{d}{2}\sin\theta)} \right]$$

Assuming $V_{v1} = V_{v2} = |V_v| e^{-j\frac{\phi}{2}}$, the equation becomes:

$$= \frac{j\omega\rho_0}{4\pi r} e^{j\omega t} \left[|V_v| e^{-j\frac{\omega}{c}(r - \frac{d}{2}\sin\theta) - j\frac{\phi}{2}} + |V_v| e^{-j\frac{\omega}{c}(r + \frac{d}{2}\sin\theta) - j\frac{\phi}{2}} \right]$$

Factoring out the common terms:

$$P(r, \theta, t) = \frac{|V_v| j\omega\rho_0}{4\pi r} e^{j\omega t - j\frac{\omega r}{c}} \left[e^{-j\left(\frac{\omega d}{2c} - \frac{\omega d}{2c}\sin\theta\right)} + e^{j\left(\frac{\omega d}{2c} - \frac{\omega d}{2c}\sin\theta\right)} \right]$$

The first part is labeled "Due to presence of one source at distance 'r'". The second part in brackets is labeled "Role of interference".

So, in the last class that is yesterday, we had developed this relation that complex pressure P as a function of r theta and t equals j omega rho naught divided by $4 \pi r$ times exponent $e^{j \omega t}$ multiplied by $V v 1$ exponent minus j omega r minus d over $2 \sin$ theta and this divided by c plus $V v 2$ e minus j omega r plus d over $2 \sin$ theta and entire thing in brackets.

Now, we had explained earlier that we are assuming that the magnitude of volume velocity 1 and volume velocity 2 are same and that is $V v$ and they are off by a phase of ϕ . So, I can rewrite $V v 1$ as $V v e^{-j \phi / 2}$ and this $V v 2$ can be expressed as $V v e^{j \phi / 2}$. So, I will make this substitution and

rewrite this. So, I get $j \omega \rho$ exponent $j \omega t$ divided by $4 \pi r V v e^{-j \phi}$ over 2 times $e^{j \omega r/c}$ minus d over $2 \sin \theta$ and there is a negative sign here plus magnitude of $V v$ times its phase $e^{j \phi}$ over 2 exponent minus $j \omega r/c$ plus d over $2 \sin \theta$. Now what we do is we reorganized all this stuff.

So, we taking $V v$ out and we also take $e^{-j \omega r/c}$ also out of the bracket. So, what we get is. So, this is $V v j \omega \rho$ exponent $j \omega t$ divided by $4 \pi r t$ minus $j \omega r/c$ and then in parentheses we are left with is exponent minus $j \phi$ over 2 plus d over $2 \sin \theta$. So, there has to be in ω here. So, I will ωd over $c \sin \theta$ and then the other term from second volume velocity source is exponent and I have $j \phi$ over 2 , there is an error here this sign; see the sign is negative here and there is a negative. So, it should become positive and if I have a negative outside then there should be a negative sign here. So, I made a mistake and the second term is e times j time ϕ over 2 plus ωd over $c \sin \theta$ actually there is also a negative sign here and in the denominator of ω over c there is a 2 term same thing is here.

So, I should have add 2 here and 2 here. Now let us look at this equation for complex pressure carefully you have the first term which is outside parentheses and then there is the second term which is inside the parentheses if you look at the term which is outside parentheses which is in green box this relation is pretty close to the pressure measured at a point r distance away which is r distance away from a single point source when we look at it; what does it say it is $V v s$ divided by $4 \pi r$. So, its $V v$ divided by $4 \pi r$ times $j \omega \rho$ exponent $e^{-j \omega r/c}$ times $e^{j \omega t}$ it is the same thing expect that here we have $V v$ units absolute magnitude that the only difference.

So, this is what it is the you can say that it is due to presence of one source at a distance r this term the second source the second term which is in the parentheses; it helps us understand the role of interference understand the role of interference why do I say that well in our original picture there was this parameter θ . So, that θ term is embedded here. So, as I change my angle this term in parentheses its value is going to change. So, θ the influence of θ is captured here also the distance between 2 point sources was d and parameter d is also here. So, if I make the distance larger or smaller all that influence will be reflected in these terms which are inside the brackets or parentheses and the third thing is that the phase difference between 2 sources was ϕ radians or ϕ radians and that parameter is also here.

So, these 3 parameters influence or the overall interference pattern and all these parameters are inside this parenthesis. So, if I have to see what is the role of interference I have to see the behavior of this term now I will just make one more observation that what does this term in the brackets or parenthesis look like it is e to the power of minus j in bracket phi divided by 2 minus omega d over 2 c time sin theta and the other is exponent and instead of negative j, I have positive j and the term in the bracket is exactly the same.

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Due to presence of one source at distance 'd':

$$= \frac{(V_0) j \omega p_0 e^{j \omega t} e^{-j \frac{\omega d}{c}}}{4 \pi r}$$

Role of interference:

$$2 \cos \left\{ \frac{\phi}{2} - \frac{\omega d}{2c} \sin \theta \right\}$$

We know $\frac{\omega d}{2c} = \frac{2 \pi f d}{2c} = \frac{\pi d}{\lambda}$

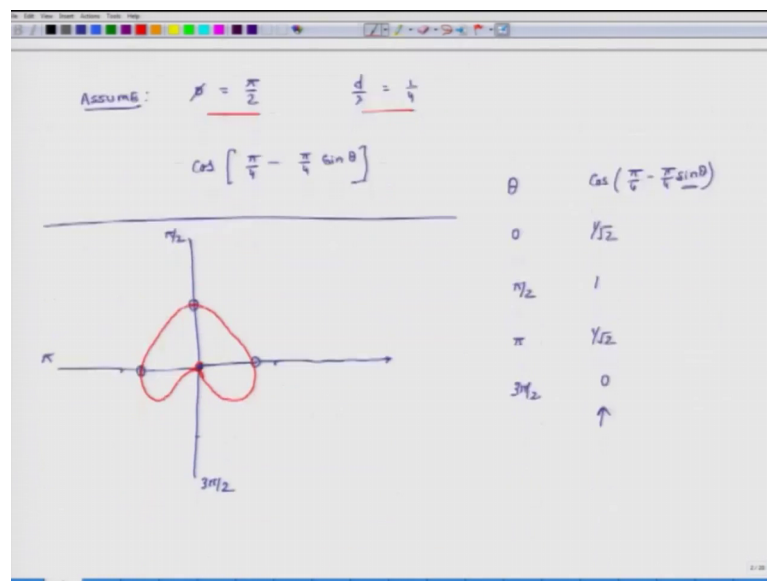
$$p(x, y, t) = \frac{(V_0) j \omega p_0 e^{j \omega (t - \frac{r}{c})}}{2 \pi r} \cos \left[\frac{\phi}{2} - \frac{\pi d}{\lambda} \sin \theta \right]$$

So, e to the power of j x plus e to the power of minus j x when you add this 2 up you get just 2 twice of cosine of x, right. So, we will just use that understanding to make this simplification. So, the term outside the bracket does not change, but what I get is 2 times cosine of phi divided by 2 minus omega d over 2 c sin theta.

So, if I have to see the dependence of this interference pattern on parameter theta it is this time which will help us understand it this term is not influenced by theta or phi or any of these parameters. So, if I have to understand the phenomenon of interference I have to see how this is influencing the overall pressure pattern. So, we will do a simple example to understand how what kind of patterns we can get last thing that we know that, but before we start doing that we will make one more simplification we know that omega d over 2 c is what omega is 2 pi f, right.

So, if you use that. So, I get $2\pi f d$ over $2c$ and c is frequency times the wavelength. So, f over c is $1/\lambda$ and 2 cancels out. So, I get πd over λ . So, my complex pressure the relation for complex pressure is finally, for the in context of the simplifying assumption which we had made earlier is $V v$ times j ω ρ naught exponent j ωt minus r over c divided by $4\pi r$ and I actually cancel out 2 from here. So, I get $2\pi r$ cosine ϕ divided by 2 minus πd over λ sin θ .

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So, now what we are going to do is we are going to develop a polar plot for this term which is in red. So, to develop a polar plot first we will find out the values of cosine of ϕ over 2 minus πd over λ sin θ for different values of θ and to do that we have to know ϕ and we have to know d and λ . So, we assume as a special case. So, what do we assume we assume that the phase difference between the 2 sources is 90 degree? So, ϕ equal π over 2 radians, how do I get that I can have a speaker which is vibrating at a particular frequency and I delay the signal to another identical speaker by π over 2 radians and it will produce this kind of a wave the second thing is I assume that d over λ is 1 by 4 what does this mean that the distance between the 2 sources is and the ratio of this distance with respect to λ 1 over 4 .

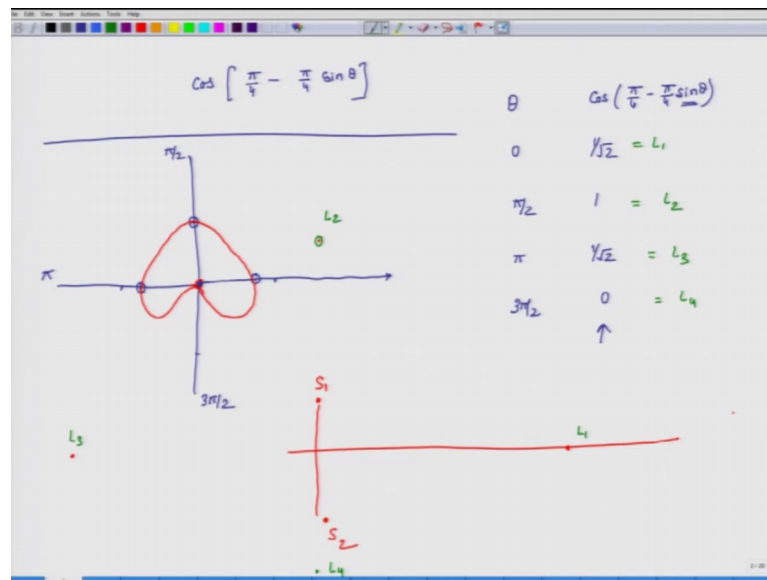
So, whatever wave length I am emitting I make sure that the distance between the 2 sources is one for the wave length. So, that is there. So, with this assumption let us calculate the value of cosine ϕ over 2 minus πd over λ . So, what do I have to

calculate I have to calculate cosine of $\frac{\pi}{4}$. So, that is $\frac{\pi}{4}$ radians minus $\frac{\pi}{4}$ radians and $\frac{d}{\lambda}$ is 1 by 4. So, this becomes $\frac{\pi}{4}$ sin of theta. So, for different value theta we will calculate this parameter. So, let us say theta and on the second column, we will calculate cosine of $\frac{\pi}{4}$ minus $\frac{\pi}{4}$ sin theta. So, let us assume theta to be 0 degree. So, when theta is 0, this term is 0 cosine $\frac{\pi}{4}$ is one over root 2, let us assume theta to be. So, we clock 90 degree each time. So, its $\frac{\pi}{2}$ radians if its $\frac{\pi}{2}$ the sine theta is 1.

So, let us $\frac{\pi}{4}$ minus $\frac{\pi}{4}$ by $\frac{\pi}{4}$ minus $\frac{\pi}{4}$ is 0 cosine of 0 is 1, then we say that we move another 90 degree and we have theta equals $\frac{\pi}{2}$ when theta is equal to $\frac{\pi}{2}$, then sin theta is again 0 cosine $\frac{\pi}{4}$ is 1 over root 2 and finally, we move to a situation where theta equals 270 degree or $\frac{3\pi}{2}$ radians. So, for $\frac{3\pi}{2}$ radians what is sin of $\frac{3\pi}{2}$ its negative 1. So, minus 1 times minus $\frac{\pi}{4}$. So, its $\frac{\pi}{4}$ by $\frac{\pi}{4}$ plus $\frac{\pi}{4}$ by $\frac{\pi}{4}$ is 90 degrees. So, this is $\frac{3\pi}{2}$ and this is 0. So, with this information we make a plot polar plot. So, what you have on a polar plot you have a reference axis and the reference axis corresponds to 0 degrees and then of course, you have all these this is your 90 degree this is. So, this is $\frac{\pi}{2}$ this is $\frac{3\pi}{2}$ and this is negative $\frac{\pi}{2}$ or $\frac{\pi}{2}$ and the distance from the origin represents this function. So, at 0 degrees; what is the value of the function one over root 2? So, let say this one. So, root 2 is one over root 2.707.

So, let say it is this point at $\frac{\pi}{2}$ it is 1 at π , its 1 over root 2 again and at $\frac{3\pi}{2}$ it is 0 degree. So, we will make a function which connects these points and let us see if I do it correct the first time. So, it will look something like that. So, what does this mean what this means is that if phi is equal to $\frac{\pi}{2}$ and $\frac{d}{\lambda}$ is 1 by 4 and what was our original problem, we had a source s 1 and we had a source s 2 and how are we measuring theta with respect to the midpoint.

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So, what this means is that if my microphone location is at theta equals 0 degrees right there will be some pressure it will be 1 over root 2 times the maximum possible value at theta equals 90 degrees which means if the microphone is located here. So, this is location L 1 this location L 2 this is location L 3 this is location L 4. So, this is location L 1, the pressure is going to be point seven times the maximum value at location L 2 it will be one times maximum value.

So, if I am placing my microphone at this location it will be the maximum measurement will be made here and if I place my microphone at location L 3, let us say here. So, I call it L 3, then again the pressure will be 0.7 times the maximum value, but if I place the microphone somewhere on the negative 90 degree line, let us say here L 4 then the contribution of the sources s 1 and s 2 they are going to cancel out and microphone will not send any sound. So, this is how my pressure is changing as I am changing the value of theta keeping the value of r fixed keeping the value of r fixed. So, this gives you some idea that if there are 2 sources and there are no reflections happening outside then these 2 sources how we can calculate the overall contribution of both the sources at a distance r and at the value of theta and the overall procedure is very straight forward.

So, if you have 3 source 4 sources whatever number of sources we have you can use same mathematics to compute the overall sound pressure level at the point of interest and what we have done today is we had made some simplifying assumption that volume

velocity of source are same and the phase is ϕ we do not necessary have to make those simplifying assumptions we made those assumption. So, that in context of teaching this concept things becomes simpler and more clear, but as you go back and if you have in your field of specialization some problems where these assumptions are not necessarily true you do not have to make this assumption and you can still get good answers by using the same mathematical principles which I have discussed.

So, that concludes our discussion for today starting tomorrow we will start talking about different noise measurement technologies and how do we make good noise related measurement. So, that is what we are going to start; we are going to start with and till then have a great day and I look forward to see you tomorrow, bye.