Noise Management & Its Control
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Lecture – 28
Radially Propagating Sound Waves in Spherical Coordinate Systems-II

Hello, welcome to noise control and management as you know this is the fifth week of this course today is the fourth day of this particular week and over this week we have covered 2 important things one is how Kuntz tube works and the second thing which we have discussed particularly in the last class was development of one dimensional wave equations; if we are using radial coordinate spherical coordinate system and for spherically symmetric propagating waves we had shown that there is an equation for pressure and the one dimensional pressure wave equation I will rewrite is 1 over r square del over del r; r square del p over del r equals 1 over C square del 2 p over del t square.

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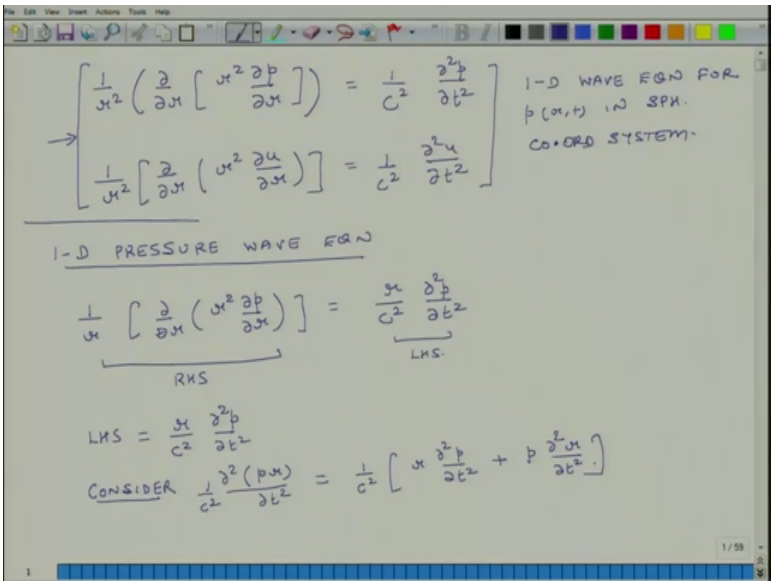

So, this is the 1-D equation; 1-D wave equation for pressure in spherical coordinate system and likewise, we can also say that the equation for velocity is 1 over r square del over del r r square del u over del r equals 1 over C square del 2 u over del t square. So, these are the 2 equations which we had described in the last class and we had also developed an equation for the momentum equation for spherical coordinate system now what I plan to do is I will change the form of these equations. So, that they are relatively easy to solve. So, ultimately we want to solve these equations. So, that we understand; how are spherical waves propagate and to do that we are going to do some mathematical manipulations.

So, that it becomes or it comes into a format which is relatively easy to solve. So, we will start with the 1-D pressure wave equation and we will transform this equation into relatively easy to solve format. So, first we will write the equation 1 more time. So, it is 1 over r square del over del r r square del p over del r equals 1 over C square del 2 p over del t square. So, the first thing; I do to this equation is I multiply both sides by r. So, if I am going to multiply both sides by r this 2 goes away from here and on the left side, I have r in the numerator. So, that is one thing; now this is my right hand side and this is my left hand side. So, we will first work on the left hand side. So, LHS equals r over C square del 2 p over del t square.

Now, we know now let us consider a term. So, let us consider a term where such that it is. So, let us consider this term. So, excuse me. So, consider this term which is second derivative of p times r with respect to t into 1 by C square if I differentiate it in parts what do I get one over C square r times del 2 p over del t square plus p times second derivative of r with respect to time.

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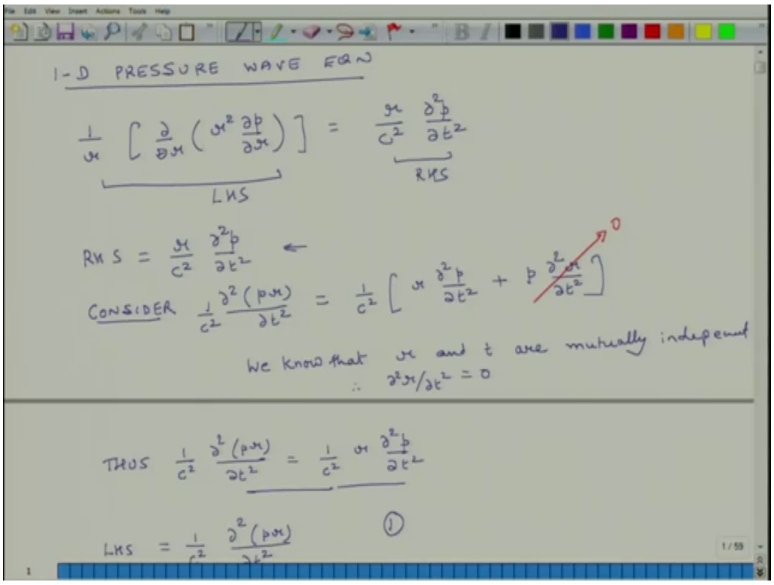

Now, we know. So, when we look at this we know that r and t are mutually independent therefore; del 2 r over del t square is 0. So, that is 0 then this term is 0, thus I can write this as. So, I can write thus one over C square into second derivative of p times r over del t square is equal to 1 over C square r del 2 p over del t square. So, we compare this equation and the relation for LHS.

So, I can go back and say that LHS of the wave equation is nothing, but one over C square times second derivative of p times r divided by del t square. So, this is equation number this is the first. Now we will play with the right hand side of I made a small mistake this should have been called LHS and this is the right hand side. So, I will make these modifications. So, this is RHS. So, we have worked on the right hand side of the equation. Now what we are going to do is we are going to manipulate the left side of the equation which is 1 over r times del over del r into r square del p over del r.

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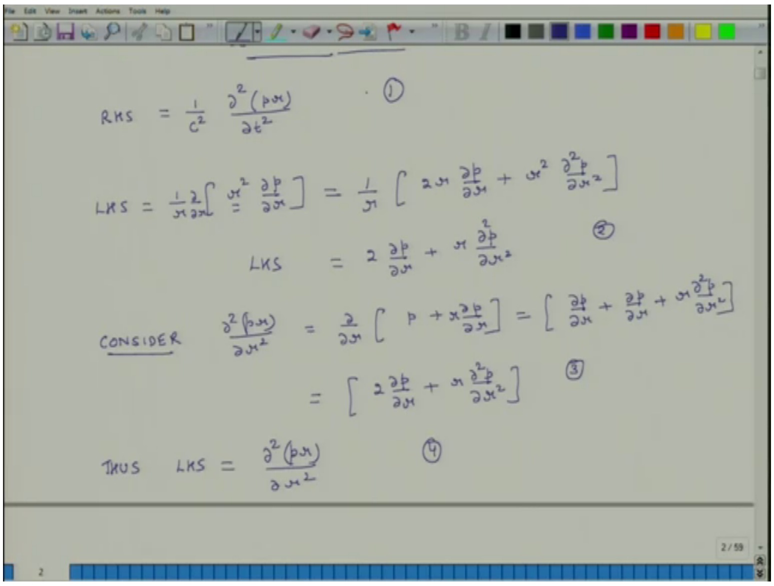

So, let us say how this is. So, the left hand side is one over r times r square del p over del r the entire things has to be differentiated with respect to r. So, if I do the differentiation what do I get one over r and then first I differentiate r square.

So, I get 2 r del p over del r plus r square del 2 p over del r square and this if I take one over r inside the brackets what I am left with is 2 times del p over del r plus r times del p over del r second derivative of pressure with respect to r. So, so this is LHS this is my second equation now we will modify this slightly more and then things will become all of a sudden simple now we consider this term. So, we consider second derivative of p times r with respect to r and let us see what it means.

So, if I am going to differentiate it what does it mean its del over del r I first differentiate it once and then I differentiate p with respect to r. So, I get P plus del p over del r times r and now I differentiate it one more time with respect to r, I differentiate it one more times. So, what do I get it is del p over del r square second derivative; this should be only the first derivative it is del p over del r and then I differentiate the second term and that gives me 2 more term.

So, I get if I work on r then I get del p over del r plus r del 2 p over del r square. So, if I simply this I get 2 del p over del r plus r times del 2 p over del r square. So, this equation 3, now, if then we look at equation 2 and equation 3 we see that they are same right which means. So, thus the left hand side of the equation I can also write it as waves mathematically same its equivalent. So, this is equal to del 2 p times r divided by del r square this is equation 4. So, now, I compare equations one and 3. So, RHS and LHS they have to be the same.

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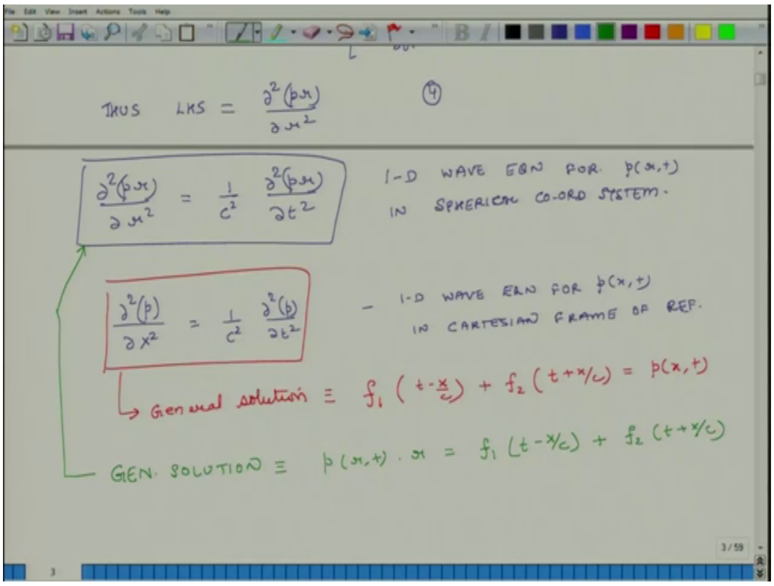

So, my modified wave equation is what it is del 2 p r divided by del r square equals one over C square del 2 p r divided by del t square. So, this is another form of 1-D wave equation for pressure in spherical coordinate system spherical coordinate system.

Now, we look at this equation and compare it to the 1-D wave equation for pressure in Cartesian frame of reference. So, what was that like it was very similar it was secondary bit of p with respect to with respect to x is equal to one over C square del 2 p divided by del t 2 this was the 1-D wave equation for p as a function of x and t in Cartesian frame of reference right. So, when you look at these 2 equations they look structurally the same the only difference is that in the Cartesian frame of reference 1-D wave equation has this the operators they work on p while in spherical system they work do not work on p, but they work on p times r. So, they work on p times r. So, the point is that we can use the solution of 1-D wave equation and use that and get inspired by that solution to come up with the solution for 1-D wave equation in spherical coordinate system.

Now, for this system we had explained that the general solution was what it was a function f 1 into t minus x over C, right plus f 2 t plus x over C and this we said is p of x t and f 1 represented in this one Cartesian frame a wave travelling forward and f 2 represents a wave which is moving in the negative x direction that is what it meant, but this was in context of p x t. So, we can use similar approach and solve for and develop a solution for this equation. So, using analogy we can say that the general solution, but here the solution will not give us value of p rather it will give us p times r. So, p times times r is equal to f 1 t minus x over C plus f 2 t plus x over c.

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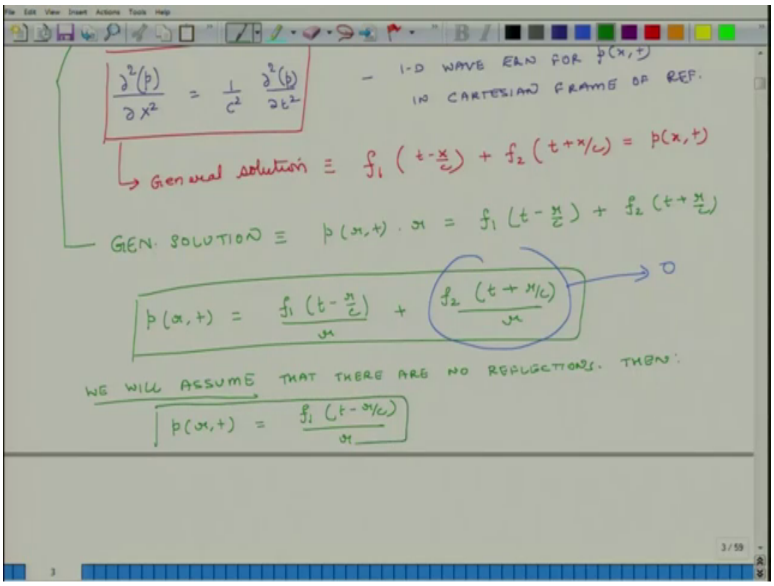

So, this is the general solution or I can say that for one dimensional spherically symmetric wave, I can say that the pressure is what it is f 1 t minus x over C divided by r plus f 2 t plus x over C divide by r.

So, this is the general solution now here f 1 represents a wave which is travelling outward. So, you have a spherical wave which has a center and f 1 represents a wave. So, excuse me. So, here it should not be x minus C. So, I made a small mistake instead of x I should have written r. So, it should be r over C and r over C here also. So, in this case for spherically symmetric wave f 1 is a function of t minus r over C and it represents a wave which is travelling outwards. So, you have a center and the wave is propagating the sound wave is moving away from the center its going on in the outward direction and f 2 represents a wave which is coming towards the center it is coming towards the center now in our in this particular course we will assume. So, what do we assume we assume that there are no reflections there are no reflections.

So, suppose there is a point in air which is generating sounds; sounds spreads in all the directions and the reflecting surface is r either at an infinite distance away or they are. So, far that by the time the wave reaches a; that infinite that the reflecting surface it is. So, weak that there is hardly any reflection that is what it means. So, we assume that there are no reflections. So, in this case then this term this term will go to 0 because f 2 represents a reflected wave. So, in that case p of r t equals f 1 which is a function of t minus r over C divided by r; this is the general solution. So, I am going to rewrite.

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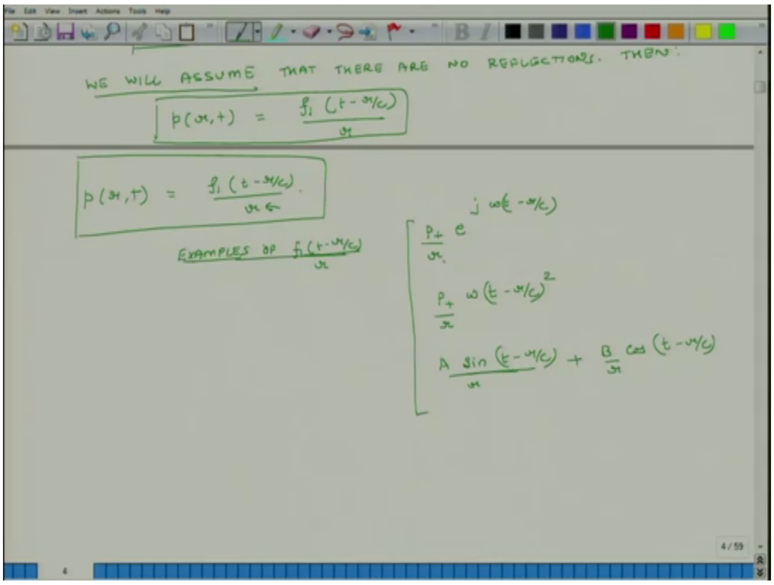

So, if there are no reflections, then p of r t equals f 1 t minus r by C divided by r. Now what does this mean; what this means is that any function which is of this form f 1 t minus r by C will represent a pressure wave which is spherically symmetric.

So, we will look at some examples. So, any function which represents this form will represent a spherical spherically symmetric wave and it will satisfy our one dimensional pressure wave equation. So, some possible examples of f 1; so, one possible example could be P plus e j omega t minus r by C divided by r; this is one possible example here wherever I am having p and r, they do not come by themselves they come in combinations t minus r by C right and there is a 1 by r term because of this things another possible another possible mathematical function which will satisfy one dimensional wave equation would be P plus omega t minus r by C square; this another possible function and then of course, I have to divided by r another possible function could be a sin t minus r by C by r plus b cosine t minus r by C by r, this is another possible function.

So, we can have infinite types of functions which will which will be valid solutions possible solutions for the pressure wave equation as long as t and r, they come in groups they come as a combo that is t minus r C, they have to appear as that and of course, the entire function should be divided by r because I have a r, here in our general solution. So, in our course in our course in this course we will assume that the function which will which we will work on will be of it will be complex exponential functions.

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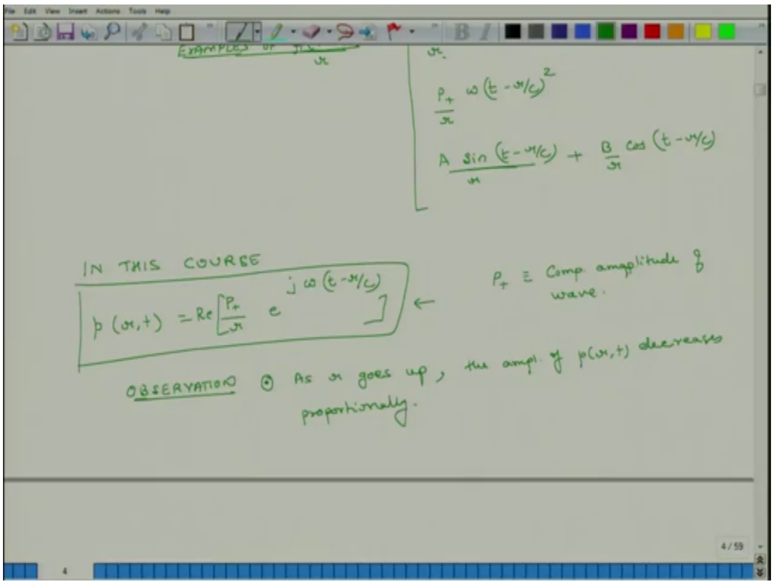

So, in this course we will say that p of r t is equal to a complex number P plus divided by r e to the power of j omega t minus r by c.

Because of the fact that most of the real complicated pressure functions you know time series functions, we can express them as sums of sins and cosines using the idea of Fourier series or Fourier transform and because e to the power of j omega t minus r over C can be expressed as a cosine and a sin function. So, this will this kind of a function will help us understand very complex waves as well. So, in this course we will see that pressure. So, in this case, we will say and because P plus is a complex number and. So, is exponent j omega t if I am interested in the real pressure then I just take the real portion of this. So, again this method of finding the solution is very similar to the method which we used in finding out the solutions for one dimensional wave equations as they are found in Cartesian reference frames.

So, in this course we will see that p of r t depends is a real component of P plus over r e j omega t minus r over C where P plus what is it? It is the complex amplitude of wave to the complex amplitude of the wave. So, as we look at this solution we make one important observation. So, this is one important observation and the observation is that as r goes up what does that mean that as we are going away from the center the amplitude of p r t decreases proportionally it decreases proportionally. So, if I double the distance between the source and the microphone where I am measuring then the pressure will go down by a factor of 2.

So, this is the very important understanding and just by understanding one dimensional spherical waves you can now appreciate that suppose you have a noise machine which is making a lot of noise. Now it is not necessarily a spherical source, but as you go far away from it in general especially for lower very low frequency lower frequencies especially if there are no reflecting surfaces as you go far away from it if you double the distance your sound pressure level or the noise which will be pursuit by the microphone or the human ear it will go down by a factor of 2 the SPL; thus the pressure will go down by a factor of 2 once the pressure goes down by a factor 2 in terms of decibels this means that it is a reduction of 6 decibels.

So, that is there. So, this is what I wanted to cover today in the next class what I plant to do is develop some sort of a similar solution for velocity and then in context of 1-D waves propagating with respect to Cartesian frame of reference we had developed an expression for z which is specific acoustic impedance and this impedance actually connects pressure and velocity, it connects pressure and velocity. So, we will develop similar expressions for spherical waves and we will figure out what is the impedance for spherically propagating waves. So, with this I want to close our discussion for today and I look forward to seeing you tomorrow.

Thank you.