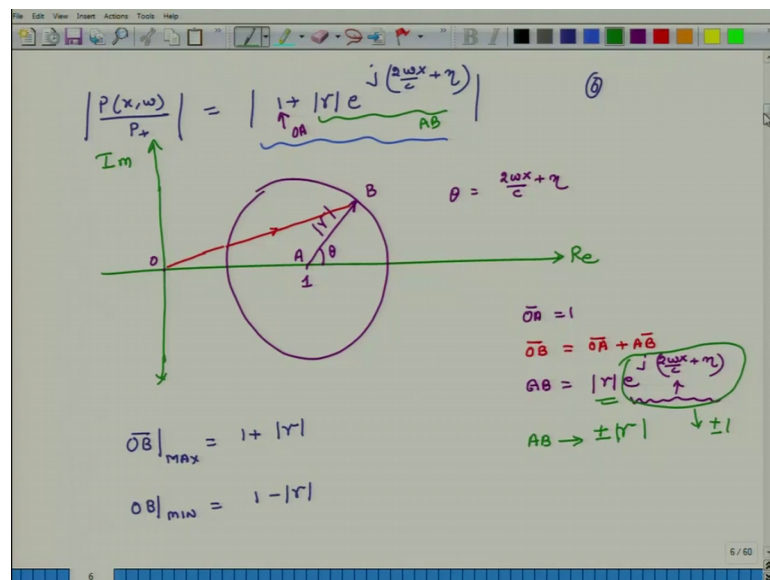


**Noise Management & Its Control**  
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**Lecture - 26**  
**1-D Sound Wave Propagation: Kundt's Tube – II**

Hello welcome to noise management and its control, today is the second day of the fifth week of this course yesterday we were discussing how a Kundt's tube works and this tube can be used to measure the impedance of any material which we want to characterize in terms of its noise absorption properties. So, in that context we had explained what is a Kundt's tube, how does it look like what is its construction and then we had done some mathematics to express impedance of an unknown material in terms of reflection coefficient gamma and if we using that relation if we can find the value of gamma, we can find the value of z L which is the impedance of the unknown material and then we had started figuring out a way to compute gamma using Kundt's tube.

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So, in that context we have arrived at this expression modulus of complex amplitude of pressure in the tube equals modulus of 1 plus modulus of gamma times e j 2 omega x over c plus eta (Refer Time: 01:41) this we had equation we had leveled it as equation number 6. Now what we will do is, we will depict this graphically the left side of this equation the right side of this equation in graphical format. So, on a complex plane, let us

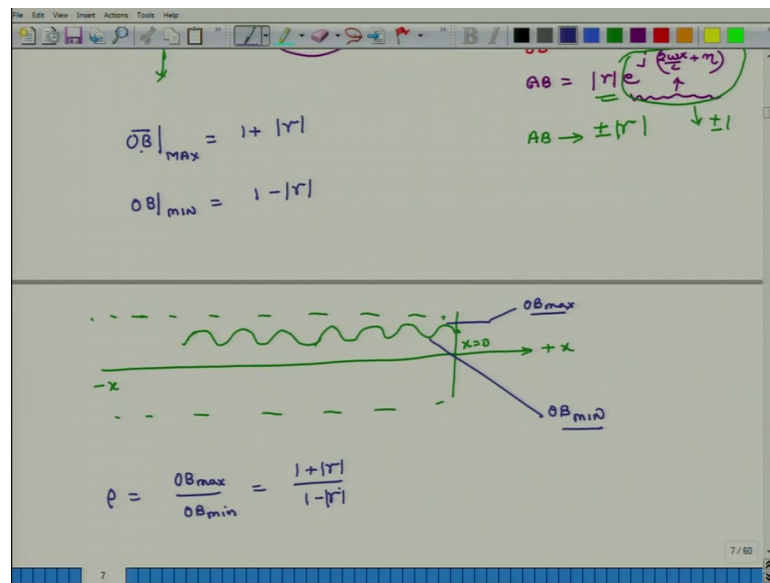
say this is my real axis this is my imaginary axis and let us say. So, let us say the origin is O the center of the circle is A and let us say this distance is 1. So, OA equals unity. So, this 1 represents OA and this term can be denoted on this graph by a vector whose length is gamma because its modulus is gamma. So, the radius is gamma and the angle this theta.

So, this angle theta which it makes with the real axis is what  $2\omega x / c + \epsilon$ . So, let us say this point is B then OB equals OA plus AB, where OA is 1 and AB equals a magnitude which is magnitude of gamma times some phase angle which is  $j 2\omega x / c + \epsilon$ . Now AB what will be the maximum value of AB? The maximum value of AB will be when exponent j to the power of  $2\omega x / c + \epsilon$ . So, as x changes because x is changing in the tube x is the position of microphone and the tube. So, as I moving in the tube x is changing. So, this angle is changing as my microphone is changing in the tube and as x is changing this term right this term if the tube is very long, then this term will fluctuate between plus 1 and minus 1 right.

When  $2\omega x / c + \epsilon$  will be equal to 0 or  $2\pi$  or  $4\pi$  or  $6\pi$  or something like that this value exponent j to the power this thing is going to be 1 and when its value  $2\omega x / c + \epsilon$  will be equal to  $\pi$ ,  $3\pi$ ,  $5\pi$  and so on and so forth. This value is going to be minus 1. So, e to the power of this exponential term its value is going to fluctuating between plus 1 and minus 1 and this is a constant right. So, what this means is that AB will fluctuate between plus gamma and minus gamma, its maximum value will be positive gamma positive of absolute value of gamma and its minimum value will be negative of absolute value of gamma. So, what does that mean that maximum value of OB will be 1 plus gamma and minimum value of OB will be 1 minus absolute value of gamma.

So, this is OB no I am sorry this is AB. So, maximum value of OB is be will be 1 plus gamma, it will be minimum 1 minus gamma and because of this and what is OB? OB is the normalized value it is the ratio of  $P \times \omega$  divided by  $P +$ ,  $P +$  is a constant for a given frequency right.

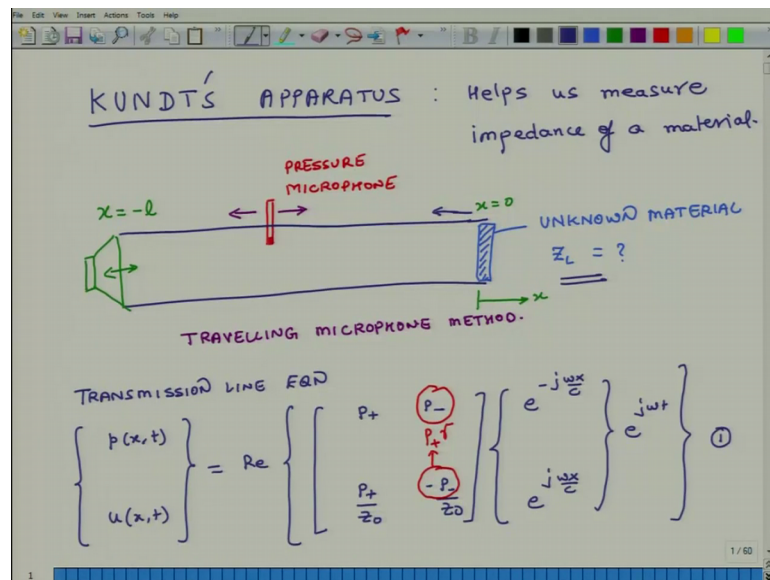
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So, what; that means, is that if I plot this is my negative x this is x is equal to 0 excuse me and this is my positive x. So, my tube exists in this direction this is my tube and tube starts from x is equal to 0 at x is equal to 0 it will have some value OB will have some value which will be what 1 plus gamma times e to the power of j eta right. If I am going to plot its value this OB will fluctuate like this and what is going to be the maximum value of this, this is going to be OB max.

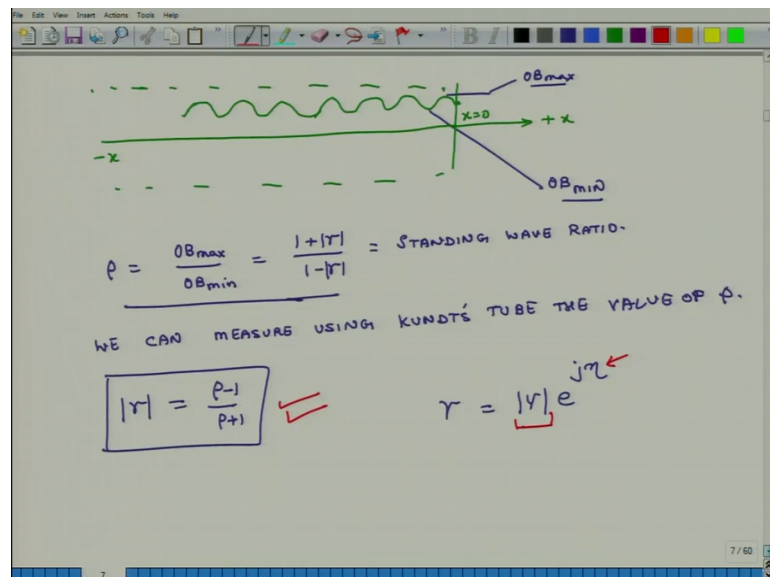
And this is going to be OB min OB min what; that means, is that if I know OB max and OB min, OB max and OB min if I know OB max and OB min let us say that the ratio let us say that rho is equal to the ratio of OB max and OB min. So, what is this ratio it is 1 plus gamma divided by 1 minus gamma.

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I can actually measure this ratio how can I measure this ratio? I go back to my Kundt's tube and I use this microphone I start sliding it from this location in this direction and wherever I find the pressure to be maximum that will be proportional to OB max and wherever I find pressure to be minimum that will be proportional to OB min.

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So using the microphone I can find out this ratio OB max and OB min, the ratio of OB max and OB min this ratio is called standing wave ratio it is called standing wave ratio.

So, we can measure using Kundt's tube, the value of rho which is standing wave ratio from our experimental set up we can measure the value of rho, once rho is known then I can calculate from this relation the absolute value of gamma which is. So, from this relation which is what? Rho minus 1 divided by rho plus 1. So, I can calculate the magnitude of gamma which is the reflection coefficient from this relation. So, experimentally I determine the value of rho standing wave ratio and then I use that standing wave ratio to compute the value of magnitude of gamma. So, in this case, but we know that gamma is having some magnitude and it also has a phase. So, we have figured out how to compute this part, we still do not know how to calculate eta the phase part.

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The image shows a digital whiteboard with handwritten notes. At the top, the magnitude of the reflection coefficient is given as  $|r| = \frac{p-1}{p+1}$  and the reflection coefficient is expressed as  $\gamma = |r|e^{j\eta}$ . Below this, the text reads "COMPUTATION OF  $\eta$  i.e. PHASE OF  $\gamma$ ". A horizontal line separates this from the next section. The second section is titled "LOCATION OF 1st MINIMA OF  $p(x,t)$  IN TUBE =  $-d_{min}$ ". It then states "When  $x = -d_{min}$ " and shows the phase calculation:  $\theta = -\pi$ ,  $\frac{2\omega x}{c} + \eta$  (for  $x = -d_{min}$ ), and finally  $-\frac{2\omega d_{min}}{c} + \eta = -\pi$ .

The magnitude part we can calculate using this relation which links the magnitude of gamma to standing wave ratio which is rho ok.

Now, we will figure out how to compute eta. So, our next step is computation of eta that is phase of gamma. Now we go back to this relation see as x increases in this picture as x increases this theta will increase theta will increase as x increases right when x is equal to 0 theta will be small, it then x then theta will be same as eta and x as x goes up theta will go up as well what happens when theta equals minus pi degrees 1 theta equals minus pi not degrees minus pi radian then this location of B comes here right. So, at that, when b, so when theta becomes pi radian or minus pi radian B is at this location add

this location what is the value of OB it is 1 minus absolute magnitude of gamma right physically what does that mean? That in this tube as I am moving my microphone from x is equal to 0 in this direction, there will be initially 1 location where the pressure is going to become minimum and that will correspond to the value when theta equals minus pi that corresponds to the value when theta equals minus pi.

So, what that means, is that to find eta we somehow related to the location of the minima of the pressure. So, let us say that the location of first minima of pressure in tube equals minus d min why is it negative because our coordinate system is such that as I moving along the length of the tube it is negative x direction right. So, when x is equal to d min then what is theta? So, when x is equal to d min minus d min, theta equals minus pi radian and what is. So, and what is theta? Theta is  $2\omega x$  by c plus eta. So, so I write down  $2\omega x$  here x is equal to minus d min. So, I write down the relation  $2\omega x$  by c plus eta equals minus pi. So, I do the mathematics and I get eta equals pi minus  $2\omega d$  min by c.

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The image shows a digital whiteboard with the following handwritten content:

- When  $x = -d_{min}$   $\theta = -\pi$ .
- $\frac{2\omega x}{c} + \eta$  (for  $x = -d_{min}$ )
- $-\frac{2\omega d_{min}}{c} + \eta = -\pi$
- A boxed equation:  $\eta = \pi - \frac{2\omega d_{min}}{c}$
- Below it, another boxed equation:  $|\gamma| = \frac{p-1}{p+1}$
- An arrow points from the boxed equations to the right, labeled  $\gamma \rightarrow Z_L$ .

So, what this expression tell that if I can find the value of d min and how do I find the value of d min I just move the microphone, gradually away from the closed end and wherever I find the first minima that is corresponds to the location d min. So, I record that number I know the value of C which is 345 meters per second and I also know what

frequency of is the of the sound I am producing in the tube because I am generating that sound using a loud speaker. So, now I know  $\omega$ .

So, in this way I can compute  $\eta$ . So,  $\eta$  is from this relation and  $\gamma$  is from this relation. So, from these 2 relations are I can compute  $\gamma$  and then from  $\gamma$  I can compute the impedance of the material which I was interested in. So, this closes the discussion for today what we will also do in next 3-4 minutes is we will summaries we have discussed several types of tubes till so far, we have discussed closed tube we have discussed open tubes we have also discussed infinitely long tubes and we will look at what kind of values  $z$   $\eta$   $\rho$  and all this things are there in these different tubes. So, we will make a table. So, we have a closed tube then we have an open tube and then we have an infinitely long tube.

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	CLOSED TUBE	OPEN TUBE	∞ LONG TUBE
$z_L = \frac{P(0, \omega)}{v(0, \omega)}$	$\infty$	0	$z_0$
$\gamma = \frac{P_-}{P_+}$	1	-1	0
$\rho = \frac{ P_{max} }{ P_{min} }$	$\infty$	$\infty$	NA
$\phi$	0	$-\pi$	NA

So, first we are going to write down the expression value of  $z_L$ ? Now what is  $z_L$  or specific acoustic impedance it is basically now in a closed tube at  $x$  is equal to 0 what is the velocity 0 in a closed tube the velocity at the closed boundary is 0. So,  $z_L$  is equal to infinity here, in an open tube in an open tube what is the pressure in an open tube at the open end pressure is 0. So,  $z_L$  is 0 in this case and in an infinitely long tube we know that is specific acoustic impedance does not change from position to position and remain same.

So, this is  $z$  naught. Next we look at reflection coefficient. So, what is reflection coefficient? It is the ratio of  $p$  negative over  $p$  positive. In a closed tube if you recall your previous lectures the value of  $p$  minus and  $p$  plus is same the ratio of  $p$  minus and  $p$  plus is same. So, this number is 1 in an open tube the reflected wave the reflect the amplitude of reflected pressure wave is negative of that of the forward travelling wave. So, this number is negative one and in an infinitely long tube there is no reflected wave. So, this value is 0 let us look at standing pressure wave standing pressure wave standing wave ratio  $\rho$ . So, we had defined it as  $P_{\max}$  by  $P_{\min}$  and the ratios of this. So, in a closed tube the minimum amplitude of the standing wave length is 0, the minimum amplitude I am talking about the modulus is 0. So, this is infinite the same thing is true, in open tube and in an infinitely long tube there is no reflected wave.

So, if there is no reflected waves I think this does not I am sorry. So, again  $P_{\max}$  this thing will be again infinite because the minimum amplitude in a travelling wave also always is also 0 as certain locations and finally, we write down the actually strictly speaking this standing wave ratio is not applicable, because there are no standing wave in an infinitely long tube it is just keeps on travelling and the phase is what? Phase is 0 in a closed tube. So, this is phase of  $\gamma$ , it is phase of  $\gamma$  in this case it is negative  $\pi$  or you can also call it  $\pi$  and then in this case it is not applicable. So, this gives you an overall view of some of the important characteristics of a closed tube and open tube and an infinitely long tube and you can add one more column if the tube is such that it is of finite length and at the end of the tube there is a material of impedance  $z_L$ . So, you can again add 1 more column to this.

So, why did I discuss this, that if you want to characterize a material and you want. So, for characterization the first step is to find out its acoustic impedance and that impedance can be calculated using a Kundt's apparatus and the mathematics for the same is something, which I have explained in last 2 lectures. What we will do in the next 3-4 lectures is we will start discussing its spherical waves and these waves are understanding these types of waves is very important because a lot of times when we encounter real sound sources, sound tends to spread is spherically in all directions and in certain cases may not spread in all direction is spherically, but it is certainly spreads in all the directions.



So, because of that its good thing to understand spherical waves because that gives us a reference point for understanding from complex sound propagation phenomena. So, that concludes our discussion for today and will continue this discussion tomorrow.

Thank you and have a great day bye.