Noise Management & Its Control
Prof. Nachiketa Tiwari
Department of Mechanical Engineering
Indian Institute of Technology, Kanpur

Lecture - 21
Impedance

Hello. Welcome to Noise Management and its Control. Today is the third day of this week, and what we are discussing this week is, our transmission line equations, specifically their solutions to one dimensional wave equations, and is even more specifically what we are addressing is, the behavior of waves of sound waves, as they travel in one dimensions with respect to a Cartesian frame of reference. So, over last two days what we have been doing is, we have develop the transmission line equations and also we have connected the constant P plus and P minus 2 U plus and U minus respectively. And today onwards we will be solving a variety of problems related to 1 d waves in tubes.

But before we start discussing that I wanted to introduce some two or three important technical terms. So, we are going to introduce a term impedance.

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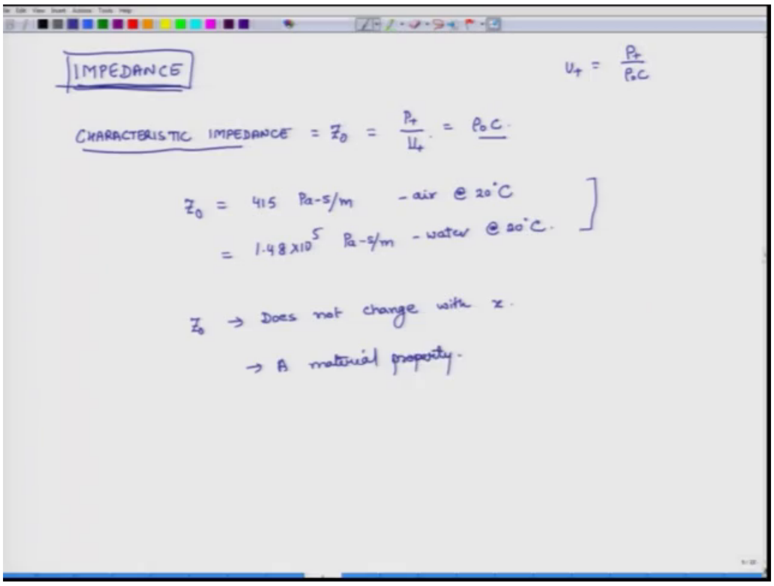

So, the origin of this impedance term, essentially it comes from electrical engineering, but in acoustics also we have this term impedance and here we have two types of impedance; one is characteristic impedance, characteristic impedance. So, this is equal to, we write it as z naught and the definition of this impedance is, that it is essentially the ratio of P plus divided by U plus.

Now we know that U plus is what P plus by rho naught C. So, this is something we had developed last time. So, z naught is equal to rho naught C. So, that is characteristic impedance and the value of z naught for air at 20 degree centigrade is 415 Pascal second per meters. This is for air at 20 degrees centigrade, and in water if we have sound waves travelling in water, it is about 1.4 8 times 10 to the power of 5, again Pascal second per meter. So, this is for water at 20 degrees centigrade.

Please note that z naught, it does not change with x, it does not depend on x. So, it is a multiple of density and speed of sound in the medium. So, it is a property, it is a material property, it is a material properties, it is a product of density and velocity of sound. Now it could be that velocity of, sound may change with respect to frequency. So, it may weekly vary with respect to frequency, but it does not change with x, but it may slightly change with frequency, its change somewhat with frequency.

So, this is our first impedance, characteristics impedance.

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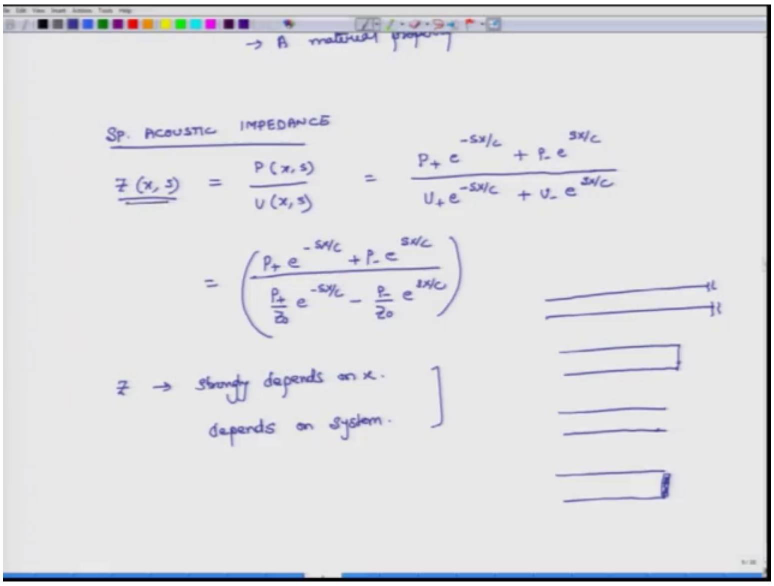

The second impedance is specific acoustic impedance. Specific acoustic impedance and that I define as z, and that is a function of x as well as s, and how do I define it? it is a function of two functions; one is complex pressure amplitude and the other one is for pressure, and the other one is complex pressure amplitude for complex velocity amplitude. Its complex velocity amplitude, and in the numerator we have complex pressure amplitude.

Now for waves in 1 dimensions what is P of x and s it is P plus e minus s x over c plus P minus e s x over c. This is how we have define complex pressure amplitude, and likewise what is velocity, it is U plus e minus s x over c plus U minus e s x over c. This I can re write it as P plus e minus s x over c plus P minus e s x over c divided by p plus by z naught e minus s x over c minus P minus by z naught e s x over c. So, z strongly varies, it strongly depends on x, and it is not a material property it. So, it depends on the system, you can have a tube which is infinitely long and you may get one function for z, you can have a tube which is closed. So, the system has changed.

And you may have another function z x s, which maybe its specific acoustic impedance. You may have a third tube, which is not infinitely long. So, the first one is infinitely long, you can have a third tube, which is not infinitely long of finite length, but its open at both ends. This may have a different function for z x a. You may have a fourth tube, which is not rigidly closed, but closed by some sound absorbing material, which absorb sound partially, and in that case the z x may again change.
So, z x s, it changes with respect to x and it is the exact nature of the function, it depends on what kind of a system we have in place, and it changes from system to system. So, that is there. The second thing is that z naught or z naught we see that, it is a product of rho naught in c. So, it is a purely real number, because rho naught is real and c is real, but z, which is expressed as this complex entity is a complex function.

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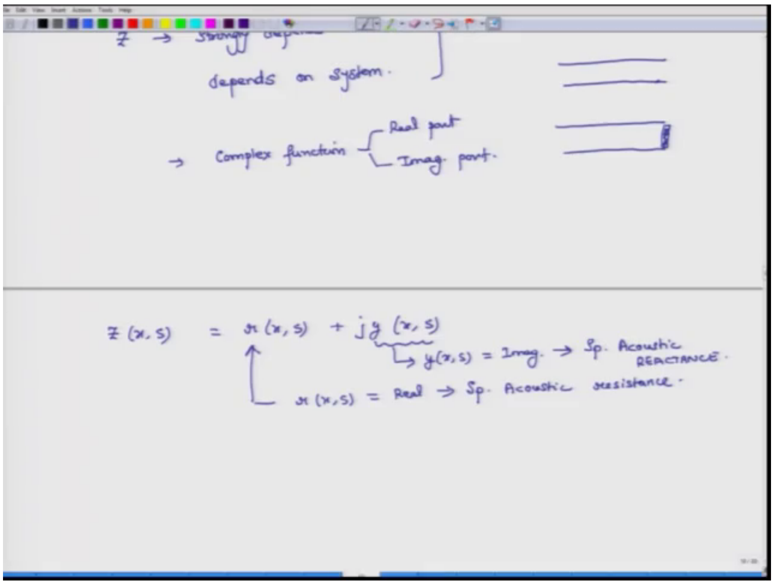

It has a real part, and it has an imaginary part. It has a real part and it also has an imaginary part. So, in general I can express z of x n s as.

A sum of two other functions are, which can also vary with x n s plus j times y which can also vary with x n s. So, what is this? This is the real portion of the function, and this is our x n s is totally real, and this is called a specific acoustic resistance, and this is the imaginary component y and. So, this is the imaginary component, and this is called specific acoustic reactance. So, z can have a real portion known as a specific acoustic resistance, and an imaginary component which will be specific acoustic reactance.

So, understanding this thing is important, because as we start talking about noise we may run into terms; like impedance, and impedance may have complex values, and then if we have this kind of a background, then we will have a better appreciation of what all these technical terms been. So, this is what I wanted to discuss in context of impedance.

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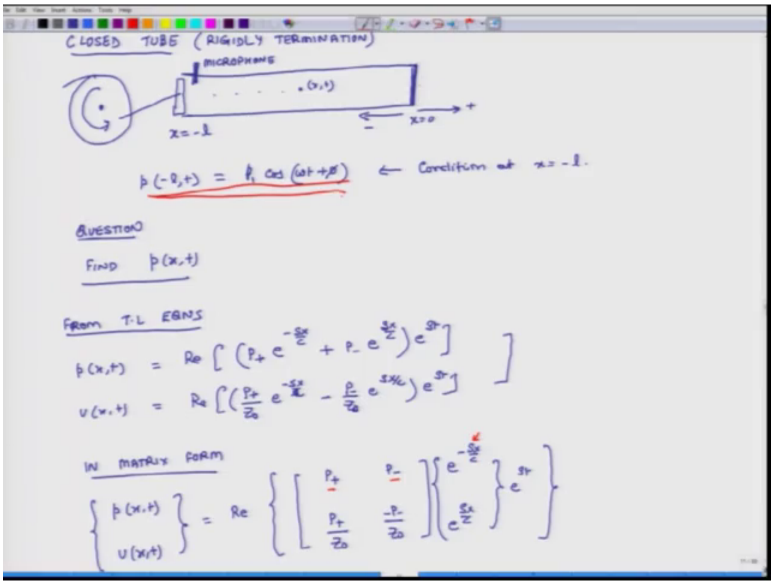

Now in the remaining part of today's lecture we are going to do one example, and that example is for a closed tube.

. So, the example corresponds to a closed tube. So, here is the problem definition. we have a tube, it is a straight tube and it is closed. So, I will qualify this, it is a rigidly closed tube rigid termination. So, what does this mean, at this end of the tube is extremely rigid, infinitely rigid. So, when sound waves hit, it does not vibrate or bend, because of that. So, this is a rigidly held closed tube, my coordinates system is such that x is equal to 0. At the rigid termination, this is my positive and this is my negative, and the length of the tube is x is l, and because I am going in the negative direction.

So, the open end of the tube is located at x is equal to minus l, and the last thing I wanted to specify is, I have a piston. So, it may be connected with a slider crank mechanism, and the piston is such that, the piston is moving back and forth, and I place some microphone here. So, I am placing a microphone at x is equal to minus l, and I measure the pressure at x is equal to minus l, and I find that the pressure is at x is equal to minus l based on my measurement from microphone, is some constant number P 1 cosine omega T plus c P 1 P 1 is real, omega is real c is really. So, this is my condition at x is equal to minus l. So, then with this problem statement the question is, that find pressure at an arbitrary position x and for all times. So, find P of x and t.

So, I want to figure out what is the pressure at this point, this point, this point and all points in the tube, at all points tube. So, how do we do? Again this is a one dimensional problem we know the conditions on both hands, and we have to go back to our transmission line equations. So, that is always our starting point; so from transmission line equations. From transmission line equations we can say that P of s and t is real of P plus e minus s x over c plus P minus e s x over c e to the power of s t.

So, this is my equation for pressure, and velocity equation transmission line equation is real of P plus by z naught e to the power of minus s x over c; so minus P minus by z naught e to the power of s x over c e to the power of s t. What I will do to these two equations is. I will reorganize them in a matrix form, because these are two equations. So, in matrix form, there is no particular advantage at least at this stage. Just looks a little more clean that is all in matrix form, I can write it as P of x t and u of x t is real of this vector which is computed by these matrices, so P plus P minus P plus P minus. Here I have z naught z naught and I am multiplying this by this column vector minus s x over c e s x over c e to the power of s t.

So, this reason we have done this, it looks a little more clean and more organized. So, again what is our aim, we have to find the values of P plus. We have to find the value of P minus, we have to find the value of s. If we know P plus P minus and s, then I can calculate the value of P x t for all values of x in the tube. So, our aim is to find P minus P plus and s. So, these are the transmission line equations, and the way we are going to figure out values of P plus P minus P s, and s is, we are going to use these conditions.

So, what is the condition that pressure at minus l is defined by this equation? So, that is one condition. We know at the first boundary of this system at x is equal to minus l. The other condition we know is, that at, there is a closed end at the, this is a closed end that the tube. So, what we will do is, we will use both these conditions and these are known as boundary conditions. One boundary condition is pressure at minus l is P 1 cosine omega t plus phi. The other boundary condition is at x is equal to 0. You have a closed tube, rigidly closed tube, and what that means? Physically we will discuss that. So, using these two boundary conditions, we will calculate P plus P minus n minus s.

So, with that I want to conclude our discussion for today. And we will complete this discussion in tomorrow’s lecture. Till then have a great day. Bye.