Noise Management & Its Control
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Lecture – 22
Pressure Wave Travels in a Closed Tube

Hello, welcome to noise management and control. Today is the fourth day of this week and yesterday we had just started discussion on how do waves travel in a closed tube; a tube which is a constant cross section finite length and which is rigidly closed at one of its ends.

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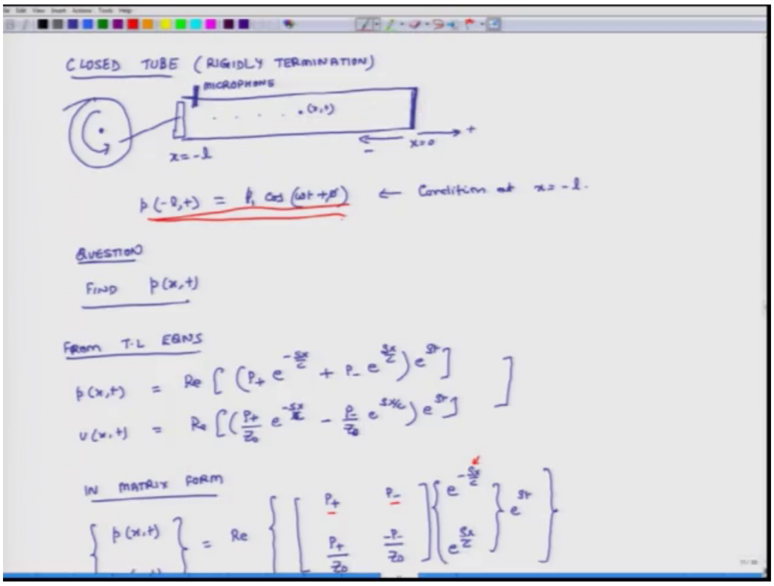

So, we will very quickly go over the problem formulation this is the picture of this tube; one end of the tube where x is equal to minus l, the pressure at the other one end of the tube is P minus l t and that is equal to p 1 cosine omega t plus phi and at the closed end its a closed end. So, it has some physical implication we will come to that just in the moment.

So, this is the tube and what we are interested in finding out is pressure at any point in the tube and the general solution for one dimensional wave equation can be expressed in terms of pressure transmission line equations. So, these are our transmission line equations this is in the regular form and this is in a matrix form. So, now, our aim is to solve this equation. So, that we know; what are the values of P plus P minus n S and the moment we know these 3 entities we have an expression for pressure at any point in the tube. So, to for this we will use our boundary conditions. So, this is boundary condition number one. So, it is B c one the other boundary condition is that the tube is closed and what does that mean. So, at x is equal to 0.

So, this is x is equal to 0 if you have an air particle at S is a if suppose this is an air particle just at the closed end because the tube is rigidly closed the end of the tube has no freedom to move back and forth and because of this the particle which is extremely closed to it just next to it that also cannot move back and forth. So, what; that means, is that at x is equal to 0 particle velocity U at 0 t is also 0 because the tube is closed and not only just closed close its rigidly closed. So, that is why the particle velocity is 0. So, I have 2 conditions at x is equal to 0 U is 0 at x is equal to minus l we know the pressure. So, now, with these 2 things we will figure out what are the values of P plus P minus n S.

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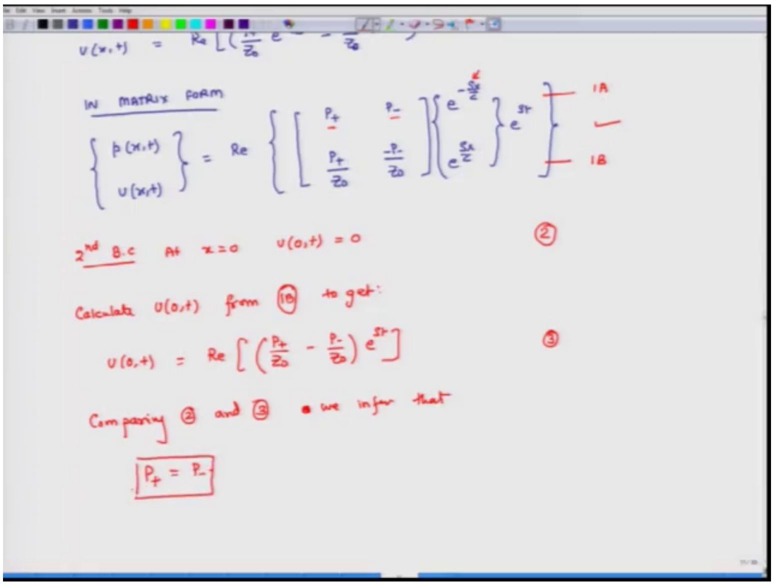

So, we again go back to our transmission line equation and we use the second boundary condition. So, second boundary condition says at x is equal to 0 U of 0 t is equal to 0. So, let us call this equation the first equation has 1 a; this equation is 1 B and this equation as 2.

Now, what we do is we calculate U of 0 t from 1 a to get. So, we are going to use oh I am sorry not 1 a 1 B from equation one B we calculate the value of U at x is equal to 0. So, U of 0 t is real of P plus by z naught minus P minus by z naught e to the power of S x over c and e to the power of minus S x over c at x is equal to 0 there one. So, they do not fade, they go away and then I have e to the power of S t now. So, this is 3. So, comparing 2 and 3 comparing 2 and 3 we will see that the value of U will be 0 only we infer that P plus equals P minus. So, using this understanding we rewrite down the transmission line equations.

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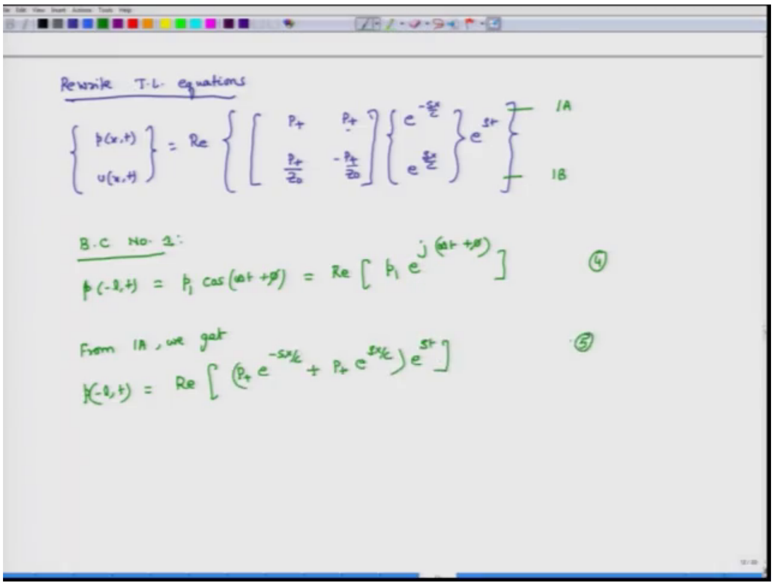

So, we rewrite. So, here we get p of x t U of x t and the first constant is P plus and the next constant is p minus, but we just said that P plus equals P minus. So, it gets replaced by P plus p plus by z naught and minus plus by z naught and then it is e minus S x over c e S x over c e to the power of S t. So, now, we have eliminated P minus, we still have figure out the value of plus and S. So, now, we look at the first boundary condition boundary condition number 1; what does it say that pressure at minus l t is equal to p 1 some constant cosine omega t plus c and I can write p 1 cosine omega t plus p as the real component of p 1 e to the power of j omega t plus c. So, this is again equation 1 a, this is equation 1 B and this is equation 4. So, this is our as a equation 4 and then from equation 1 a, we again calculate what is the value of pressure at minus l. So, from one a we get P minus l comma t is real of P plus e minus S x over c plus P plus p S x over c e to the power of S t. So, this is 5 and equation 4 and 5 both are the expressions for pressure at minus l x is equal to minus l.

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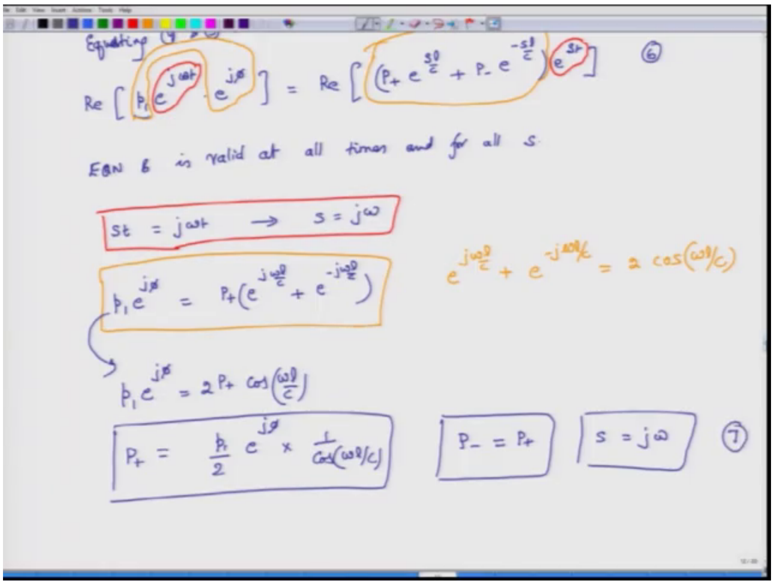

So, equating 4 and 5 what do we get real of p 1 e j omega t e j c is equal to real of P plus e. So, I am going to put x is equal to minus l here. So, it will be e S l over c plus P minus e minus S l over c e to the power of S t.

So, this is equation 6; now once again equation 6 is valid at all times and for all S, it is valid for all S and for all times. So, that can be true only if the time component here which is this is same as the time component here and simultaneously the other component simultaneously this other component should be equal to this other component simultaneous both these equations should con inequalities should hold. So, if that is the case. So, from this we get that S t equals j omega t which tells us that S equals j omega. So, this is from the equalities of terms in read and then once we substitute it, then we get the other equality is p 1 e j phi equals P plus and now I am going to substitute instead of S the value of j omega.

So, e j omega l over c plus e minus j omega l over c this side; so, this is I am getting because I am equating terms indicated in orange. Now we know that e to the power of j omega l plus e to the power of minus j omega l over c is nothing, but 2 cosign omega l over c. So, I get from here p 1 e j phi equals P plus twice cosine omega l over c. So, P plus equals p 1 by 2 e j phi into 1 by cosine omega l over c. So, this is my P plus my P minus.

We had already calculated and that is equal to same as p plus. So, this is the second thing and S we have figured out is equal to j omega. So, these are equations seven. So, once we have these equalities I can put equation 7 back into equation 1 a and 1 B. So, that is what I am going to do.

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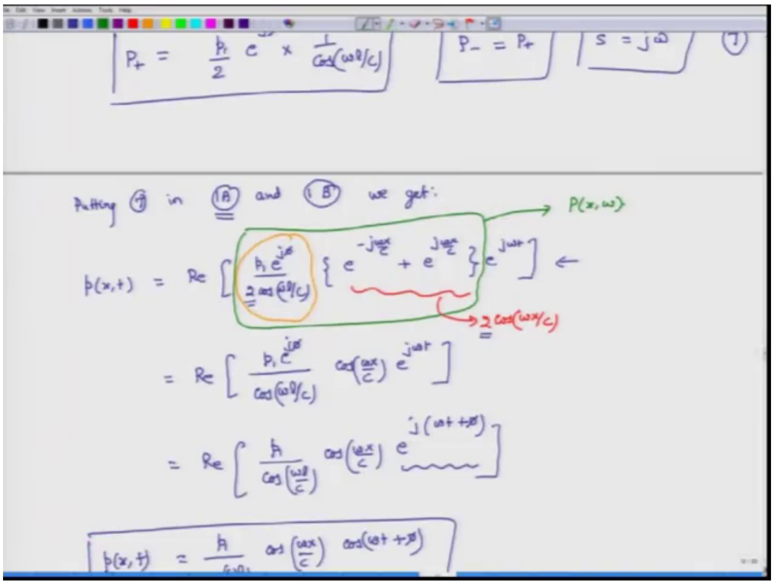

So, putting 7 in 1 a and 1 B phi get; what do we get we get transmission line equations for pressure and velocity in specific terms. So, my equation for pressure p of x t is real of P plus and P plus is what this complicated thing. So, it is p 1 e j phi by 2 cosine omega l over c and in parenthesis I get e minus j omega x over c plus e j omega x over c times e j omega t. So, this is my equation for pressure and I know that this is what cosine omega x over c times 2. So, my modified equation of pressure becomes or my transformed equation becomes p 1 e j phi by cosine omega l over c and this 2 term cancels out this 2 term.

So, I get cosine omega x over c e j omega t and I manipulate this further and I get p 1 by cosine omega l over c cosine omega x over c e j omega t plus phi. So, my pressure relation becomes. So, if I take the real power portion of this; this is the only complex portion an exponent or to j omega t plus pi and its real portion is cosine omega t plus v. So, p of x and t is what p 1 divided by cosine omega l over c cosine omega x over c cosine omega t plus phi. So, this is the expression for pressure and this express using this expression I can calculate the values of pressure at all the points in this tube at all the locations in this tube and I know p 1 because that number is given to me. So, this is the expression for pressure and now what we will do is we will develop an expression for velocity.

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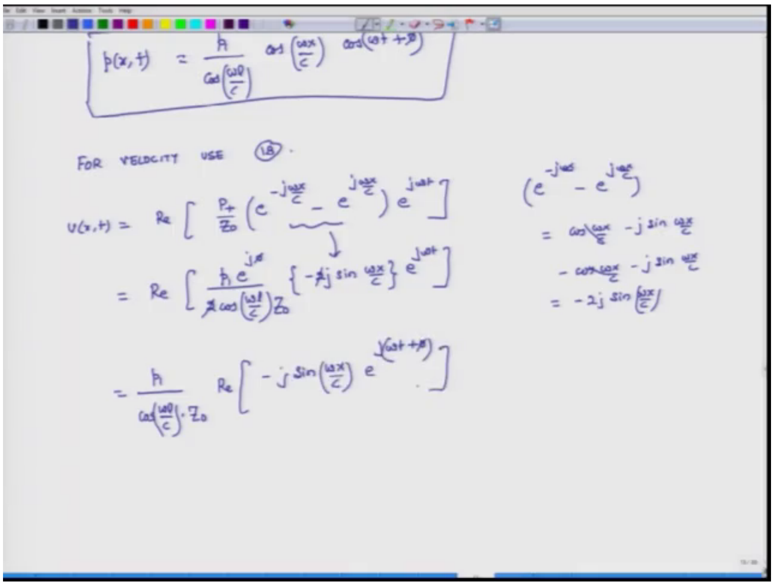

So, for expression for pressure we had used equation one a now we will develop an expression for velocity. So, we will do that. So, for velocity we use 1 B.

So, my expression for velocity is U x t equals real of P plus by z naught e minus j omega x over c minus e j omega x over c e j omega t and now what we are going to do is we are going to put this thing as the value of p plus. So, this is real of p 1 e j phi divided by 2 cosine omega l over c that is my P plus and we also know that e minus j omega x over c minus e j omega x over c; if I take the real and imaginary components and subtract one from other, I get cosine omega x over c minus j sin omega x over c minus cosine omega x over c minus j sin omega x over c, right. So, what I end up is these 2 cancel out, I get minus 2 j sin omega x over c, this is what I get. So, this thing is nothing, but minus 2 j sin omega x over c e j omega t and there is also a z naught here.

So, I can take all the real terms outside the bracket. So, it is p 1 by and then t this to 2 cancel out. So, what I left with this cosine omega l over c times z naught real of minus j sin omega x over c and I have e j omega t plus phi.

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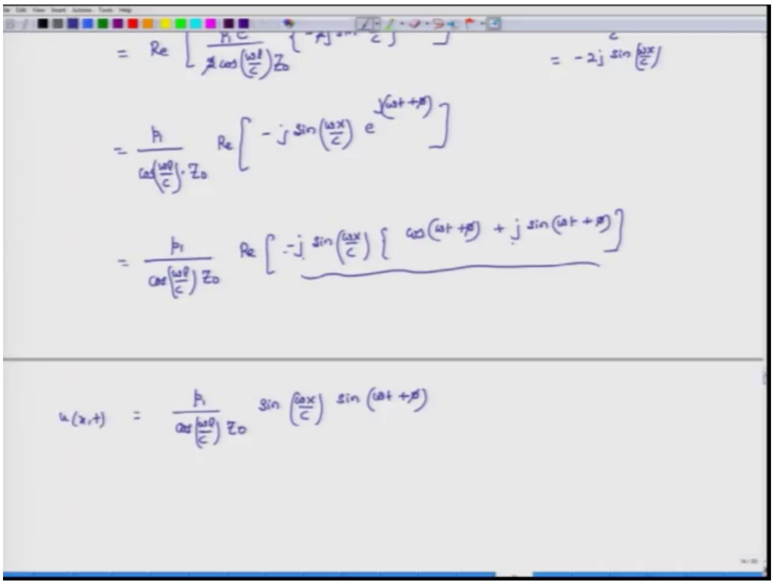

And this is equal to p 1 divided by cosine omega l over c z naught real of minus j sin omega x over c and this thing is cosine omega t plus phi plus j sin omega t plus phi. So, if I take the real portion of this what I am left with this I have to multiply j with j, I get minus 1 and then there is also this negative one negative already there. So, what I get is p 1 by cosine omega l over c z naught times sin omega x by c sin omega t plus phi. So, this is my velocity.

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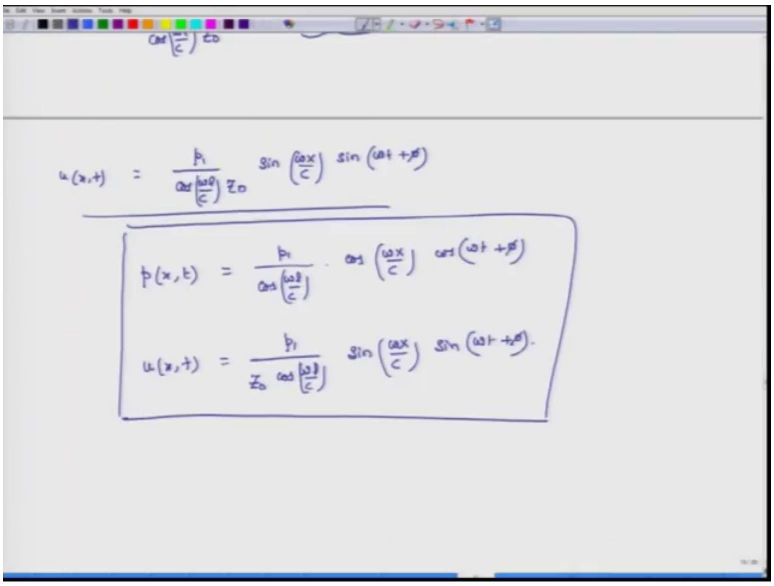

So, eventually lets summaries the expression for pressure is p 1 by cosine omega l over c times cosine omega x over c times cosine omega t plus phi and velocity is 1 divide by z naught cosine omega l over c sin omega x over c sin omega t plus phi. So, these are the expressions for pressure and velocity in a tube as shown here which is a closed tube rigidly closed tube and the pressure at x is equal to minus l which is the opponent is defined by p 1 cosine omega t plus phi this is the; so, if I use these boundary conditions and the transmission line equations and if I patiently run through all the mathematics I can actually predict pressure and vela particle velocity at all the points in this system at all the points in the system.

So, this is important to understand, now let us look at these equations a little bit more carefully, but before we look at this set of equations we find what does. So, this is the equation we got from equation one a and when we look at this entire thing this green box what does it represent this represents complex pressure amplitude because when you go back to our original definition of complex pressure amplitudes we had expressed it in 3 forms. So, this is where we had. So, there is this term e to the power of S t and whatever is multiplied to e to the power of S t everything else goes into complex pressure amplitude right everything if you just remove e to the power of S t whatever you are left with is the complex pressure amplitude of the wave similarly complex velocity amplitude of the wave inform if we look at form 3 is also shown here.

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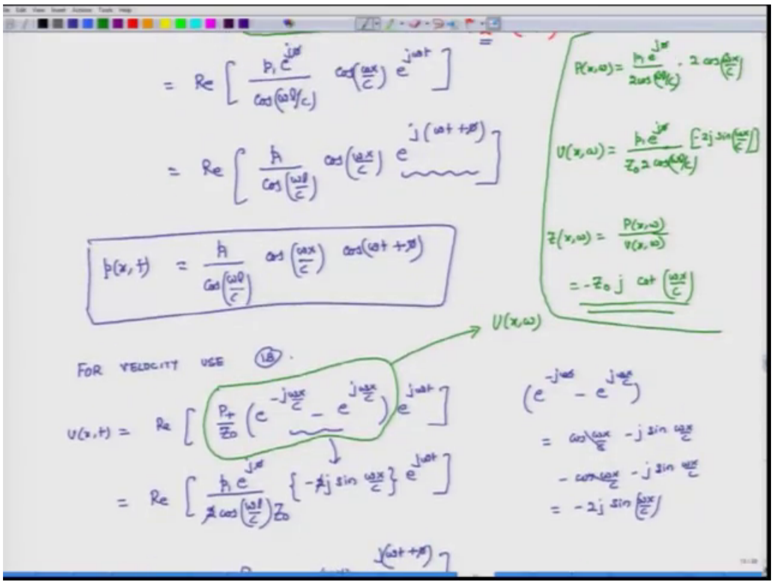

So, now, we go back this expression right here it is real of some complicated expression multiplied by e to the power of j omega t and we know that S is equal to j omega in our case. So, if I remove e to the power of S t from here whatever I am left with is the complex pressure amplitude which is p x omega and so, p x omega is this thing and what does that mean p x omega is same as this thing I mean of course, this is the real portion of it this is a real portion of it similarly the complex velocity amplitude for velocity wave is this entire thing complex velocity amplitude is this thing U x omega and. So, we will just make a small box here.

So, p x omega is what is equal to p 1 e j phi divide by 2 cosine omega l over c times this entire thing is 2 cosine omega x by c right and complex velocity amplitude U x omega is equal to p 1 e j phi divided by z naught 2 cosine omega l by c and.

What else and then of course, I have to multiply by minus 2 j sin omega x over c this thing. So, if I have to compute the specific characteristic impedance of this system then specific characteristic on sorry specific acoustic impedance of this system that is a function of P x w and U x w. So, that is equal to p x omega over U x omega and if you do the mathematics what you find is p 1 e j phi 2 cos omega l over c they all cancel out essentially what you end up getting is z naught j cotangent omega x over c and of course, there is a negative sin for this system the complex the specific acoustic impedance of this system which is a tube with rigidly closed end.

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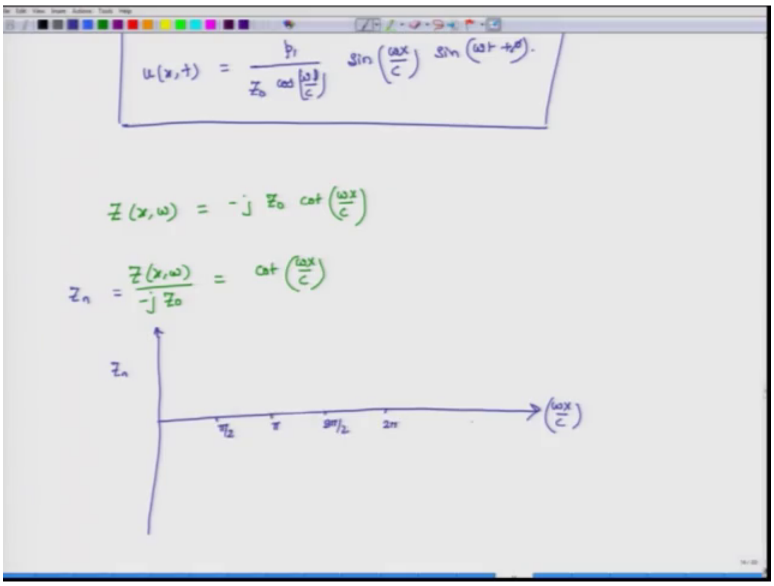

Z is equal to minus j z naught cotangent omega l over c this how we calculated it, right; this is what we calculated. So, I am just rewriting it later this omega x over c and if I just plot this thing z x omega divided by minus j z naught. So, I am left with cotangent omega x over c. So, if I plot this on x y plane and on the x axis; let us say; I am plotting omega x over c. So, either I can increase by omega or I can increase x and on the y axis let us call this function z n. So, it is basically cotangent of omega x over c normalize z that is why I am calling it and let us say that this is pi over 2 pi 3 pi over 2 2 pi and so on and so forth, then at x is equal to 0.