Noise Management & Its Control
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Lecture - 20
One Dimensional Example Problems

Hello. Welcome to Noise Management and its Control. Today is the second day of the fourth week of this course and what we plan to do today is an extension of the discussion we initiated yesterday and specifically; what we plan to accomplish today is we will cover a couple of examples of how pressure waves travel in one dimensional equation. And how now we can actually using transmission line equation for pressure; how actually we can calculate the variation of pressure as a wave moves in a very long tube and then later in the second half of today's lecture, we will develop interrelationships between com these constants P plus P minus u plus and u minus. So, these interrelationships will link the transmission line equation for pressure and transmission line equation for velocity.

So, we are going to start with an example and the question is that suppose, we have a long tube. So, it is going to be an infinitely long tube.

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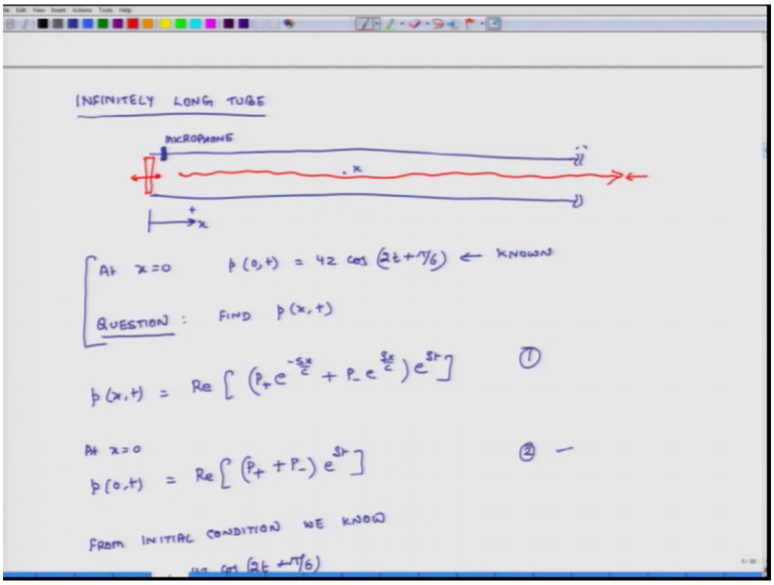

So, it is an infinitely long tube and because it is an infinitely long tube I signify it using these symbols and it is a straight tube and what we have at the one end of the tube is a piston and this piston is moving back and forth such that. So, what is my coordinate system this is my coordinate system x. So, this is my positive direction and in the other side is negative direction.

So, if I place some microphone here. So, this is a microphone I am using this microphone I can measure the pressure at the beginning of the tube and the tube is infinitely long. So, it has a beginning, but we cannot figure out what its end is going to be because it is an infinitely long tube. So, at the beginning of the tube at x is equal to 0 I take a microphone and measure the pressure. And I find that the value of pressure at x is equal to 0. So, p 0 and for all times is 42 cosine 2 t plus pi over 6.

So, this is known and how do we know it we know it by placing a microphone at the beginning of the tube and recording the signals and then developing a plot of the signal. So, then the question is. So, this is the information we know and then the question is that at any given location let us say; so, this location is x. So, it is a distance x away from the beginning of the tube. So, the question is find pressure at any given position x and at any time; time t. So, find p of x and t. So, this is the question. So, we know the initial conditions that p of 0 and p of 0 t is 42 cosine 2 t plus pi over 6 and that the question is that how do we find p of x and t and how do we do it.

So, the solution for this is resides in transmission line equations and because we are interested in finding pressure we go to the transmission line equation for pressure. So, first we write it down. So, pressure we know that our transmission line equation for pressure is equal to real P plus e minus s x over c plus P minus e s x over c e to the power of st. So, this is equation number one also. So, at x is equal to 0 if I use equation 1 at x is equal to 0 we get p at 0 t is equal to real of P plus and e to the power of s x over c is 1 because x is 0 e to the power of minus s x over s is also 1.

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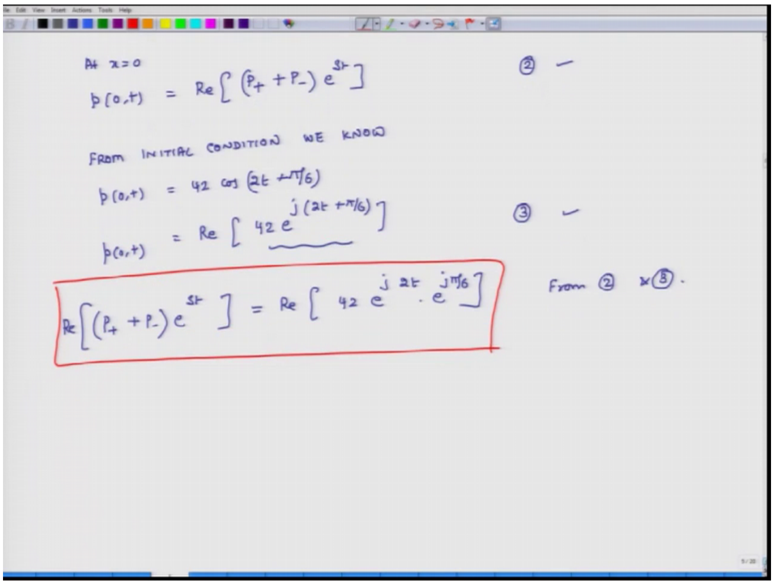

So, I get c plus plus P minus e to the power of s t. So, this is equation 2 now from our initial condition we know that p 0 t is equal to 42 cosine 2 t plus pi over 6 this is something we already know because we used the microphone and measured it and I can express this as real of 42 e to the power of j times 2 t plus pi by pi over 6, because if I take the real of this thing. So, this is my equation 3. So, if I take real of this thing in the bracket I get 42 cosine 2 t plus pi over 6. So, I have one equation for p 0 t equation number 2 have another equation; equation number 3 I can equate them. So, I get P plus plus P minus e to the power of s t is equal to. So, real I have to equate real quantities on both sides real of 42 e to the power of j 2 t times e to the power of pi over 6 times j.

So, this I am getting from 2 and 3, now I look at this equation and this equation has to be true at all values. So, this equation has to be true. So, I call this equation 4.

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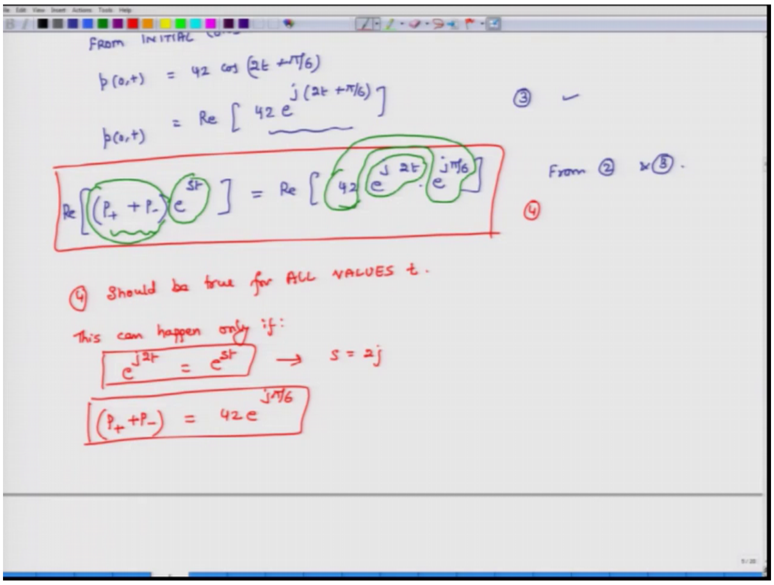

So, I say that equation 4 should be true for all values of t it is not that it should be true only at t is equal to 0 or t is equal to it has to be valid true for all at all times because our transmission line equation it comes from the pressure wave equation which is valid for all values of x and all values of time. So, this helps to has to be value true for all values of t and that can be true only if the time dependent term e to the power of st should be equal to time dependent term in this case e to the power of 2 t and then this entire thing should be equal to. So, this entire thing should be equal to this thing.

So, this can happen only if e to the power of j time 2 t equals e to the power of st and; that means, s is equal to 2 j and the second thing is that P plus plus P minus is equal to 42 e to the power of j pi over six. So, we get 2 relationships. Now, I know that s is equal to 2 j and P plus plus P minus is equal to 42 times e to the power of j times pi over 6, but still we do not know the specific values of P plus and P minus we know the sum of P plus p minus. Now, we figure out what is the value of P plus and what is the value of P minus and to understand that we have to go back to our original picture, what P plus represents is the amplitude of the wave as it is travelling in the forward direction and what P minus represents is the wave amplitude of the wave travelling in the reverse direction and this is a infinitely long tube.

So, once a wave starts, it will just keep on going forward only because it will never get a chance to hit a wall and reflect back which means. So, based on this understanding of the physics of the problem or how this picture is made out we can say that the amplitude of the reflected wave or the backward travelling wave is 0.

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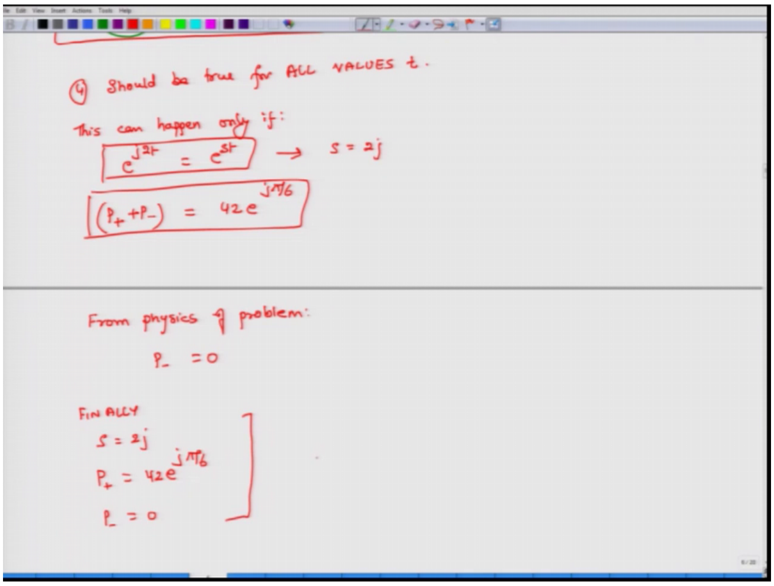

So, from the physics of the problem P minus equals 0. So, finally, we can say s is equal to 2 j P plus equals 42 e to the power of j pi over 6 and P minus equals 0.

So, let us call this equation 5. And now what we do is we put equation 5 back into equation 1.

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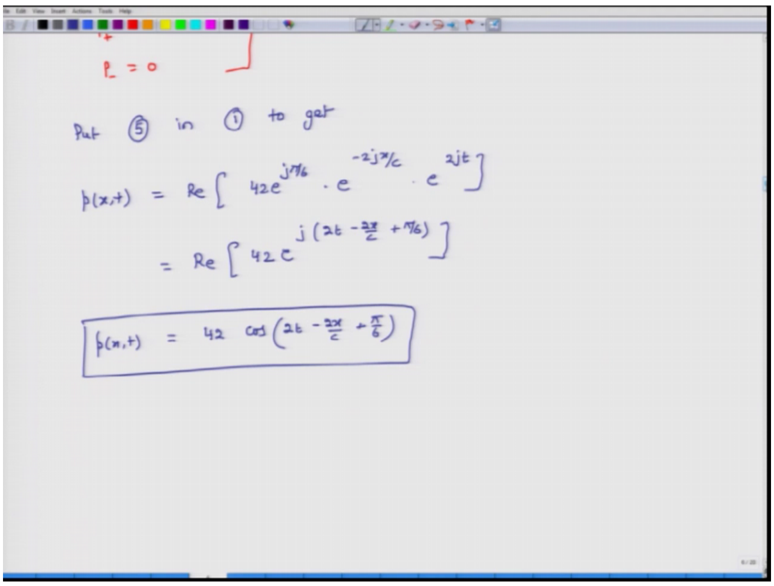

So, now we put equation 5 in equation 1 to get p of x t equals real of p plus. So, P plus is 42 e to the power of j times pi over 6 times e to the power of s minus s x over c and s x over 2 j. So, I get minus 2 j x over c and then there is a P minus term which is 0. So, all that goes away and then I have to multiply by e to the power of st. So, again let us 2 j t or it is equal to real of 42 e to the power of j 2 t minus 2 x over c plus pi over 6.

Now, this is a complex exponential function with complex powers. So, this becomes nothing, but 42 cosine if I take its real portion only cosine of 2 t minus 2 x over c plus pi over 6. So, this is the solution. So, what we have done in last 10-15 minutes is that we have considered all the information which was made available for this problem, one information which was made available was what is the value of pressure at x is equal to 0 at different times and then the other information based on the physics of the problem is that there is no reflected wave in the system.

Based on these 2 pieces of data what we did in our problem is that we identified the value of P minus we identified of P plus we identified the value of P minus and we also identified the value of s. So, we have to whenever we solve the transmission line equation, we have to figure out; what is the value of P plus; what is the value of P minus and what is the value s and once we know that then we have figured out the exact solution of the system. So, that is what we have done and using this approach, we have figured out that the pressure wave represented by p of x and t is 42 cosine 2 t minus 2 x over c plus pi by 6.

So, that is one part of the discussion which I wanted to have today. So, now, the next thing is this is how we solve problems by applying the physics of the problem boundary conditions or the physics of the problem and also the initial conditions and then we compute the values and P plus P minus and s and we solve the problem.

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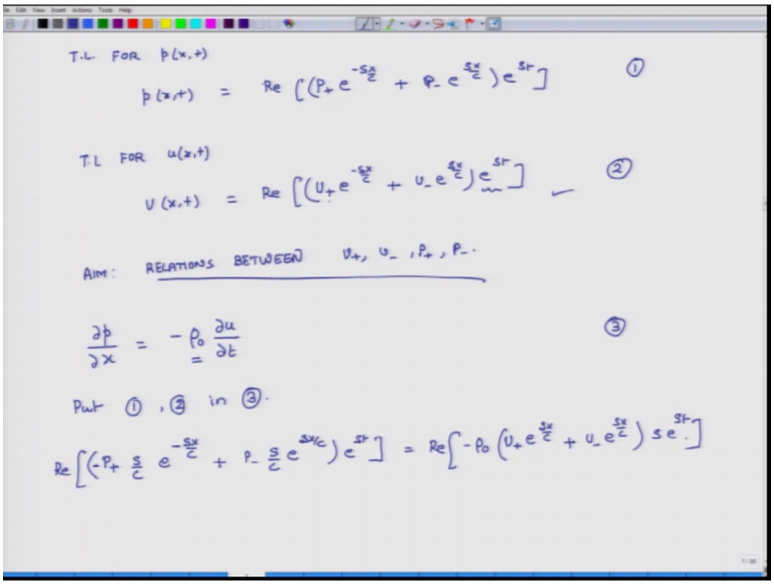

Now, in last class we had developed 2 different transmission line equations. So, the transmission line equation for pressure was p of x t equals real of P plus e minus s x over c plus P minus e s x over c e to the power of s t and the transmission line equation for velocity is u of x t equals real of u plus e minus s x over c plus u minus e s x over c e to the power of s t. So, what we plan to do today is find out the relationship between u plus u minus P plus and p minus. So, our goal is or our aim is to find relations between u plus u minus P plus P minus this is what we planned to do.

Now the way we are going to do it is that we know that when we were developing our one dimensional wave equation we used 3 different equations to come up to the one dimensional wave equation the first equation we used was Newton’s second law of motion which is which we called as the what was that equation it is the momentum equation then the second equation which we used was for law of conservation of mass that is the continuity equation in the third equation was the gas law. So, what we will do is we will use the momentum equation because in momentum equation it connects; what does momentum equation say? It says that the force on a system equals rate of change of momentum rate of change of momentum.

So, on the right side of the equation, you have rate of change of momentum which is mass times velocity its differential and on the left side you have force and force is nothing, but pressure times area. So, the momentum equation connects pressure with velocity. So, we will rely on that equation to connect P plus P minus u plus and u minus. So, let us write down the momentum equation. So, the momentum equation is partial derivative of pressure with respect to x is equal to minus rho naught del u over del t.

So, let us call this equation 1, let us call this equation 2, let us call this equation 3. So, what we do is we put 1, 2 in 3 we substitute equation 1 in on the left side equation 2 on the right side and we do the mathematics and what we get is. So, the left side is del p over del x. So, what I get is real of P plus minus s over c. So, I am differentiating it with respect to x e to the power of minus s x over c plus P minus s over c e to the power of s x over c this entire thing multiplied by e to the power of st. So, this is the left side and on the right side I get real of and here I have a density term. So, minus rho naught and then I differentiate u which is this equation u with respect to t.

So, I get u plus e to the power of s x over c plus u minus e to the power of s x over c and this is the term which relates to time. So, it is s e to the power of st. So, I now rearrange. So, one thing I do realize is that I have e to the power of now in this equation I have e to the power of st here and e to the power of st is here.

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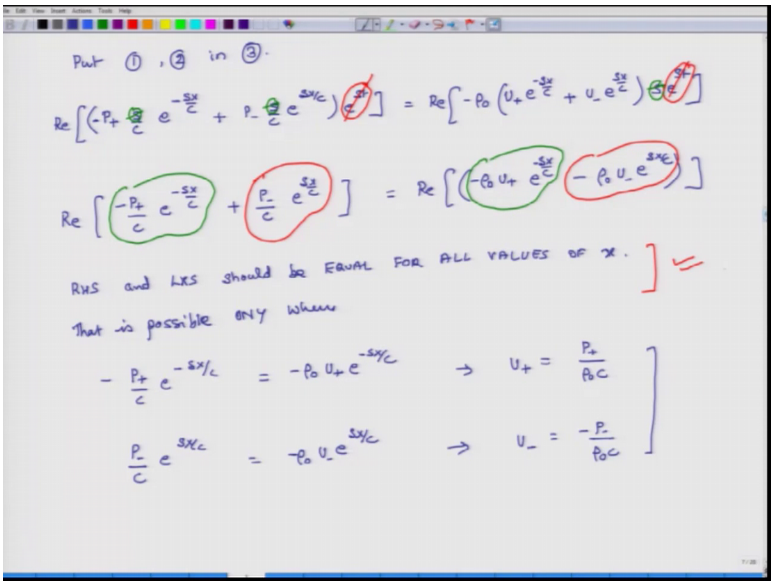

So, I cancel them out other thing is that there is s here and there is s in both the terms here. So, I cancel this s s also. So, what I am left with is real of minus P plus over c e to the power of minus s x over c plus P minus divided by c e to the power of s x over c this equals on the right side minus rho naught u plus e to the power of s x over c.

There should be minus here and there is a minus here minus rho naught u minus e to the power of s x over c and brackets closed now this momentum equation is again valid for all times and all values of x all times and all values of x. Now it just happens that when we apply momentum equation to transmission line equations the time term goes away because we have cancelled it out, but x term is lying on both the sides. So, RHS and lhs should be equal for all values of x for all values of x and that is possible only when individual terms are equal; what do I mean by that; that is possible only if this term in the green which is multiplied by e to the power of s x over c is same as this term on the right side and similarly this term in red equals this time in red on the right side.

Only then this condition will be true otherwise we cannot make sure that for all values of x this remains true. So, if that is the case what; that means, is that minus P plus over c e to the power of minus s x over c should be equal to minus rho naught u plus e to the power of minus s x over c or u plus equals P plus by rho naught c and similarly from the terms which are encircled in red, I get P minus divided by c e to the power of s x over c equals minus rho naught u minus e to the power of s x over c and from this I get u minus is equal to minus P minus by rho naught c.

So, let us call this equation what is it equation 4. Now, what we do is we put these equations equation 4 in equation 2. So now, we put 4 in 2 and once we do that what we get.

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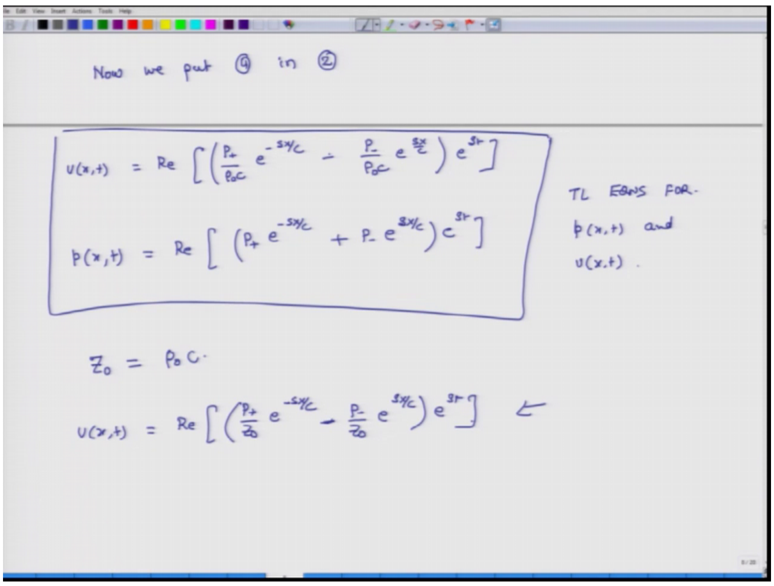

So, if we do that then u of x and t equals real of P plus by rho naught c e to the power minus s x over c; so, this is actually P plus minus and then the reflected wave term is P minus divide by rho naught c and this entire thing is multiplied by e to the power of st and we have already the equation for transfer transmission line equation for pressure and that is real of P plus e to the power of minus s x over c plus P minus e to the power of s x over c e to the power of s t.

So, these are the 2 transmission line equations in the final form. So, these are transmission line equations for pressure and velocity respectively now there is a term known as z naught and this is defined as rho naught c. So, it is a constant z naught. So, I can rewrite the transmission line equation for velocity as real of P plus by z naught e minus s x over c plus P minus by z naught excuse me this is a negative sign here e s x over c e to the power of st. So, this is another alternative form where I am just defining rho naught c as z naught.

So, this completes our discussion for today. And tomorrow onwards we will start looking at tubes, where we will actually calculate both the pressure wave as well as velocity wave in the entire tube using these transmission line equations.

So with that I will like to close it, and have a great day. Bye.