Noise Management & Its Control
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Lecture – 24
Pressure Wave Travels in an Open Tube

Hello, welcome to noise management and its control, today is the last day of 4 th week of this course and what we plan to do today is conclude our discussion on transmission lines equations, is specifically we will do one more case for one dimensional waves, as they are travelling in straight lines using a Cartesian frame of reference and here what we will discuss is how do waves travel in an open tube. So, the problem statement is as follows.

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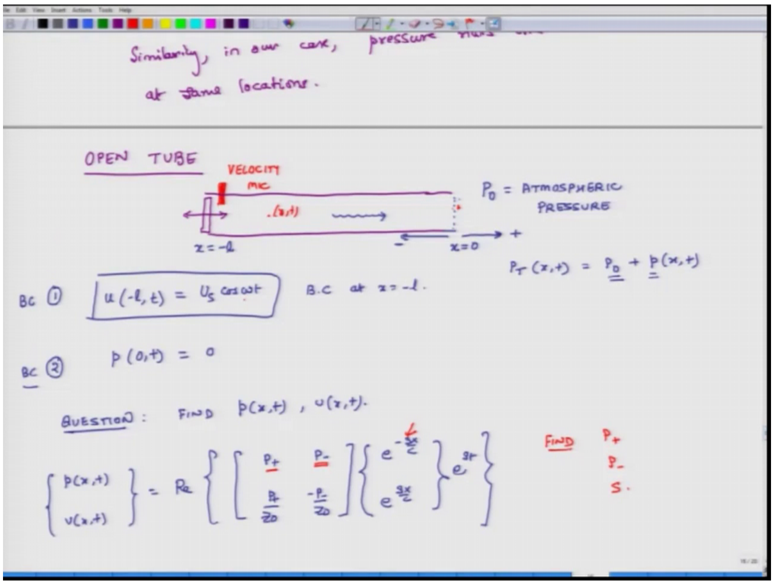

Suppose we have an open tube and at one end of the open tube, I have a piston and piston is moving back and forth and it is generating sound, to measure sound; I have a velocity microphone. So, they are 2 types of microphones one microphone is known as a pressure microphone and it measures pressure and then there are velocity microphones.

So, this is a velocity microphone. So, it can measure velocity of the air particles, where sound is getting propagated and then there is the other end of the tube and the tube is open. So, it is opened to the outside larger atmosphere and what is the pressure outside it is a very big infinite amount of air is there. So, we have the tube opens up into atmosphere so the pressure here is P 0 which is atmosphere pressure. So, whatever pressure fluctuations are coming from the tube and they reach the boundary of the tube, they just get dissipated into the overall atmosphere and the overall and because this is infinitely large there are no pressure fluctuations in the outside atmosphere so that is the situation. Once again our coordinate system is such that x is equal to 0 at the open end, this direction is positive and this direction is negative and that length of the tube is l meters.

So, the other end of the tube is x is equal to minus l, now my velocity mic is going to measure the velocity of sound waves travelling in the tube and what it is when I do the measurements, I find out that the value of velocity at x is equal to minus l is cosine function and I can write it as u minus l t; this is velocity at x is equal to minus l is equal to velocity of the piston, it is amplitude times cosine omega t. So, it is a simple function Us cosine omega t. So, this is boundary condition at x is equal to minus l, now so this is boundary condition x is equal to minus l. So, this is by first boundary condition, the second boundary condition is that if the tube is open into the atmosphere. So, whatever pressure fluctuations are there they do not create any pressure fluctuations and the on the outside of the tube.

Which means that? So, what that means is, so remember in our 1 of the earlier lectures, we had said that the total pressure when sound travels in atmosphere PT, total pressure is equal to atmospheric pressure which is constant, plus small changes in pressure which is small p. So, if the pressure in the atmosphere is fixed and it is not fluctuating that means, that this p in the atmosphere is 0, which means that a point very close to the boundary extremely close at 0 distance to the boundary, the value of P is 0 and I from that I can infer that also at just inside the boundary also the pressure fluctuations are 0. Which means that a consequence of this open boundary condition is that pressure at x is equal to 0.

So, this is the second boundary condition. So, this is boundary condition number 1 this is boundary condition number 2, so this is the problem formulation open tube velocity at x equals minus l is u s cosine omega t, pressure at x is equal to 0 is 0 and then the question is find P as a function of x and t and u as a function of x and t this is what we have to find. So, once again we start with transmission line equations.

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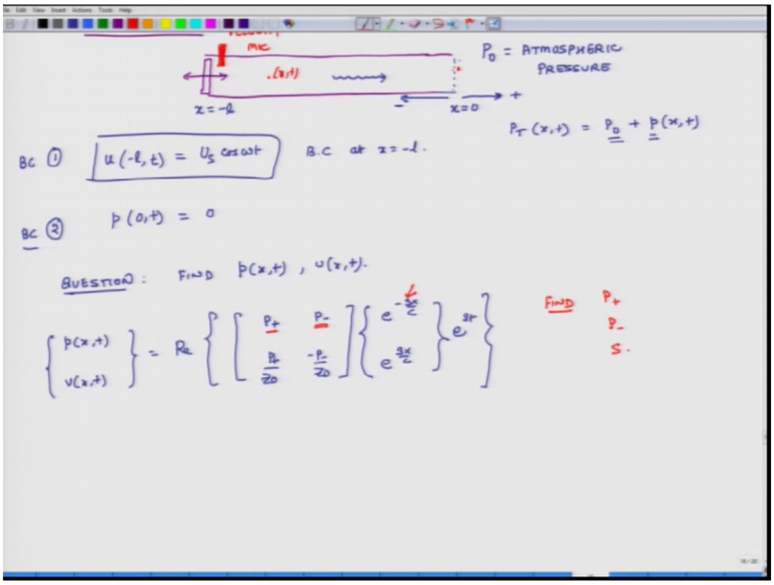

So, P of x and t and u of x and t in the matrix form is real of P plus P minus P plus by z naught, minus P minus by z naught, e minus s x over c e s x over c e s t closed. So, these are the transmission line equations for pressure and velocity and on the right side of these equations, we do not know P plus we do not know P minus and we do not know s.

So, we have to find what we have to find P, plus we have to find P minus and we have to find s if we know these 3 things, then I can predict the pressure and velocity at all the points in this tube, I can find pressure and velocity. So, whatever mathematics we are going to do in next 10 15 20 minutes, we will be devoted towards figuring out the values of P plus P minus and s. Now in the last 2 cases we have done till so far, the case for an infinitely long tube and we have also done the case for a close tube and what we found was that the value of s is the complex frequency, whenever we do the mathematics it comes to the frequency of the source. So, in this case this frequency of the source is omega.

So, that will be the value of s. So, we can say based on that otherwise we can do all the mathematics and again we will get to the same answer, but just for purpose of gravity.

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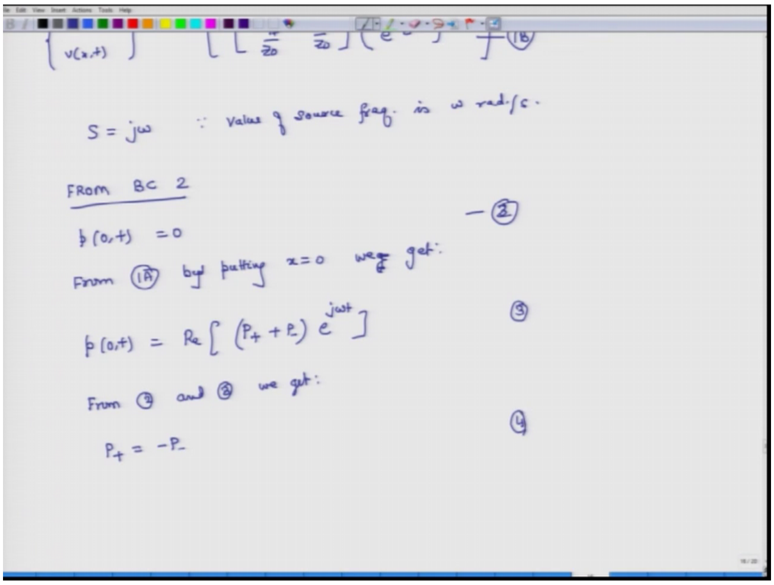

We can say that s is equal to j omega because value of source frequency is omega radians per second. So, right away, but if we do not feel comfortable with right away this kind of a statement and we can go through all the mathematics which we have described earlier and we will still come to the conclusion that s is equal to j omega. So, out of P plus P minus and S, we have taken care of S. Now we want to find P plus and P minus and the wave we are going to find out is that we are going to use these 2 boundary conditions b c 1 and b c 2 and with this information we will be able to find P plus and P minus.

So, now for starters what we will do is we will look at the second boundary condition, which says that at x is equal to 0 which is the open end of the tube pressure is 0 from B C 2 P 0, t is equal to 0 and let is call this 1 A and let is call this equation 1 B. So, from 1 A what do we get by putting x is equal to 0, we get P of 0 t is equal to so will number this equation as 2. So, we get P of 0 t is equal to real of P plus and S, I am going to put j omega and x is 0. So, e to the power of minus s x over C e to the power of s x over c both of them are 1. So, essentially I get P plus P minus e to the power of j omega t. So, this is 3 and when I compare 2 and 3, from 2 and 3 we get P plus is equal to P minus. So, this is my equation 4. So, now, what I do is I rewrite my transmission line equations.

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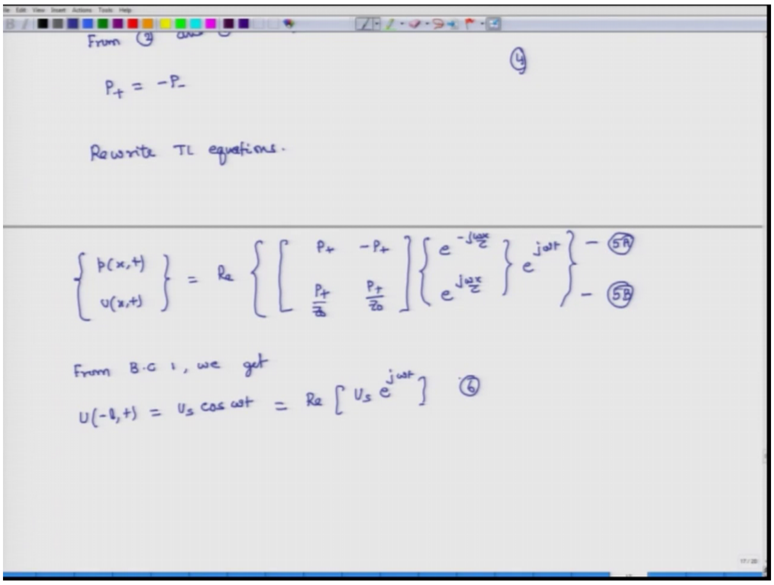

So, we will rewrite t l equations. So, what do I have P x t, u x t equals real of P plus and P minus is negative of P plus. So, it is minus P plus P plus P plus by z naught by z naught, e to the power of minus s x over c and s I know is j omega. So, I say minus j omega x over c e to the power of j omega x over c, e to the power of j omega t. So, let is call this equation 5 actually I will split it up; I will call this equation as 5 A and 5 B. So, now, in 5 A and 5 B on the right side, the only thing which I do not know is P plus. So, now, we will calculate the value of P plus and the way we are going to do is we are now going to use our first boundary condition, which is this 1 that at x is equal to 0 no x is equal to minus l velocity is u s cosine omega t. So, from BC 1 we get velocity at minus l t is equal to u s cosine omega t and I can express it in exponential format as real of us e to the power of j omega t.

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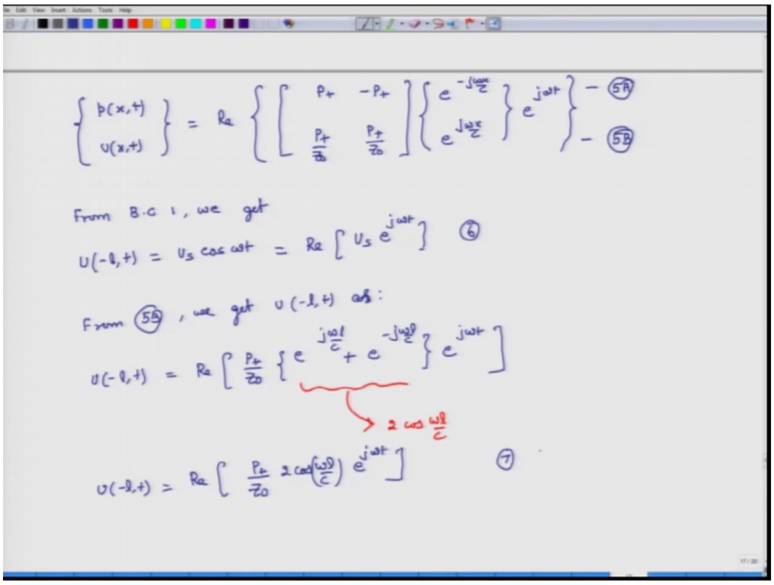

So, this is equation 6 and we can also get the value of velocity at x is equal to minus l from 5 b. So, from 5 b we get u at minus l t as u minus l t equals real of P plus by z naught, e to the power of minus j omega and when I put x is equal to minus l, this minus sign goes away. So, this goes away omega l over c plus e to the power of minus j omega l over c and then I have e to the power of j omega t. So, this term in bracket is nothing, but 2 cosine omega l over c right. So, what I get is u minus l t is equal to real of P plus by z naught 2 cosine omega l over c e j omega t.

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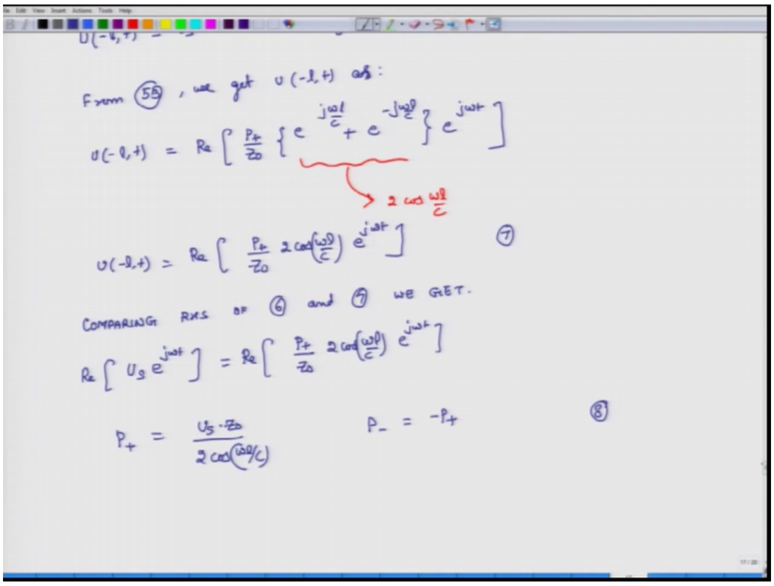

So, this is 7 and then we compare RHS of equation 6 and 7 we get, what do we get real of U s e j omega t is equal to real of P plus by z naught 2 cosine omega l over C e j omega t.

Which means P plus is what Us times z naught divided by 2 cosine omega l over c and we also know that P minus s minus P plus. So, this is equation 8 and the next thing we do is we put this equation 8.

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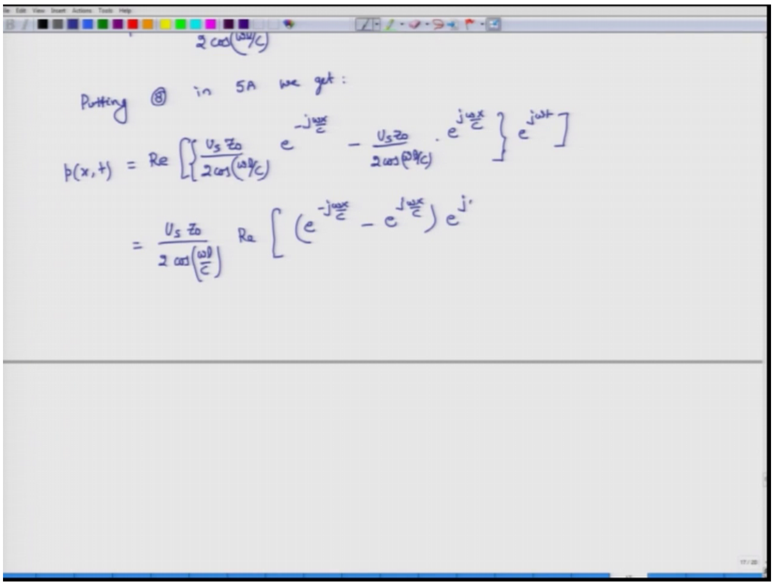

Where I have values for P plus and P minus back into my transmission line equations for pressure and velocity which is in equation 5. So, putting 8 in 5 A and 5 B we get. So, first we will write down the expression for pressure. So, pressure. So, actually I am not going to do 5 B right now. I am just going to be do 5 A, we get real of P plus. So, what is the equation P plus, so P plus is u s z naught by 2 cosine omega l over c times e to the power of minus j omega x over c minus P minus no plus P minus and P minus is negative of P plus.

So, it is minus u s z naught by 2 cosine omega l over c times e j omega x over c, this entire thing multiplied by e j omega t. So, if I take the common terms out, what I am left with is u s z naught by 2 cosine omega l over z real of e minus j omega x over c minus e j omega x over c e j omega t. Now what is e minus j omega x over c this thing.

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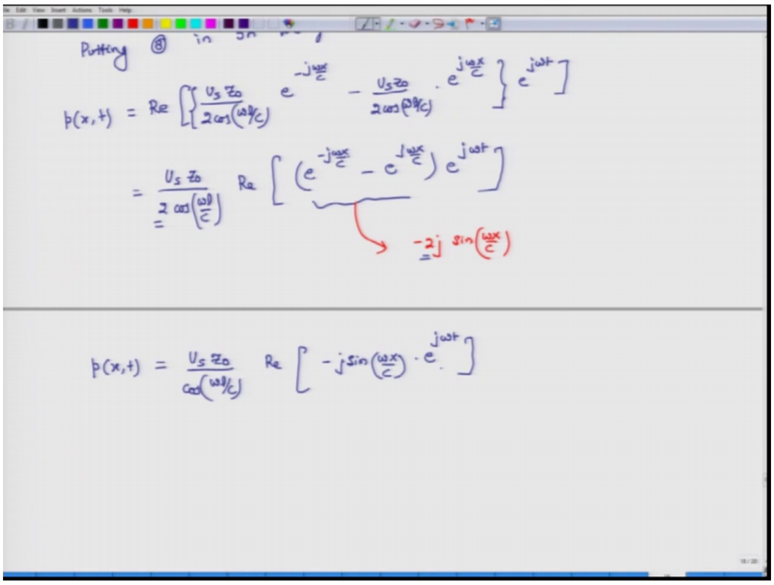

If we go back to our concepts of complex algebra and we do the math, this works out to be minus 2 j sin omega x over c. So, P of x t is and this 2, 2 cancels out. So, what we are left with is u s z naught by cosine omega l over c times real of minus j sin omega x over c times e j omega t.

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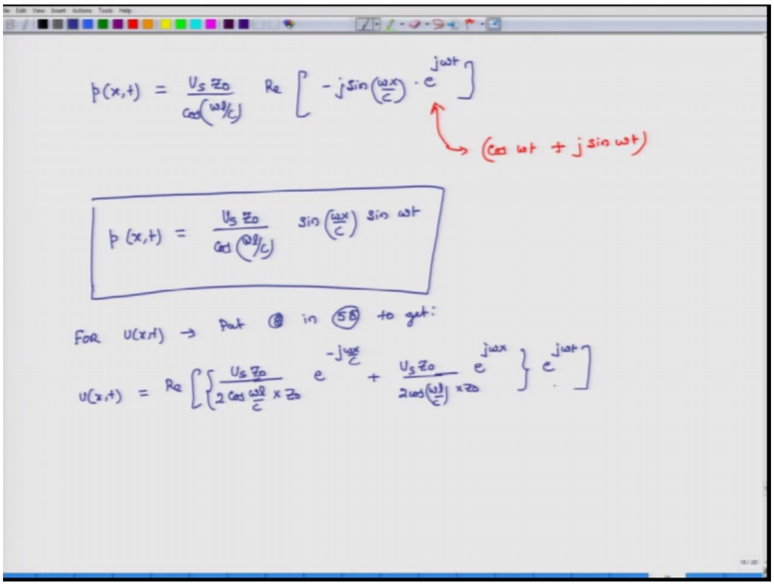

Now, once again e j omega t is what, it is cosine omega t plus j sin omega t. So, if we substitute this in e j omega t and we take only the real portion of the entire expression.

What we are finally left with is u s z naught divided by cosine omega l over c times sin omega x over c sin omega t. So, this expression for pressure and then now we do similar mathematics for velocity. So, for u of x t we put 8 in 5 B and what do we get. So, we will do the math. So, u of x t equals real of, so first term is P plus divided by z naught and P plus is Us divided by times z naught cosine omega l over c and then I have to divide this by z naught. So, it is this times e minus j omega x over c plus P plus again divided by z naught. So, is the same term e j omega x, this entire thing multiplied by sin omega t; no I am sorry, e to the power of j omega t. So, I z naught cancels out and I take all the constant terms constant real constant terms outside.

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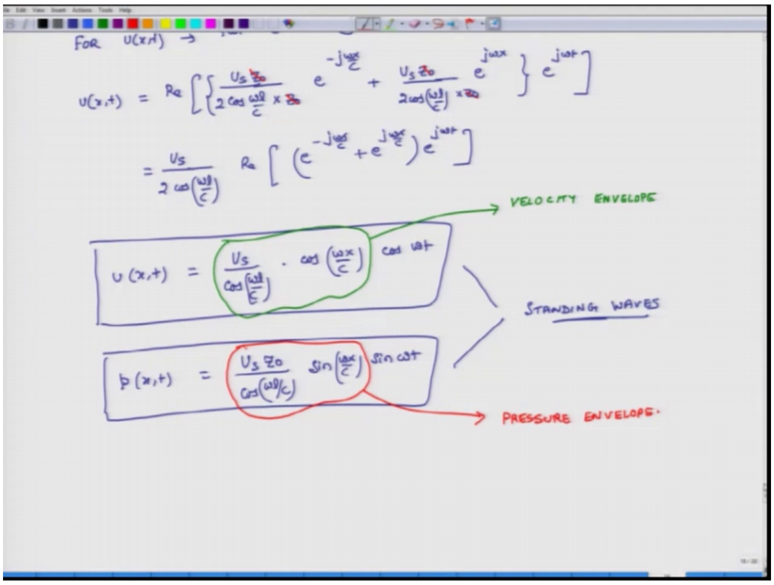

So, I am having Us divided by 2 cosine omega l over c real of e minus j omega x by c plus, e j omega x by c, e j omega t. So, if I do all the math what I am left with is the equation for velocity, is u s by cosine omega l over c times cosine omega x by c, cosine omega t. So, this is my expression for velocity and I will just re write my expression for pressure. So, that we can look at both of them once again at the same time, so that is Us times z naught divided by cosine omega l over c sin omega x over c sin omega t, this is the expression for pressure couple of things.

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These once again represent standing waves, the pressure amplitude for the amplitude of the pressure wave is this thing and it again changes from position to position, the amplitude for velocity is the term encircled in green.

So, this green thing is the velocity envelope, if I plot this thing on an x y plane then it will give me the velocity envelope and the red thing is going to give me my pressure envelope, in case of closed tubes; if you come look at the closed tube the pressure envelope had a value which was maximum at the closed end at x is equal to 0. Here the pressure envelope gives as a value of 0 at x is equal to 0. So, it is just the opposite velocity envelopes value was maximum was minimum at the closed end at x is equal to 0, here velocity envelopes value when x is equal to 0 is maximum. So, the standing waves are there, but the nature of pressure envelope and the velocity envelope they just get inversed, they just become opposite to each other relative to the closed tube.

Last thing I wanted to tell is if you remember the impedance, the impedance for waves in a closed tube; were some constant time were somewhat, it was for closed tube it was.

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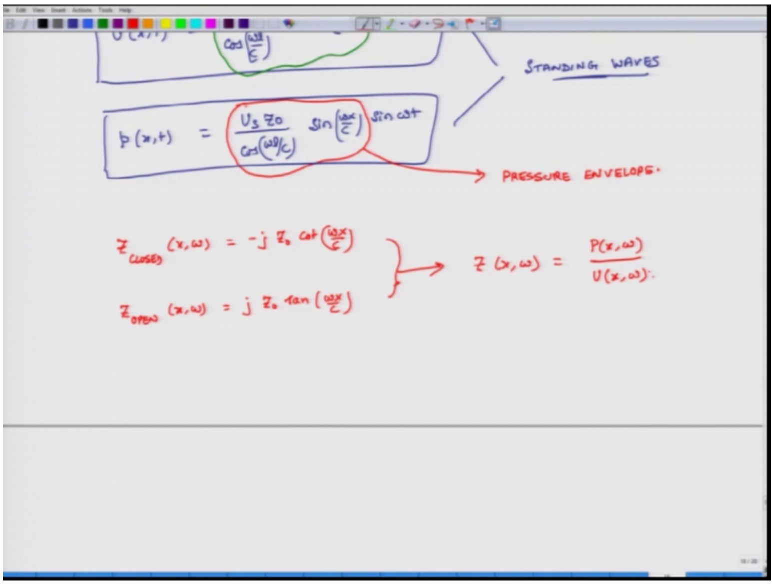

So, impedance closed was a function of x and omega and if I remember it correctly it was minus j z naught cotangent omega x over c right. If you do the same mathematics in open tubes, you will find that z open and how is equal to j z naught tangent of omega x over c and how do you find these impedances, is specific acoustic impedance by using this relation z x omega equals complex pressure amplitude, divided by complex velocity amplitude. Now in case of these waves what is the complex pressure amplitude; see; what is the complex pressure amplitude it is this term multiplied by this term, which is same as this thing.

So, it will be u s z naught divided by 2 cosine omega l times minus 2 j sin omega x over c, that is the pressure amplitude complex pressure amplitude and when you look at complex velocity amplitude, you get this term multiplied by this which is cosine omega x over c and when you take the ratio you get this thing. So, once again I mean here velocity. So, this is what I wanted to talk about standing waves in open tubes. So, overall what we have done over this week is we have developed transmission line equations, using those transmission line equations we can now predict how sound travels in straight tubes and we have also introduced the concept of characteristic impedance, which does not depend on the system it is a property of the material and also we have introduce the concept of specific acoustic impedance, which can be a complex entity it depends on the system.

So, it will have 1 function for an infinitely long tube for a closed tube, it will be a cotangent function and for an open tube it will be a tangent function and with this we conclude our discussion for today. Next week we will discuss similar waves, but in spherical frame of reference and with that discussion we will proceed into the next phase of this course, where we will start talking about noise measurement, noise characterization and solving noise related problems. So, with that we close our discussion.

Thank you and have a great weekend, bye.