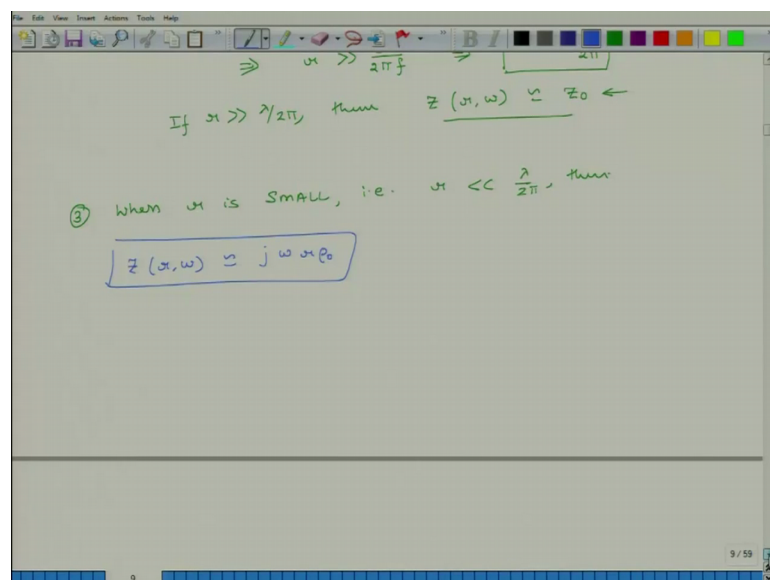


Noise Management & Its Control
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Lecture – 30
Volume Velocity – I

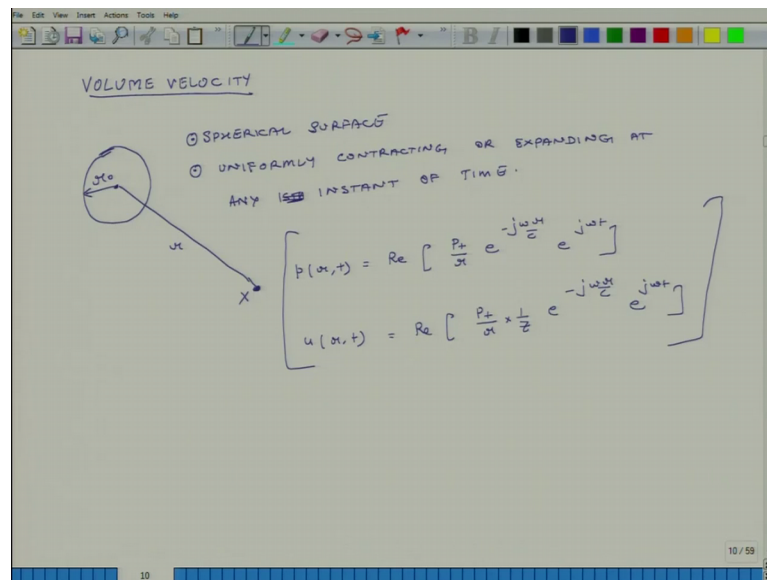
Hello, welcome to Noise Control and its Management. Today is the last day of the fifth week of this course. And what we plan to do today is introduce a new concept known as volume velocity and this particular concept will be; you can; we will find it very useful in terms of dealing with noise, especially noise which is coming from surfaces which are vibrating.

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For instance, you are in a room or in outside in the open, and you are interested in trying to figure out the magnitude of noise as well as the frequency content of noise from a piece of sheet maybe a metallic sheet which could be a part of the enclosure of a big machine maybe an engine or a generator which and there it has an enclosure and that encloses sheet metal is vibrating rapidly and that is generating a lot of noise. And if you; so you would like to get a feel that suppose, I reduce the amplitude of these vibrations by a factor of 2, what will be the impact of my on my noise levels and so on and so forth. In that context this notion of volume velocity comes in very handy.

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So, that is why I wanted to talk about volume velocity in context of this course. So, the topic we are going to cover is volume velocity, and what we will do is we will relate this volume velocity to the pressure which is generated by a vibrating object. Now at least in today's lecture, we will talk about vibrations from a surface which is spherical in shape which is spherical in shape, but some of the insides we will learn by talking about volume velocity will be pretty general and you can apply it to even more complicated shapes as well.

So, suppose we have a sphere. Suppose we have a sphere. So, that the center of the sphere and imagine you have a sphere and this sphere outside surface is vibrating and how is it vibrating? At a given time, either it is expanding uniformly in all the directions or it is contracting uniformly in all the directions. You may think about like a spherical ball and when the pressure goes in; pressure in the ball increases; the ball increases in diameter uniformly in all the directions and suppose you use reduce the pressure then the ball strings to a smaller diameter uniformly.

So, here we are assuming that the surface which is vibrating is spherical surface, that is, one thing; second thing is it is uniformly either contracting or expanding at any instant of time. So, it was not that a part of the ball is expanding another part is contracting essentially what it means is that every point on the outside surface of the ball is moving in phase is in phase with respect to each other the phase difference of motion of all the

points on the surface of the ball is 0. All the points are in the same phase. So, they are uniformly expanding or uniformly contracting.

In real problem; in real shapes in real geometry is this may not be always the case it could be that suppose you have this piece of sheet it could be possible, that when the sheet vibrates maybe this part is moving like this in the positive Z direction and this part it could be moving in the negative Z direction at a given point of time, but at least in the context of today's discussion we are not assuming the presence of these kind of phase differences. What we are assuming? Once again as I had mentioned earlier is all the points are moving in phase to each other are in phase with each other. So, for this type of a spherical thing.

If I am interested let us say at a far distance at some distance r away and let us assume that r is very large compared to the radius of the ball which is r_0 . So, suppose I am far away from the ball, then this ball appears like a small point right and I can consider it as a small point. And in that case, what is the pressure? which will be generated will the pressure which is going to be generated is $p(r,t) = \text{real of } P \text{ plus divided by } r \text{ e minus } j \omega r \text{ over } c \text{ e } j \omega t$ this is the equation which we had developed yesterday. Similarly, velocity of the particle of air at location x right is what is real of $u \text{ plus or I do not have to talk about } u \text{ plus I can express it has } P \text{ plus by } r \text{ times } 1 \text{ over } Z$, where Z is the specific acoustic impedance for spherical spherically propagating waves times $e \text{ minus } j \omega r \text{ over } c \text{ e } j \omega t$. So, these are the two equations one is for velocity and the other one is for pressure. And what is Z we have already developed an expression for z . So, we will just write that.

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ANY INSTANT OF TIME.

$$p(r, t) = \text{Re} \left[\frac{P_+}{r} e^{-j\omega t} e^{j\omega r} \right]$$

$$u(r, t) = \text{Re} \left[\frac{P_+}{\rho_0 c r} e^{-j\omega t} e^{j\omega r} \right]$$

$$\bar{U}_+(r, \omega) = \frac{P_+}{\rho_0 c r} e^{-j\omega r} \left(\frac{1}{\rho_0 c} + \frac{1}{j\omega \rho_0 r} \right)$$

So, this term is what this is P bar r omega and this term this is U bar r omega. So, u bar and because it is propagating in the positive direction I will put a plus sign here. So, U bar plus r omega equals P plus over r e minus j omega r divided by c times 1 over Z. So, and one over Z we had seen that relation earlier. So, I will just write it down directly 1 over rho naught 1divided by rho naught c plus one divided by j omega rho naught r. Now I will now what; why I will do is; I will calculate the value of U plus bar on the surface of the sphere which is generating the sound.

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$$\bar{U}_+(r, \omega) = \frac{P_+}{\rho_0 c r} e^{-j\omega r} \left(\frac{1}{\rho_0 c} + \frac{1}{j\omega \rho_0 r} \right)$$

On the surface of sphere, $r = a_0$, thus:

$$\bar{U}_+(a_0, \omega) = \frac{P_+}{\rho_0 c a_0} e^{-j\omega a_0} \left[\frac{1}{\rho_0 c} + \frac{1}{j\omega \rho_0 a_0} \right]$$

$$P_+ = \bar{U}_+(a_0, \omega) a_0 e^{j\omega a_0} \frac{j\omega \rho_0 a_0 c}{j\omega a_0 + c}$$

$$P_+ = \left[\bar{U}_+(a_0, \omega) a_0^2 \right] e^{j\omega a_0} \frac{j\omega \rho_0}{j\omega a_0 + c}$$

So, on the surface of the sphere, what is the value of r ? It is r_{naught} . So, on the surface of sphere r is equal to r_{naught} , and thus the value on the surface of the sphere. So, what am I doing I am essentially computing the velocity, the complex velocity magnet amplitude on the surface of the sphere \bar{U} plus bar is what it is the amplitude of the velocity at location r and I am computing it on the surface of the sphere. So, this is equal to P_{plus} divided by r_{naught} $e^{-j\omega r_{\text{naught}}}$ divided by c times one over ρ_{naught} c plus one over $j\omega \rho_{\text{naught}} r_{\text{naught}}$.

So, if I manipulate this relation, I can write it as that P_{plus} equals \bar{U} plus which is a function of r_{naught} and ω . So, I am computing this P_{plus} in terms of \bar{U} plus at location r_{naught} ω times r_{naught} and then exponent $j\omega r_{\text{naught}}$ divided by c and if I moved this entire thing to the other side you can do the math you will get $j\omega \rho_{\text{naught}} r_{\text{naught}} c$ divided by $j\omega r_{\text{naught}} + c$ is what I get and I can reorganize this all this good stuff. So, I get \bar{U} plus bar evaluated on the surface of the sphere times r_{naught}^2 . This is one thing times exponent $j\omega r_{\text{naught}}$ divided by c .

So, r_{naught} I have taken out this is this was the r_{naught} that r_{naught} I have moved in the sphere right. So, and then I am left with $j\omega \rho_{\text{naught}}$ and what I am doing is I am dividing the numerator as well as the denominator by c . So, c from the numerator goes away and in the denominator I am having $j\omega r_{\text{naught}}$ divided by c plus 1.

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The image shows a whiteboard with handwritten mathematical derivations. At the top, the pressure P_+ is expressed as $\bar{U}_+(\alpha_0, \omega) \alpha_0^2 e^{\frac{j\omega \alpha_0}{c}}$ multiplied by $\frac{j\omega \rho_0 \alpha_0 c}{j\omega \alpha_0 + c}$. Below this, a boxed equation shows $P_+ = [\bar{U}_+(\alpha_0, \omega) \alpha_0^2] \cdot e^{\frac{j\omega \alpha_0}{c}} \cdot \frac{j\omega \rho_0}{j\omega \frac{\alpha_0}{c} + 1}$, with a circled '1' next to it. A note 'SURFACE AREA OF SPHERE' with an arrow points to the $4\pi \alpha_0^2$ term in the next equation. The volume velocity is defined as $V_{vs} = \text{VOLUME VELOCITY} \equiv \bar{U}_+(\alpha_0, \omega) \cdot 4\pi \alpha_0^2$, with a circled '2' next to it. This leads to $\frac{V_{vs}}{4\pi} = \bar{U}_+(\alpha_0, \omega) \cdot \alpha_0^2$. Finally, a boxed equation shows $P_+ = \left[\frac{V_{vs}}{4\pi} \cdot \frac{j\omega \rho_0 e^{\frac{j\omega \alpha_0}{c}}}{1 + j\omega \frac{\alpha_0}{c}} \right]$.

So, this is my expression for P plus. Now, I define the term which I said I will introduce today, and I will say that volume velocity. Volume velocity is equal to, what is it equal to? It is the amount of volume of air which the surface which is uniformly expanding or contracting it is displacing the magnitude amplitude of that volume of air. So, what; So, that is essentially what? The way if the ball is uniformly expanding or contracting, then if I multiply the velocity of the point on the surface of the ball times the surface area of the ball that is the amount of air is it will displace right at a given point of time so this is equal to U plus r naught omega bar times 4 pi r square; 4 pi r square is what?

It is the surface area; surface area of sphere. And this I am going to call it as V v s; volume velocity of source, why source because ball is the source of sound. So, this is V v s. So, I can say that V v s divided by 4 pi equals U plus bar evaluated on the surface of the sphere times r square. So, let us call this equation 1. Let us call this equation 2. So, if I plug 2 in 1, I get P plus equals V v s divided by 4 pi times j omega rho naught e j omega r naught divided by c divided by 1 plus j omega r naught divided by c plus 1.

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The image shows a whiteboard with handwritten mathematical derivations. At the top, the equation for complex pressure amplitude is given as:

$$p(r, t) = \text{Re} \left[\left\{ \frac{V_{vs}}{4\pi r} \cdot \frac{j\omega\rho_0}{1 + j\frac{\omega r}{c}} \right\} \times \frac{1}{r} e^{-j\frac{\omega r}{c}} e^{j\omega t} \right]$$

Below this, a condition is stated: $\text{If } \frac{\omega r_0}{c} \ll 1 \Rightarrow \frac{2\pi f r_0}{c} \ll 1 \Rightarrow r_0 \ll \frac{c}{2\pi}$ then:

The terms $1 + j\frac{\omega r_0}{c} \approx 1$ and $e^{j\frac{\omega r_0}{c}} \approx 1$ are shown to be approximated to 1. The final boxed equation is:

$$p(r, t) = \text{Re} \left[\frac{V_{vs}}{4\pi r} j\omega\rho_0 e^{-j\omega\left(\frac{r}{c} - t\right)} \right]$$

So, this is the equation number 3 and what does equation number three says? That, I can calculate the value of complex pressure amplitude for an outwardly travelling wave, which is P plus in terms of the volume velocity right and what is volume velocity physically what does it represent? It is a multiple of U plus and the surface area. So, if the ball is expanding and contracting very rapidly; then the corresponding value of

pressure will be very high and vice versa; what I will do is I will now use this expression for P plus and I will plug it back into our original equation which is this one right; because, ultimately I am interested in finding out pressure as a function of time and position. So, here I am going to plug the value of P plus in this equation.

So, I put this 1, 2 times. So, eventually what I am getting is the expression for pressure at any position. So, pressure is equal to P plus and P plus is this long thing. So, it is the real component of P plus which is $V v s$ divided by $4 \pi e$ to the power excuse me; times $j \omega \rho$ naught divided by $1 + j \omega r$ naught divided by $c e^{j \omega r}$ minus r naught by c or actually I will just keep it in the original shape. So, I will still have $e^{j \omega r}$ naught divided by c . So, this entire thing is P plus and now I have to multiplied it by 1 by; I have to multiplied by 1 by r so, into 1 by $r e^{j \omega r}$ naught divided by $c e^{j \omega r}$ naught.

So, once again so this is the equation for pressure this is the equation for pressure. Now, if ωr naught divided by c if ωr naught divided by c is very small compared to 1, what does that mean? So, what are we looking at? We are looking at this term you are looking at this time and here also we see this; the same term ωr naught divided by c is appearing and here it is being added to 1.

So, if ωr naught divided by c is very less small; very less compared to 1, then what; that means, is what is ω ? It is $2 \pi f r$ naught by c is very small compared to one which implies; now f over c is λ which implies that r naught is very small compare to λr over λ over 2π . So, if that is the case, then tell what can I can simplify this equation the first equation for pressure. So, if that is the case, then I can make two simplification $1 + j \omega r$ naught divided by c ; I can approximated it as 1 and the second approximation is $e^{j \omega r}$ naught divided by c is also approximately equal to 1. So, if I use these two simplifications, then my relation for pressure is real of $V v s$ divided by $4 \pi r$. So, this term it goes away and this term goes away. So, $V v s$ divided by $4 \pi r$ times $j \omega \rho$ naught and then of course, $e^{j \omega r}$ over c minus t .

So, this relation is valid. If the radius of the sphere which we are talking about is very small compared to λ over 2π which means it is very small compared to roughly the one-sixth of the wavelength. So, if that is the case, then I can consider this thing as

kind of a point object and in this case I can say that pressure is directly proportional to the volume velocity of the source and of course, it is inversely proportional to the distance which is r times $j \omega \rho_0 \text{naught} \text{time } e^{-j \omega r / c}$ minus t .

So, this is where volume velocity comes in and later you will see and also in your professional life if you try to apply it you will find that this concept of volume velocity will find it to be extremely useful, if you are trying to reduce noise from vibrating bodies sheet metal bodies or if there is an engine or something which is generating noise and lot of vibrations are happening and those vibrations are creating noise then the thinking should be to somehow reduce the amplitude of those vibrations and if you reduce the amplitude of those vibration by a factor of two the volume velocity goes down by a factor of two and as a consequence the pressure which it will create for sound it will also go down by a factor of 2.

So, one thing you can do to reduce volume velocity is reduce the amplitude of those vibrations, but volume velocity is not just amplitude of the vibrations it is also related to the surface area. So, either you reduce the velocity amplitude of the velocity of the vibrations or reduced the vibrating surface area or you do both and that kind of a strategy will help you reduce the overall noise level in your system which can be attributed to vibration you know which can be attributed to body is which are vibrating. So, that is pretty much what I wanted to talk about today next week we will continue in one or two more lectures this discussion on volume velocity spherical sources and so on and so forth and then we will start talking about some other things. So, that I conclude our discussion and I look forward to seeing you next week which will be the sixth week of this course.

Thank you, bye.