Noise Management & Its Control
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Lecture – 27
Radially Propagating Sound Waves in Spherical Coordinate System-I

Hello, welcome to this course once again. Today is the third day of this particular week which is the fifth week and what we plan to do starting today and also over next three four more lectures is discussion of waves spreading in a radially symmetric wave. So, what do I mean by that.

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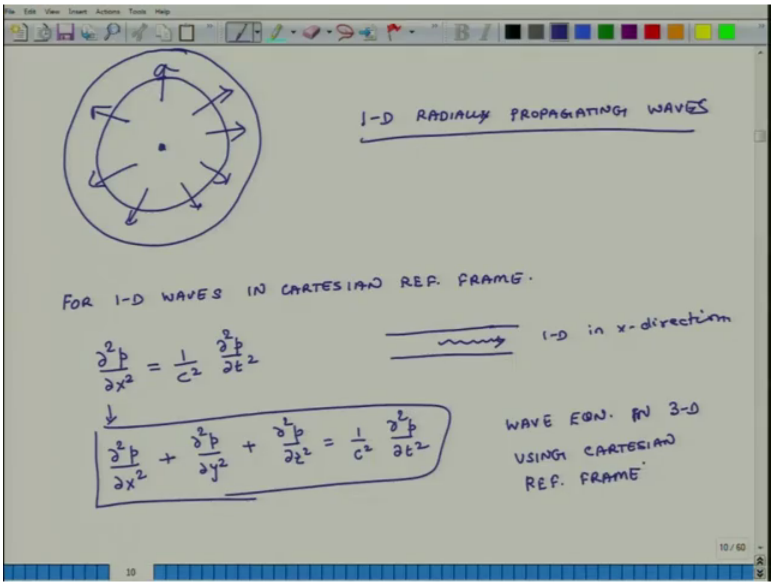

So, suppose you have a very small source of sound and the source of sound is such that it is producing sound of equal intensity in all directions and also the medium in which sound is propagating is homogeneous. So, it is not changing from point to point in a radial way. So, what will happen is that, sound waves will spread out uniformly in all directions in a radial fashion. So, they will just keep on spreading out in all directions in a radial fashion.

A lot of times we encounter sources of sound, where sound spreads in all directions. In tubes when we generate sound travels only in 1 direction along the length of the tube, but when we generate sound in say air suppose there is a bird on a tree and tweets that sounds spreads readily in all the directions or there is lightning that sound also spreads readily in all the directions. Now here we are talking about radial propagation of sound which is also uniform in all the directions. So, at theta equals zero, theta equals 30 degrees and so on and so forth the spread of sound is uniform. Lot of practical sources of sound the spread of sound is not uniform with respect to this angle, but here what we will discuss are ideal sign sound sources where sound spreads radially not only in radially, but also uniformly in all the directions.

One dimensional radially propagating waves and what we will do is first we will develop through logic governing equation for these types of waves, and then we will start solving that equation to see how actually sound spreads in a radial fashion. Now for 1 dimensional waves in a Cartesian frame of reference Cartesian reference frame are equation was what for pressure it was second derivative of p with respect to x equals 1 over c square del 2 p over del t square this is the equation for spreading of waves in Cartesian reference frame. So, here sound will spread only in x direction. So, it is 1-D in x direction.

If I make this equation more general using exactly the same steps which we had used when we were developing 1 dimensional spread of waves, but here we are not worried about 1 dimension you know in x, but sound is spreading in x y and z and we are still using a Cartesian frame of reference then what happens. Then using the same methodology which we had used in week 1, we can extent this equation to three dimensions and what does it look like. So, it look likes second derivative of p with respect to x square plus second derivative of p with respect to y, plus second derivative of p with respect to z and that equals 1 over c square del 2 p over del t square. So, this is the wave propagation equation wave equation in three dimensions and what is the reference system we are using? Cartesian frame of reference using Cartesian frame of reference frame.

So, here is the sound is spreading in all the three directions and we are using Cartesian frame of reference. So, what is a Cartesian frame of reference? This is let us say this is x axis, this is y axis and this is z axis. So, this is a Cartesian frame of reference.

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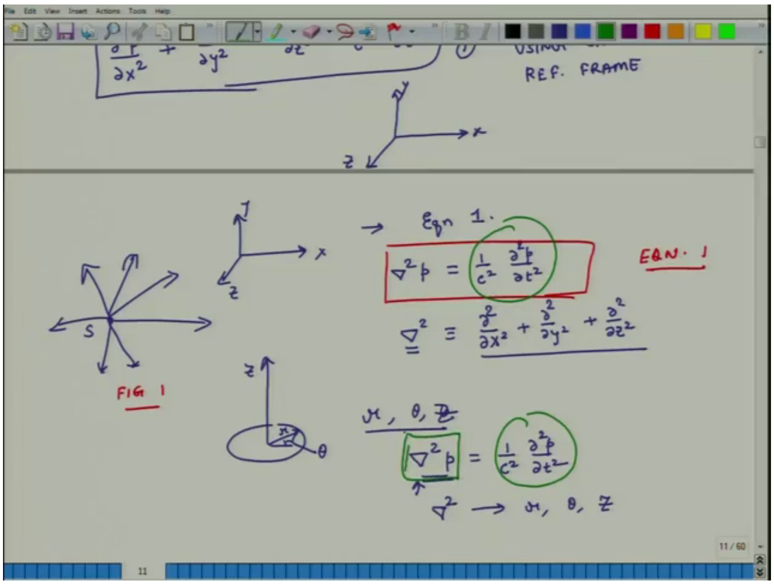

Now consider a fact that suppose there is a sound source and it is emitting sound. So, sound is spreading in all directions right and it also a travelling in the z direction. So, it is travelling in all the three directions and I can use this equations let us call this equation 1, to figure out how sound is spreading because here sound is spreading in all the three direction. So, I can use this equation, equation number 1 to figure out how sound is spreading right it is not just 1 dimensional propagation its spreading in all the directions.

So, I can use this equation and if I use Cartesian frame of reference, which is this 1 x z y then I use equation 1. I can rewrite this equation 1 as grad square p equals 1 over c square times second derivative of pressure with respect to time. Where this operator what does it correspond to? It corresponds to del over del x square del 2 over del y square plus del 2 over del z square. Now look at this picture again. So, let us call this figure 1 let us call. So, let us look at this figure 1 again and when we use Cartesian frame of reference I can use equation 1 to figure out how sound spreads in x direction y direction and z direction using equation 1, but there are several coordinate systems, I can also use cylindrical coordinate system right I can also use a cylindrical coordinate system.

So, what are the cylindrical coordinate system with respect to origin what are the three parameters. So, Cartesian frame of reference the three parameters are x y z in cylindrical coordinate system it is r theta. So, this is my r, this is my theta and this is my z right here also. So, so you r theta and z right these are this these are the cylindrical in a cylindrical coordinate system three independent parameters are r theta z. Same source I should be able to use a cylindrical coordinate system to also predict how sound spreads because the source does not know what reference frame I am interested in. So, it will just a spread in the way it has to spread.

Now, the equation for this is. So, what is the general equation? The general equation the general equation is del square p equals 1 over c square del 2 p over del t 2 this now. So, this is my general equation, this now in a in a Cartesian frame of reference this operator grad square means this thing in a cylindrical frame of reference it may mean something different this thing it depends on what these three parameters r theta and z depends on r theta and z in Cartesian frame it depends on x y and z. But the value of this operator will be same because the right side of the equation you see the right side of the equation here and the right side of the equation here is the same right it is second derivative of pressure with respect to time on both in both the equations.

So, the value of this grad square will not change whether you are using Cartesian frame or a cylindrical frame or for that sake even a spherical frame.

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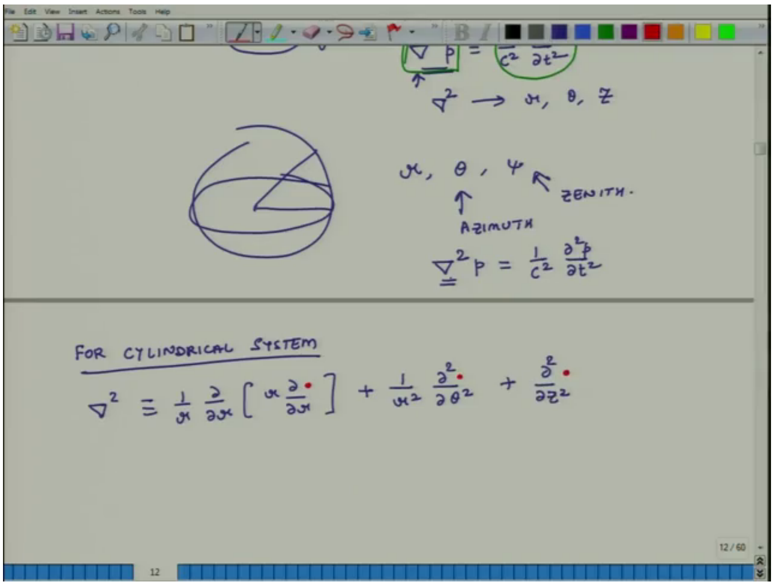

So, in a spherical frame of reference, another option could be I could have a spherical frame of reference right. So, here my parameters are r. So, r is radius and I just want to make sure my terminology is correct theta and psi. So, this is called Azimuth angle and this is called Zenith angle or elevation. So, if I use a cylindrical frame of reference or spherical frame then also my overall governing equation is the same, but the operators mathematical expression will change. So, I will write down the operators mathematical expression for cylindrical system what is it? Grad square corresponds to 1 over r del over del r, r del over del r, so whatever.

So, here we put pressure plus 1 over r square del 2 over del theta square plus del 2 over del z square. So, wherever we have a red dot we will put it replace it by pressure when we actually want to use it.

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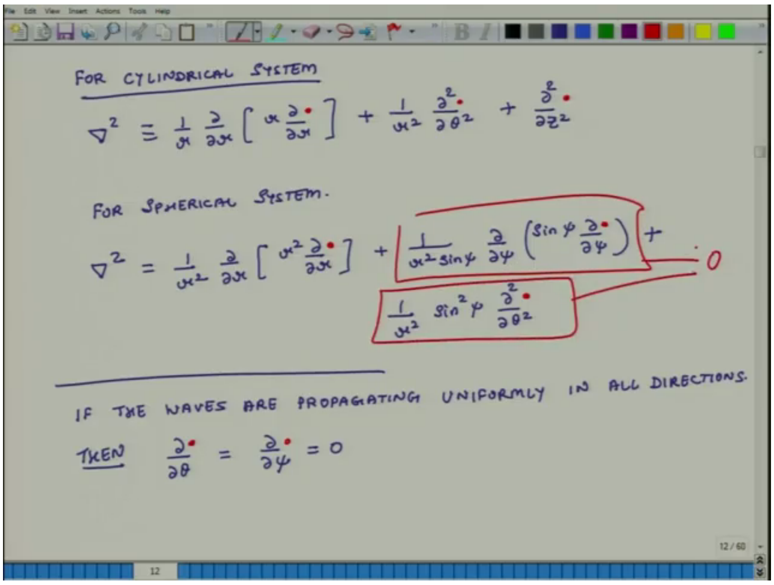

And for spherical system this operator means this longish expression 1 over r square times del over del r, r square del over del r oops plus 1 over r square sin phi sin psi del by del psi, sin psi del over del psi plus 1 over r square sin square psi del 2 over del theta square this is for a spherical system. Now if the waves are propagating uniformly in all directions then. So, its waves are propagating uniformly in all direction then what does that mean? Then what it means is del over del theta equals del over del psi equals 0 right.

So, if that is the case then this term goes away and so does this term 0. So, for a spherical system when waves are propagating uniformly in all the directions then my equation governing equation for the system becomes grad square p equals 1 over c square del p over del t square and actually this is this gets simplified.

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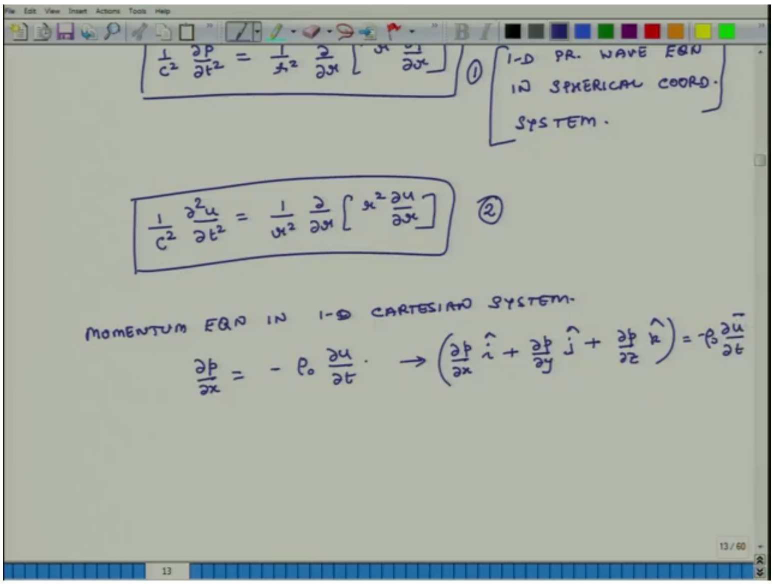

So, this is equal to 1 over r square del over del r, r square del p over del r this is what it based on to. So, this is the 1-D pressure wave equation in what? In spherical coordinate system and this equation is valid when is it valid if all the sound is coming from a point and it is spreading out in all the directions uniformly. If it is not uniform spread of sound then we have to use more general operator.

So, the pressure waves is spread and they follow this law as they are spreading out uniformly. Similarly velocity waves can wave also spread out uniformly. So, the equation for 1 dimensional velocity wave equation is 1 over c square del 2 u over del t 2 equals 1 over r square del over del r, r square del u over del r. So, that is my equation for velocity waves equation 1 2. When we were developing one dimensional pressure wave equation for Cartesian reference frame we had used three important basic equations 1 was Newton's second law which we when we developed it we using Newton's second law we developed a momentum equation and the other equation which we used was based on the principle of conservation of mass and we called it continuity equation and then the third one was gas law.

So, using that analogy when we recollect the momentum equation in 1-D Cartesian system in 1-D Cartesian system, what was it del p over del x equals minus rho naught del u over del t. You can generalize it if it is a three dimensional Cartesian frame you can generalize it as del p over del x i plus del p over del y j these are what direction code you know unit vectors and del p over del z k equals minus rho naught del u and here it is a vector like this and this thing can be expressed as a gradient operator operating over p equals minus rho naught del u over del t and just as the gradient square does not change whether we change the coordinate system this gradient operator also does not change. So, when we want to have a similar equation for a spherical system the equation does not change the gradient of pressure equals minus rho times derivative of u with respect to times.

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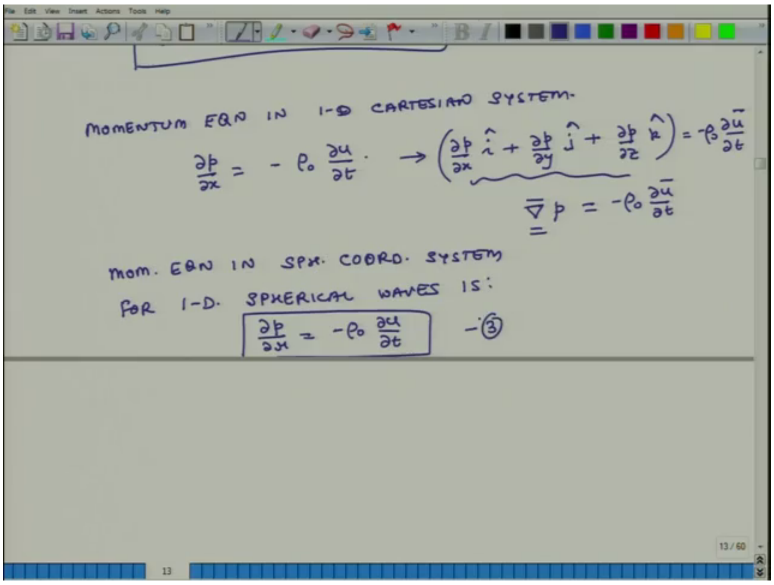

So, momentum equation in spherical coordinate system for 1-D is spherical waves, what is it is? Partial of pressure with respect to radius equals minus rho naught del u over del t. So, this is third equation. So, this 3 equations are important as we start looking at readily propagating waves which are spreading out uniformly. So, the first one is equation 1-D pressure wave equation, the second equation is for velocity and the third equation is for it is the momentum equation for one dimensionally is one dimensional spherical waves in spherical coordinate system. So, these are three important equations and we will use these 2 develop solution for spherical waves in our next lecture and also subsequent lectures.

So, with this we conclude our discussion and we will continue discussion tomorrow as well.

Thank you.