Noise Management & Its Control
Prof. Nachiketa Tiwari
Department of Mechanical Engineering
Indian Institute of Technology, Kanpur

Lecture – 29
Complex Impedance for Radially Propagation Sound Waves in Spherical Coordinate Systems

Hello, welcome again to noise control and its management. Today is the fifth day of this particular week and what we plan to do today is very quickly in next 4-5 minutes develop the solution for the one dimensional velocity wave equation for the spherical propagating waves and later; what we will do is developed relation for complex impedance as it relates to spherical propagating waves.

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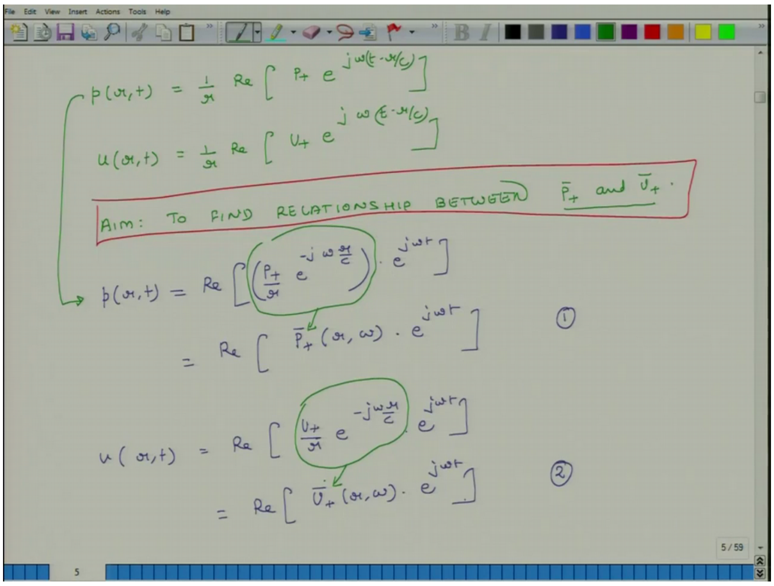

So, we had shown that the solution for one dimensional pressure wave equation is p r t equals 1 over r real of P plus e j omega t minus r by c. So, this a solution and this is the solution, this is a valid solution as long as there are no reflecting surfaces around the source and because the nature of the equation for velocity 3. This is the velocity pressure wave equation and if we develop the velocity, this is the pressure wave equation and if we develop the velocity wave equation it will come in exact same form. So, with that understanding I can directly write that velocity for spherical symmetric waves when there are no reflections could also be expressed as u plus e j omega t minus r by c and now are concern is how are P plus and u plus mathematically related and the answer to that will come from the momentum equation.

So, now our aim is to find relationship between P plus bar and u plus bar and what are those P plus bar and u plus bar actually I will write them. So, that is separately. So, before, what I will do is I will define what is P plus bar and u plus bar. So, from this I get prt equals real of P plus bar P plus over r e j omega r over c negative of that times e j omega t and this entire expression I call P plus bar similarly u r t equals real of. So, I can I am sorry for this confusion, but I will make it more explicit.

So, this is equal to real of P plus bar which is a function of r and omega times e j omega t where P plus bar is what. So, this entire thing is P plus bar this entire thing is P plus bar and what P plus bar represent is the amplitude of the wave at location r P plus represents the maximum amplitude for all locations P plus bar represents the amplitude of the wave at location r because it changes from place to place. So, that is there and similarly u r t is equal to real of u plus over r e minus j omega r by c times e j omega t and that equals real of u plus bar r omega times e j omega t.

So, let us call this equation 1 and this is equation 2. Now what our aim is to figure out the relation between P plus bar and u plus bar and we have understood that P plus bar r omega is this entire thing encircled in green and u plus bar r omega is this entire thing which is circled in green. So, we want to find out the mathematical relationship between the complex amplitude of pressure and the complex amplitude of velocity at location r that is what we are interested in. Now this relationship we will be able to figure out, if we use the momentum equation and 2 days back we had developed the momentum equation for one dimensional spherical propagating waves as.

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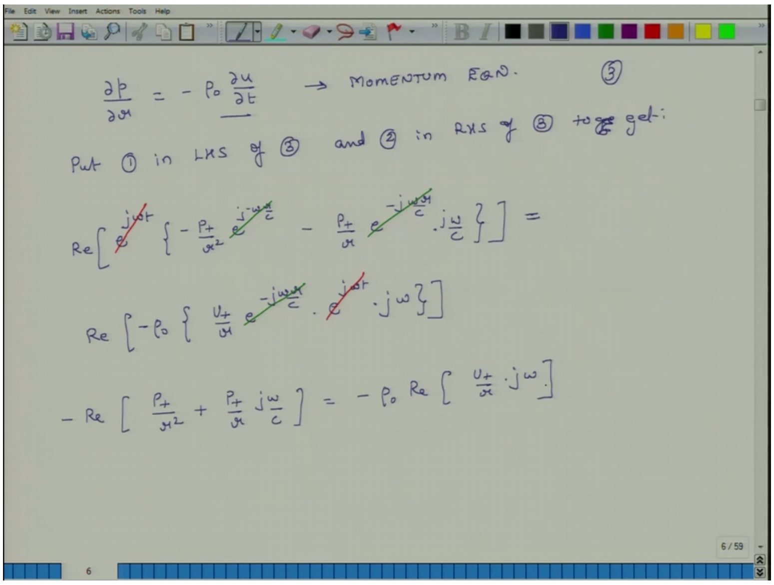

So, what is the momentum equation was second derivate first derivative of pressure with respect to r is equal to minus rho naught del u over del t this is the momentum equation this is momentum equation. So, let us call this equation 3. So, what we will do we will plug 1 and 2 in equation 3 and that will help us relate the 2 things. So, what we do is we put 1 in LHS of 3 and 2 in RHS of 3 to get and then we what we and then we do all the mathematic and this is what we get. So, I have to do; I have to differentiate pressure with respect to r.

So, I use this equation and I put it on the left hand side and on the right hand side I have to differentiate velocity with respect to time. So, I use a second equation and put it there. So, what do I get? So, what I; So, what we get is. So, first we will differentiate pressure with respect to r. So, pressure with respect to r is e times j omega t times. So, first there is a P plus divided by r term and if differentiate with respect to r I get minus P plus by r square e j omega r over c and then I also have to differentiate the exponential form. So, that is P plus divided by r and minus e minus j omega r by c times omega over c and this is the left side and of course, there should be a j also and on the right hand side, I have minus rho naught.

So, this is minus rho naught times partial of u with respect to time. So, the time component is only e to the power of j omega t. So, I have u plus over r e minus j omega r over c times e j omega t times j omega. So, this is what we now I make some simplification. So, I see that e j omega t is present on both sides it is common to both sides.

So, they cancel out each other the second thing is that there is e j omega r over c in each term on the left side as well as on the right side. So, e j omega r over c e to the power of j omega r over c e to the power of j omega r over c they all cancel out each other. So, what I am left with real of minus P plus over r square plus P plus over r j omega divided by c equals minus rho naught real of u plus over r times j omega I can remove the real operator from both the sides and also this negative sign goes away from both the sides. So, there is a negative sign here and there is a negative sign here.

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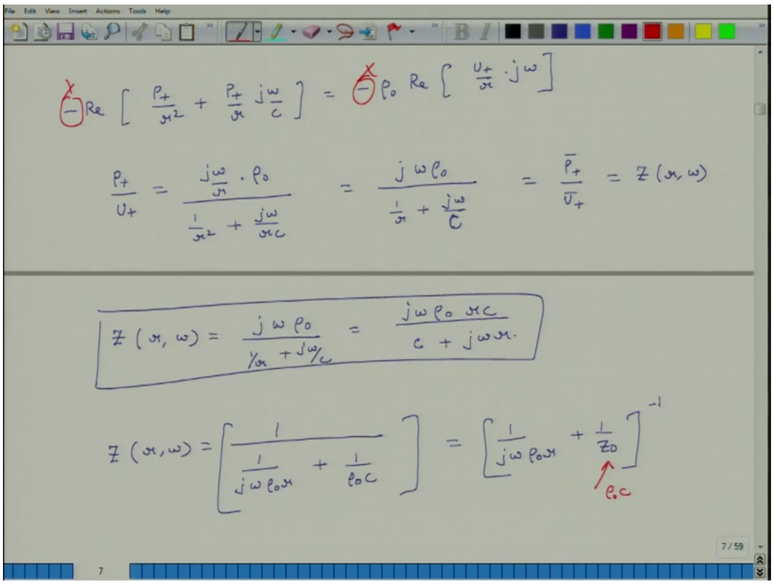

So, they get cancelled out also I can remove the real operator from both the sides because this equation has to be valid for all values of rr not for some particular value of r. So, that will be true only if the parameters and parentheses themselves equal each other.

So, I can write from this I can write P plus over u plus equals j omega over r time rho naught divided by 1 over r square plus j omega over r c. So, the r r r in the numerator and r in the denominator they also get cancelled out. So, what I am left with j omega rho naught divided by 1 over r plus j omega by c now this is also same as P plus bar over u plus bar and why is it because when you see this is the ratio of P plus over u plus if I multiply P plus and also multiply u plus by this term e minus j omega r over c divided by

r. I will get P plus bar and if I multiply u plus by e minus j omega r over c divided by r I will get u plus bar.

So, the ratio of P plus and u plus in our case is same as the ratio of P plus bar and u plus bar and this we call compare impedance for the spherically propagating wave and it changes with r and omega it changes with r and omega it changes with r and omega. So, I can say that impedance specific acoustic impedance for a spherical propagating wave outwardly propagating wave not wave which also as reflected component it changes with r and omega and that is equal to what it is equal to j omega rho naught divided by 1 over r plus j omega over c and if I do the mathematics I make this little more simple.

So, I get j omega rho naught r c divided by c plus j omega r. So, this is one form for Z I can also mathematically manipulate this further I can say that Z of r and omega equals. So, what do I do I divide the numerator and denominator by j omega rho naught r c. So, what do I get here 1 over j omega rho naught r plus 1 over rho naught c this is another form and this can be written as 1 over j omega rho naught r plus 1 over Z naught inverse an what is Z naught? Z naught is rho naught c, it is the characteristic impedance of air which we had define earlier. So, we have 2 mathematically equivalent expressions for z.

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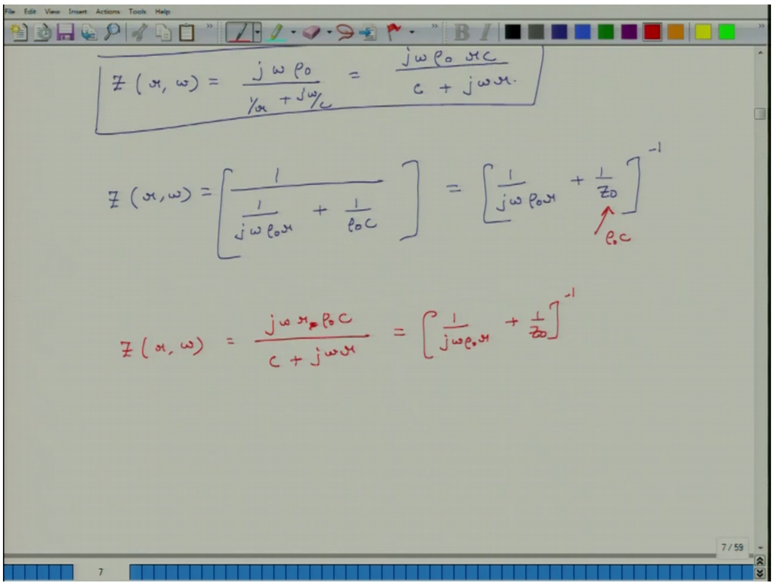

So, we will just write it for purposes of clarity one expression is j omega r naught r rho naught c divided by c plus j omega r or I can also express it as 1 over j omega rho naught r plus 1 over Z naught inverse. Now make some important observation at this. So, this Z is the specific acoustic impedance for spherically propagating waves where there are no reflection where there no reflections.

So, if there are no reflection these wave continue to travel outwards and go up to infinity and never get reflected back and their specification acoustic impedance is define by this equation and it depends the specification acoustic impedance it depends on omega and it also depends on r, right. So, now, we will make some observation.

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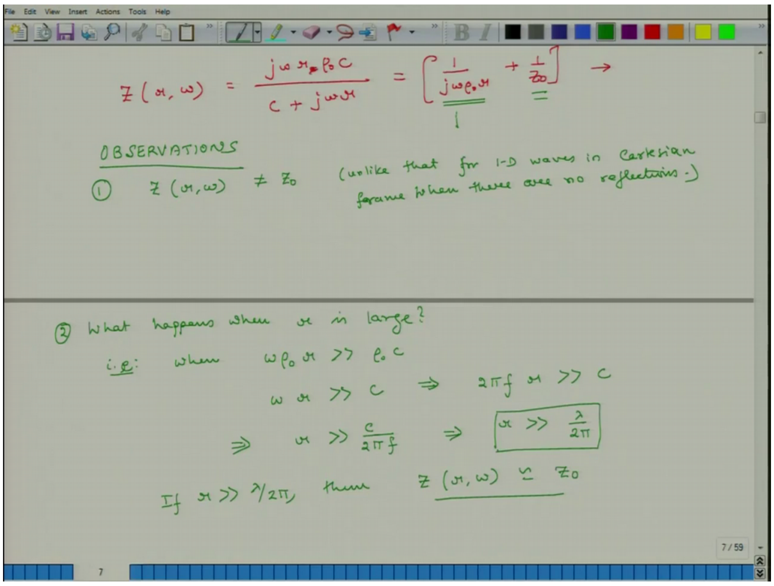

So, the first observation is that for one dimensional wave which was travelling in a Cartesian frame of reference say for instance in a tube we had found that it is specification acoustic impedance if the tube is infinitely long if its infinitely long there is no reflection for such a tube we had found that its characteristics impedance Z naught and its specific impedance were same, but in this case Z r omega is not equal to Z naught unlike that for 1-D waves in Cartesian frame when there are no reflections this is first important observation. So, spherical waves even though there are reflection if there no reflection they behave differently compared to linear waves travelling in a straight line second thing we will work on the second relation thing.

So, what happens when r is large what happens when r is large when r is large and what do we mean by large r is large in the sense that this term is if r becomes very large this term becomes very small compared to 1 over Z naught. So, what happens when r is large. So, so I can say that that is when omega rho naught r is very large compared to Z naught and Z naught, we had defined it as rho naught c. So, I can also say that omega r is extremely small large compared to c or what; that means, is 2 pi f times r is very large compared to c or equivalently I can say that r is very large compared to c over 2 pi f and what is c over f; f is the frequency. So, or I can say that when r is very large compared to c over; now excuse me. So, c over f is lambda wavelength lambda over 2 pi. So, if r is large in the sense that r is very large compared to lambda over 2 pi so.

So, if r is very large compared to lambda over 2 pi then Z of r and omega it is approximately equal to Z naught because this term becomes negligible. So, what does this mean physically what it means is that suppose there is a point source which is emitting spherically symmetric waves and you are going very far away from it and if you are very far away in the co sense that you are several factors distant compare to lambda over 2 pi, then the waves the sound waves which are been emitted by this source will be similar to one dimensional waves moving in a straight line because the impedance of a one dimensional wave moving in a straight line is Z naught. So, how is it important for people who are working in noise what; that means, is that if you are far away from the source then if you are interested in managing that noise you have to manage the noise in a sense that it is of a similar nature as that of one dimensional way moving in a straight line.

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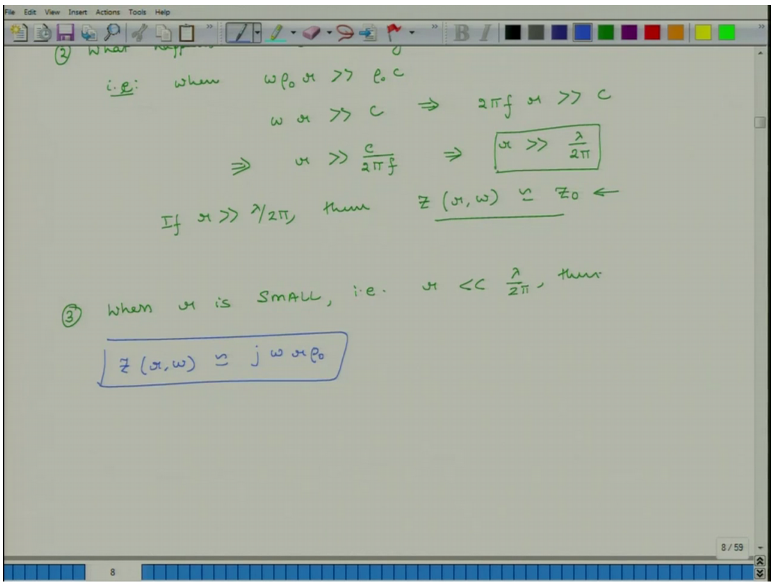

This is the second important thing and the third thing is when r is small and what do I mean by small that is r is very small compare to lambda over 2 pi, then now we look at this expression 1 over time once again. So, if r is small, then this term 1 over Z naught will be negligible compare to this term because r is a small.

So, denominator is small and if denominator is small, then 1 over Z naught can be neglected. So, in that case then Z of r n omega you can approximate it as j omega r rho naught this is the other 1. So, these are some of the important characteristics of waves which are travelling in readily symmetric fashion and tomorrow which is the last day of this week we will continue this discussion and we will also do probably all this something similar in the next week also, first couple of lectures; we will continue our discussion on spherically symmetric waves; tomorrow, specifically we will be talking about a new concept called volume velocity and this is a very use full concept especially people like you who are interested in understanding noise of surfaces noise coming from surfaces which are vibrating. So, we will introduce a new concept call volume velocity which will be very useful for noise.

Thank you and I look forward to see you tomorrow, bye.