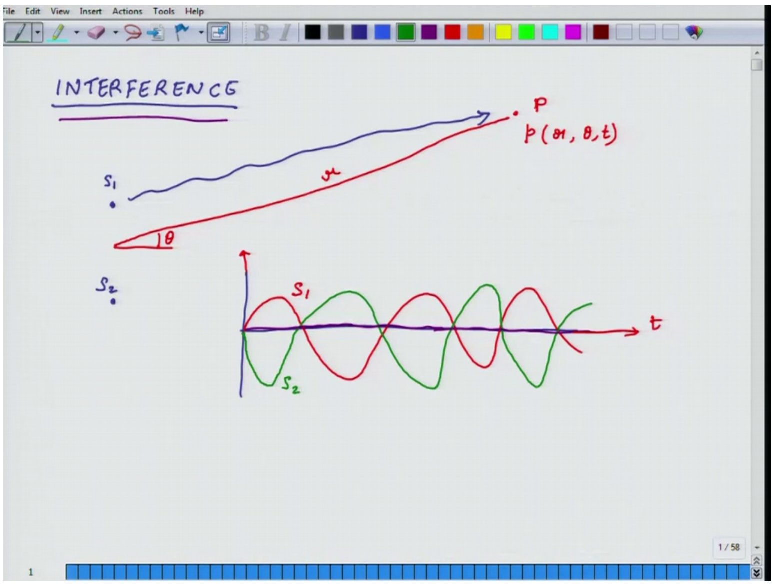
Noise Management & Its Control

Prof. Nachiketa Tiwari
Department of Mechanical Engineering
Indian Institute of Technology, Kanpur

Lecture - 31
Interference of 1-D Spherically Propagating Sound Waves - I

Hello, welcome to noise management and control this week we will continue our discussion on one dimensional waves one dimensional acoustic waves which are propagating spherically. So, that is one part of the week and in the subsequent part of the week we will start discussing about various technologies and issues related to noise measurement. So, today which is the first day of this week we will specifically talk about interference.

(Refer Slide Time: 00:47)



So, the problem is or the question which we will try to address today and maybe also tomorrow is that if there are 2 point source of sound and both these 2 sources are emitting some sound which is spherically symmetric. So, each source is emitting sound in all the directions and its propagating uniformly in all the directions. So, let us call this S source S 1 and this as S 2, then what we are interested in is that suppose, there is a point far away from these 2 sources let us call this point P. So, at this point we are interested in find the finding the sound pressure level. So, p and that is a function of. So, if the midpoint of this sources is this point and the distance between p and this midpoint is r and the angle with the horizontal axis is theta, then what we are interested in finding out is that what is the pressure at point p and how does it change when I change r or when I change theta or when I change time.

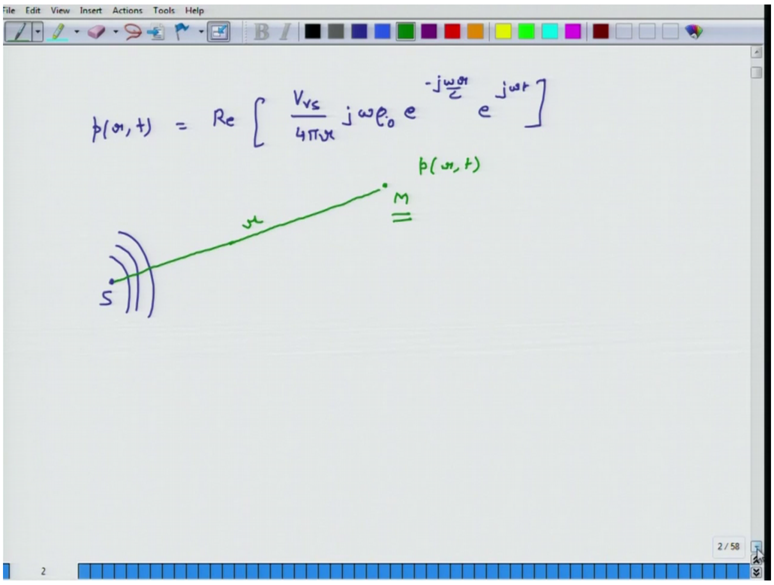
So, once again the problem which we are trying to addresses that if there to sound sources each of these sources are point sources of sound and they are producing a spherically symmetric wave patterns then how do this wave patterns add up at a far away distance point p, and the that is the question at hand and that is what is implied by interference. So, what could possibly happen is that. So, first we will talk about some basic concept, suppose the wave coming from source S 1 is depicted by let us say this red curve. So, what am I plotting I am plotting on the x axis time, and on the y axis I am plotting the nature of wave; so, from S 1.

So, this is the nature of wave coming from S 1 at point p, now a possible situation could be that there could be another wave coming from S 2 and because of the distance between S 1 and S 2 it may be arriving a little later and if it just arrives just late enough up. So, that when S 1 is maximum S 2 is at a minima and vice versa, then ultimately what will happen is that the total wave the nature of the way which is experienced at point p will be such that you add up these 2 and essentially you may get a flat horizontal line.

So, you may not hear any sound at all this is one possibility; another possibility could be that when both these waves are in face then they will add up each other and what you will hear is twice the pressure level you know. So, the sound waves pressure level will become amplified, and it will be 2 x if the magnitude or the amplitude of both the waves is same and of course, there could be all sorts of intermediate possibilities where the waves are either more or less adding together or maybe they are trying to destroy each other.

So, this phenomena is known as interference and in today’s lecture and probably also in tomorrow’s lecture; this is what we will try to understand from the stand point of physics. So, we know. So, before we start discussing interference, we know that if there is a point source if there is a point source and that emits sound in all the directions uniformly in all the directions.

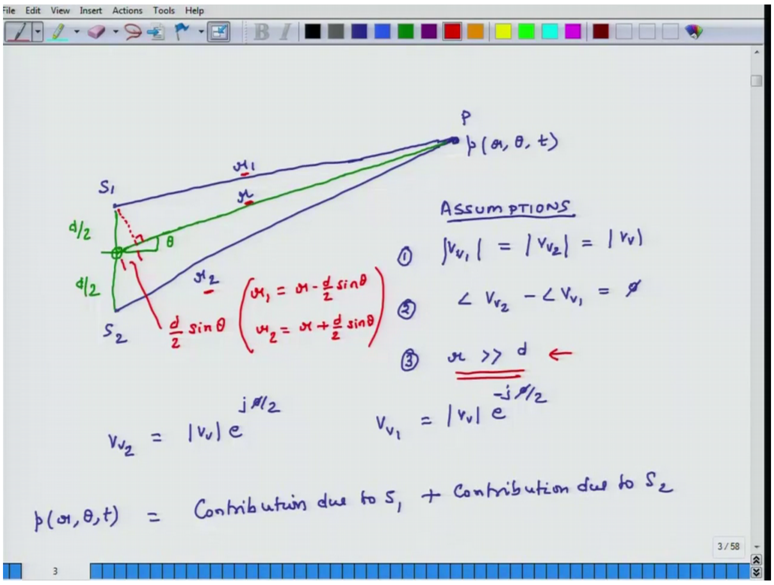
(Refer Slide Time: 05:06)



Then the pressure of the wave at a point distance r is equal to real component of Vvs divided 4 pi r j omega rho naught exponent minus j omega r divided by c, times e j omega t. So, what does what does this expression tell us that if I have a point source right let us say we call this source S and its emitting sound in a spherical fashion, then if you have a point let us say you have a microphone located here let us call this point m and the distance between S and m is r, then if I want to know what is the sound pressure level at location m what is it. So, I can signify it as pressure at location r.

So, that is at point m and at time t. So, then the pressure can be expressed as real component of Vvs. Vvs is the volume velocity of the source and Vvs is a complex entity complex entity divided by 4 pi r times j omega, omega is the angular frequency of the source. So, if it is just emitting one single frequency then that is the value of omega times rho naught e minus j omega r time e j omega t. So, this expression we had developed in our earlier lectures. So, this is an important relation which we will use to figure out the total pressure for a problem like this, where we have 2 sources and we want to figure out the total pressure at point p which is located at a distance r and it an angle theta away from the midpoint of sources S 1 and S 2.

(Refer Slide Time: 07:26)



So, with this background we will again reformulate the problem. So, let us say that there are 2 points S 1 and S 2 at point S 1 there is the first source and then the second source is rebuild as S 2, and what we are interested is in finding out the pressure at a far distant point and let us call this point as capital P and the pressure here is p which varies with distance r angle theta and t. The distant between p and S 1 is r 1 the distance between p and S 2 is r 2 and the distance between S 1 and S 2 is d.

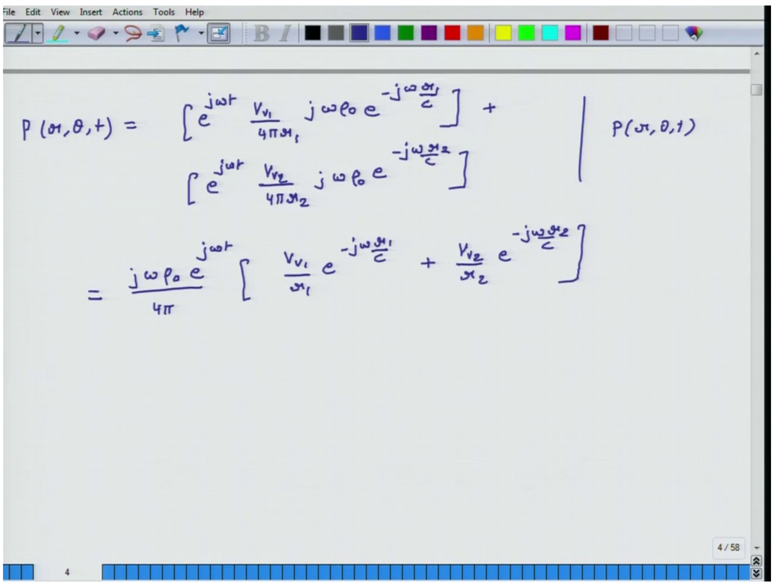
So, this is d over 2 and this is another d 2 and this is the midpoint. So, from midpoint if I draw a line connecting the midpoint to this far distance point t, then the angle between this line and the horizontal axis is theta. Now for purposes of today's class we will make some simplifying assumptions, but once you have learned this trick of how to solve this interference problem you can use these; this method to solve any problem related to interference and it need not be require it need not require all the simplifying assumptions which I am going to state. So, what are the assumptions? So, let us write them down. So, the first assumption is that the volume velocity of source S 1 and the volume velocity of sources S 2 they are same in magnitude. So, if volume velocity of source S 1 is V v 1 and its magnitude is this then it is equal to volume velocity of 2, I mean the magnitude of second volume velocity and let us call this parameter is because its common Vv.

So, this is the first assumption, the second assumption is that the phase difference between the second source and the first sources is 5. So, phase of Vv minus phase of Vv one is some number phi in radians. So, this is the second assumption and the third assumption is that this distance r, r is very large compared to distance d which is the distance between the 2 point sources. So, now, we will start working on the problem. So, we can express Vv 2 and Vv 1 in terms of Vv in such a way in this way. So, Vv 2 equals its magnitude times e to the power of j phi over 2 and Vv 1 the volume velocity of the first source is what its magnitude times exponent j times phi over 2 negative.

So, if I write these relations both of these relations tell us that the magnitude of first source and magnitude of second source is same that is mod of Vv and the phase difference between these 2 sources is phi divided by 2 minus of phi over 2 which is phi radians. So, this is one thing now let us look at pressure. So, pressure at capital point t I am expressing it p as a function of r theta and t, will have 2 components one component will be due to source S 1 and another component will be due to source S 2. So, I will just write that in words.

So, I will call it contribution due to S 1 plus contribution due to S 2 and we have already we know how to calculate contribution due to S 1 and S 2 using the relation which I explained earlier.

(Refer Slide Time: 12:33)



So, we will just use that relation to compute prt. So, p r theta t is equal to real of exponent j omega t times Vv 1 divided by four pi r 1 j omega rho naught e minus j omega r 1 divided by c. So, this is the contribution due to the first source and plus the contribution due to the second source is exponent j omega t, V v 2 divided by 4 pi r 2 j omega rho naught e minus j omega r 2 divided by c. So, these are the 2 contributions and. So, this is the actual pressure prt, if I want to find out complex pressure. So, this is lowercase p is the actual pressure if I want to find complex pressure. So, complex pressure I express it as capital P r theta t, then all I have do I have just erase it and excuse me and just replace small p by capital P and here these real components go away.

So, these are my complex pressure and if I want to find actual pressure I just take the real value of it and I find the actual pressure at point p. So, that is not a problem. So, we will work on the complex pressure and then later if you want we can always get its real component and find out its actual pressure at point p. So, now, we will process this equation further what we will do is, will take j omega rho naught e j omega t out. So, j omega rho naught exponent j omega t divided by four pi all this is common into Vv 1 divided by r 1, e minus j omega r 1 divided by c plus Vv 2 divided by r 2 e minus j omega r 2 by c now we know that r.

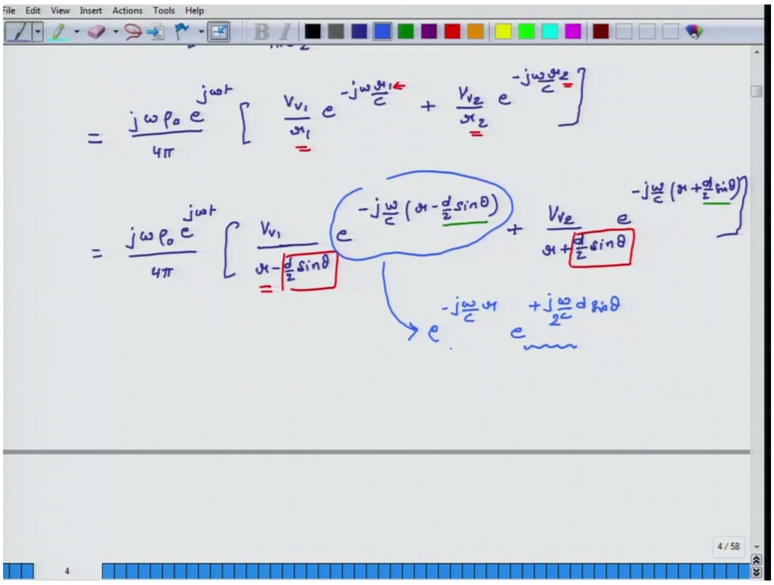
So, we have to find a relation between r 1 and r. So, for this what I will do is I will go back to the picture which I have drown earlier and let us look at this picture. So, this is r 1 this is no I am sorry this is r this is r 1 and this is r two. So, I have to relate find the relationship between r 1 and r and r 2 and r. To do that what I will do is I will draw a perpendicular from here from S 1 on this line which connects the midpoint to point p ok.

The other thing is we have to realize that this point p is extreme very far from the sources S 1 and S 2 or other terms we already have assume that r is extremely large compared to d now this distance this distance is how much? It is d over 2 sin theta and if the mid if the point if point p is extremely far away very very far away from the midpoint, in such a way that r is extremely large compare to d, then I can say that r 1 equals r minus d over 2 sin theta why?

Because these vectors r and r 1, they will be almost parallel the difference in distance will be just d by 2 sin theta similarly I can also say that r 2 equals r plus d over 2 sin theta. Now if you run into a situation where this particular assumption is not true then you have to actually use the actual value of r 1 and r 2. So, that depends on your geometry. So, since you know the geometry you can always figure out the value of r 1 and r 2, but in this case we are trying to make things simpler. So, we are making this simplifying assumption, but you do not have to do, if you want to get more exact solution.

So, with this simplification what we are going to do is we are going to replace r 1 here and here and r 2 here with r minus d over 2 sin theta and r plus d over 2 sin theta.

(Refer Slide Time: 18:24)



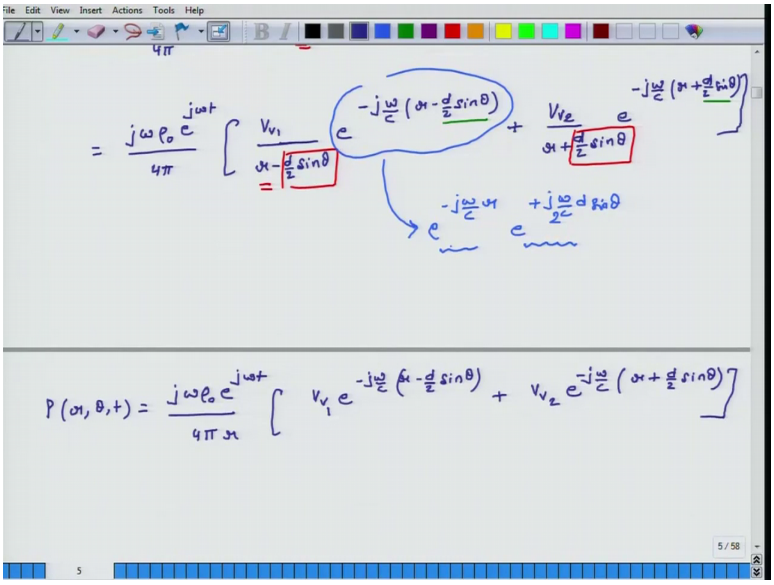
So, that is it what we are going to do next. So, I have j omega rho naught exponent j omega t divided by four pi Vv 1 divided by r 1. So, it is r minus d over 2 sin theta exponent minus j omega over c, r minus d over 2 sin theta plus Vv 2 over r plus d over 2 sin theta times exponent minus j omega over c, r plus d over 2 sin theta bracket close. So, this is where we are.

Now, what we going to do is we are making going to make this a little simple. So, we know that r is very large compared to d over 2 and the maximum value of sin theta can be 1. So, compared to r d over 2 time same sin theta will always be very small. So, I can drop this term out from the equation, similarly I can drop this term from the equation. Now the question is can I also drop this term and this term from the equation and the answer to that is no and the reason for that is here I am comparing r with d over 2 sin theta, in the numerator it is exponent j omega over c times r minus d over 2 sin theta.

So, let us look at this term what this term means is that it is a multiple of 2 terms, it is exponent minus j omega over c r times exponent plus j omega over c d sin theta. If this term was very small compared to this, I could have drop this, but that is not the case because what is e j omega r over c, it will be sin of omega r over c plus j times cosine of omega r over c and what is e j omega over 2 c times d sin theta it is again there is sin term and cosine term. So, both these terms involve one sin term and one cosine term and there is no way I can make a successful argument that this sin term and cosine term is going to be extremely large compared to the other sin term and cosine term. So, I cannot make this simplification in the numerator, I cannot drop the d over 2 sin theta term in the numerator, but in the denominator because I am directly adding r and d over 2 r and d over sin theta, I can ignore its contribution. So, with this understanding I make a simpler relation and I say that complex pressure P r theta t equals j omega rho naught exponent j omega t divided by 4 pi Vv and now because I have dropped r from here.

So, I cannot r d over 2 sin theta I can take r out and I am left with an on the inside e minus j omega over c, r minus d by 2 sin theta and this is Vv 1 times Vv 2 e j omega over c excuse me there should be negative sign here r plus d over 2 sin theta.

(Refer Slide Time: 21:54)



So, this is where we are today, what we will do tomorrow is we will continue this discussion and will make this equation even simpler and then try to make some physical sense out of this particular equation. So, that concludes our lecture for today and I look forward to seeing you once again tomorrow till then have a great day, bye.