Noise Management & Its Control
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Lecture - 19
Transmission Line Equations

Hello. Welcome to Noise Management and its Control. This is the fourth week of this course and today is the first day of this particular week. Throughout this week, we will be expanding on the 1-D wave equation which we developed in the last week and what we will specifically, we doing is we will be actually providing solutions to this equation in context of one dimensional wave.

And in that context we will introduce Transmission Line Equations which are essentially general solutions for one dimensional waves as they travel in a single straight line.

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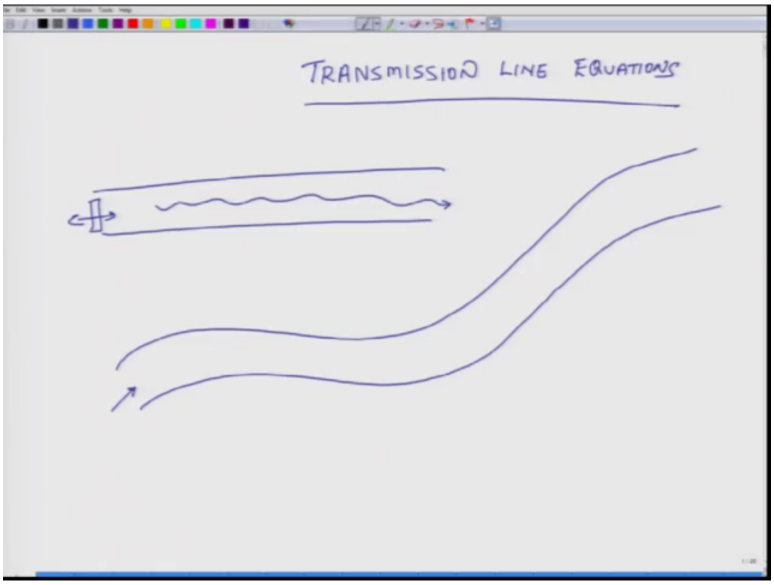

So, what we are going to talk about a transmission line equations. So, what are these equations all about essentially they are solutions to one dimensional wave equation; for sound waves as they travel in a single dimension. So, what could be examples of such waves? So, you can have a long tube and I am producing some sound waves at one end of the tube and the sound travels along the length of the tube and I can figure out how sound travels along the length of this tube which is not varying in cross section and it is a straight tube.

So, by using transmission line equations another example could be a waveguide. So, wave guide is essentially a similar tube, but it could be a little bit of curved, but the cross sectional area remains uniform and if sound enters at one end of the tube it follows and its path is parallel to the length of the tube and this also I can figure out using transmission line equations. Now these equations were originally developed in electrical engineering field of study. They were developed to explain how electromagnetic waves travel in one dimension.

And what we are doing here in this lecture as well as in subsequent next 4, 5, 6 lectures will be will be essentially using similar equations which are used in electrical engineering, but there the context is electromagnetic waves here, we will be discussing acoustical waves travelling in one dimensions.

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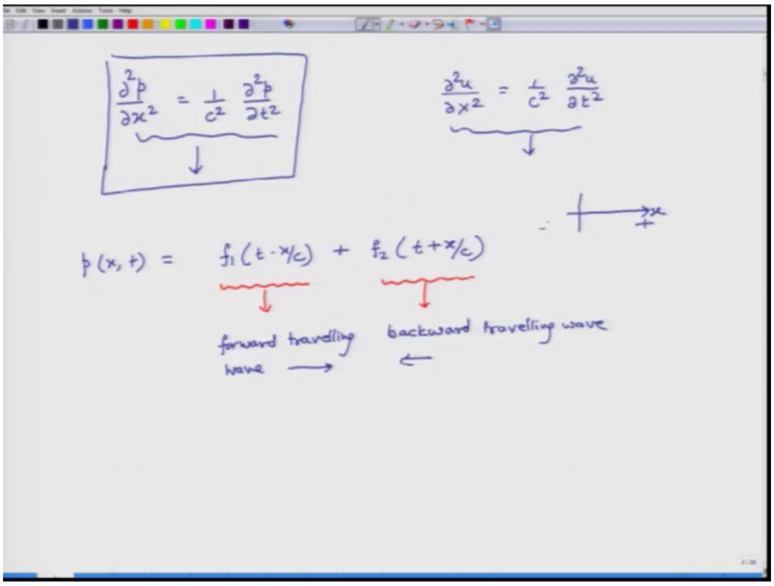

So, before we start developing these transmission line equations we will rewrite the one dimensional wave equation for pressure and the one dimensional wave equation for velocity particle velocity. So, the one dimensional wave equation for pressure is second derivative of incremental pressure with respect to x that equals one over c square del 2 p over del t square and the one dimensional wave equation for velocity is second derivative of u which is particle velocity differentiated in space which is x and that equals one over c square del 2 u over del t square.

So, what we are trying to do is we will first develop a transmission line equation for the pressure wave equation and what that will tell us is how does pressure change in time as well as along the length of the one dimension. And similarly, we will develop a one dimensional wave equation for velocity also.

Now, earlier in one of the earlier classes we had said that a very general solution for pressure wave which satisfies this one dimensional wave equation is it can be written as p of x t and that is some function f 1 of t minus x over c plus f 2 of t plus x over c. So, this is a general equation. Now if you recollect what we had said was that f 1 which depends on t minus x over c this is a wave travelling. So, this is a forward travelling wave forward travelling wave. So, it is moving in positive x direction and f 2 is a wave which is moving in negative x direction. So, this is a backward travelling wave. So, this is moving in negative x direction where are positive x is this. So, this is our positive x and this is our negative x.

So, p of x t is sum of f 1 and f 2 and I can have any type of a function as long as it depends on t minus x over c it is a valid function.

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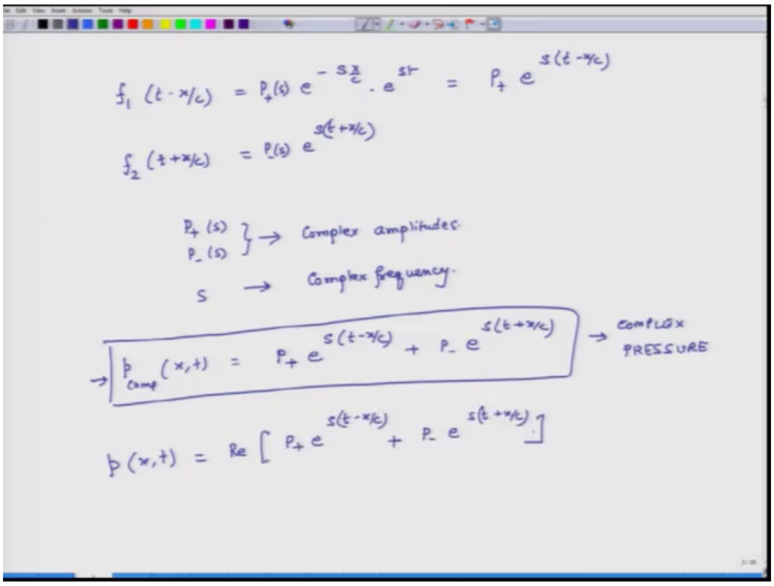

So, here what we will do is we will say that f 1 is which depends on t minus x over c, we will define it as a complex number P plus. So, it is P plus times e minus S x over c times e S t and it actually is a valid form of f 1 because I can rearrange these things and I can say that this is equal to P plus e S t minus x over c similarly f 2 is a function of t plus x over c. So, this is equal to P minus e S t plus x over c. And I will actually modify these 2 equations a little bit I am not going to modify, but I am going to make some things more explicit than I had written earlier. So, P plus is a constant, but it can change with S and P negative is also a constant, but it can vary with respect to S. So, that is there. So, P plus as we had discussed earlier in one of our introductory lectures these are complex amplitudes and S is the complex frequency.

So, the complex pressure can be written as p complex which depends on x and t, I can write it as P plus e S t minus x over c plus P minus e S t plus x over c and so, this is an equation which gives us complex pressure. So, this gives us complex pressure and it is complex pressure as a function of x which is position and time x and t, but what we are interested in finding out is the actual pressure the real pressure. So, if I have to find the real value of this basically I take the real value of the right hand side. So, I will call this as p x t is equal to real of P plus e S t minus x over c plus P minus e S t plus x over c.

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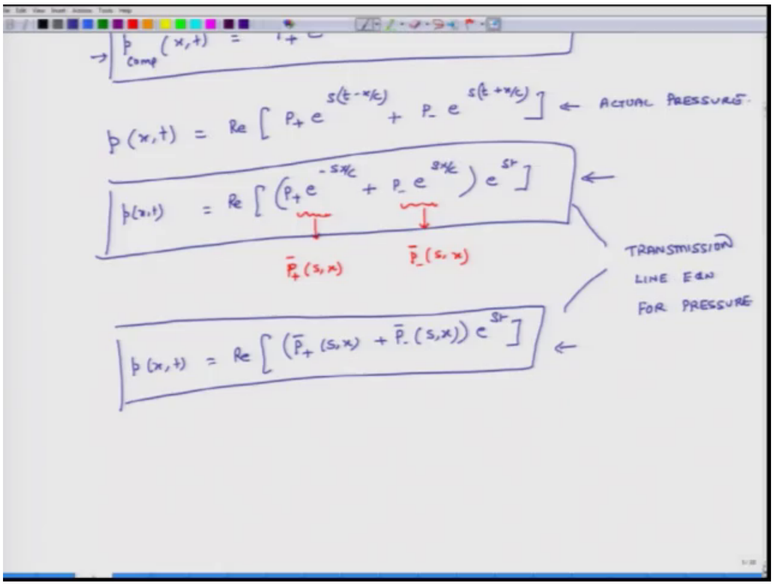

So, this is my relation for actual pressure and I can further rearrange this thing as real of P plus e to the power of minus S x over c plus P minus e to the power of S x over c.

I can put it in parenthesis and I can take e to the power of st out. So, this is another way to organize our results I now define this entire term as P plus bar and this depends on S and t and I define this entire thing as P minus bar and this again depends on S and t. So, with these definitions I can rewrite it as p x t is. I am sorry. So, you are right. So, this depends on S and x and this also depends on x and x.

So, I can rewrite this entire thing as P plus bar which is a function of S and x plus P minus bar which is a function of S and x e to the power of St. This equation or I can also call this equation. So, I can look at if these 2 equations they are mathematically equivalent these are known as transmission line equations. So, I can take the first one or I can take the second one does not matter if this is the transmission line equation for pressure and essentially what this equation says is that if the complex frequency of the wave is S then the pressure wave in the tube, it changes according to this equation or alternatively it changes according to this equation.

Now, you may wonder that why did I choose this form I choose f 1 as P plus S times e to the power of St minus x over c and f 2 as this form I could have chosen any other arbitrary form as well.

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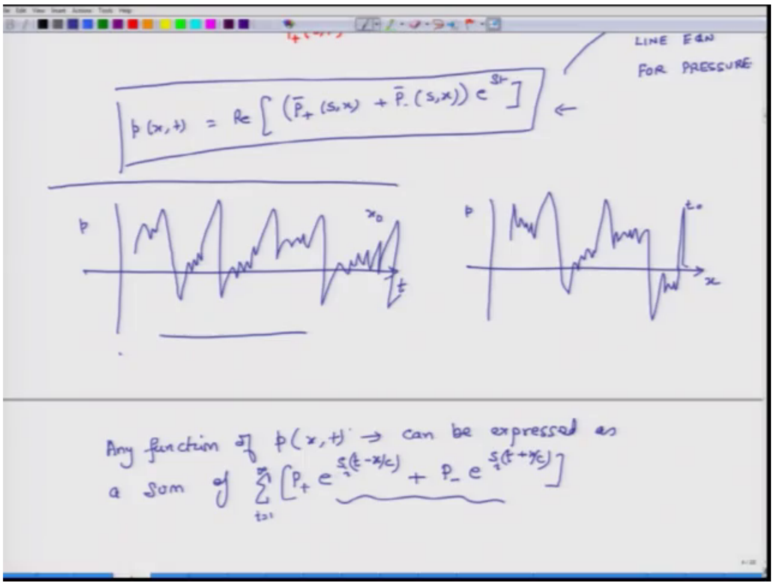

So, why did I why did I choose this and the relation answer to that is it is important that you all understand and the reason is that in reality in reality the wave if I; if I plot pressure. So, if I at lower if I plot pressure as a function of time at a location x naught then the pressure may be doing something arbitrary it may be doing something arbitrary. And similarly if I plot pressure as a function of x at a given instant t 0 again the pressure may be doing something arbitrary.

So, the waveform which exists in real situations need not be a simple function like this or this it could be a complicated function, but the reason we choose these functions is that advanced mathematics tells us that I can take any function any function of p which changes with respect to x and t; it can be expressed as is sum of you know P plus e to the power of S t minus x over c plus P minus e to the power of S t plus x over c and here I am putting an index I.

So, I can express any function p of x and t as a sum of all these different P plus I is equal to I can say from one to infinity and as I keep on changing my complex frequency and I sum them up I can arise if I use a mathematics correctly I can express any function p of x and t as a sum of different P plus e S t minus x over c and P minus e S t plus x over c using a method known as Fourier transform using Fourier transform.

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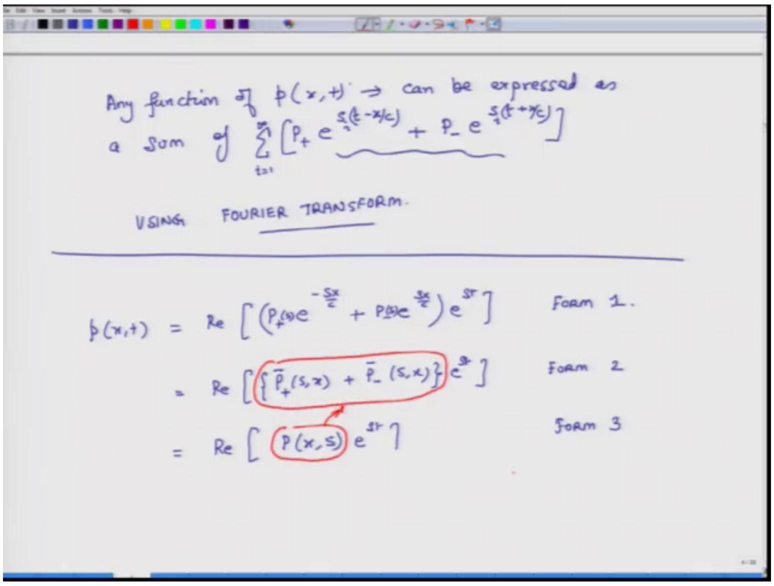

So, remember this function need not be periodic p of x t may not be periodic, but I can express any complicated function which may be periodic may not be periodic as a sum of different complex frequencies using the method of Fourier transform.

So, my general equation if I have to figure out how it changes it will be essentially as sum of p one xt plus p 2 x t plus p three x t and so on and so forth. So, that is why right now I am just assuming that there is just one single complex frequency which is passing through the system and it can be expressed as a forward travelling wave P plus e to the power of S t minus x over c and a reflected wave of backward travelling wave which is P minus S t plus x over c and if I sum them up this is the overall expression for P plus p of x and t.

So, this is my transmission line equation for pressure when I have just one single frequency going through the system if I have a complex wave, then I add up all these different frequencies which it is made up of and I can get a complex wave pattern in the tube. So, that is the reason I am using a simple frequency.

So, to recap we will once again right that if there is a single complex frequency or single frequency passing through the system then p of xt what are transmission line equation says is real of P plus excuse me e to the power of minus S x over c plus P minus e to the power of S x over c times e to the power of S t. So, this is form one of transmission line equation I can write this transmission line equation in another form and that is P bar.

So, these are functions of x S and p bars are function of S and x; so this P plus bar and then this P minus bar. So, this is form 2 and there is a third form also. So, this is capital p of x and S e to the power of St, this is form three where p capital p of x and S is nothing but. So, this thing is nothing but this entire thing and this p x and this type of form is used especially when we try to compute impedance of system.

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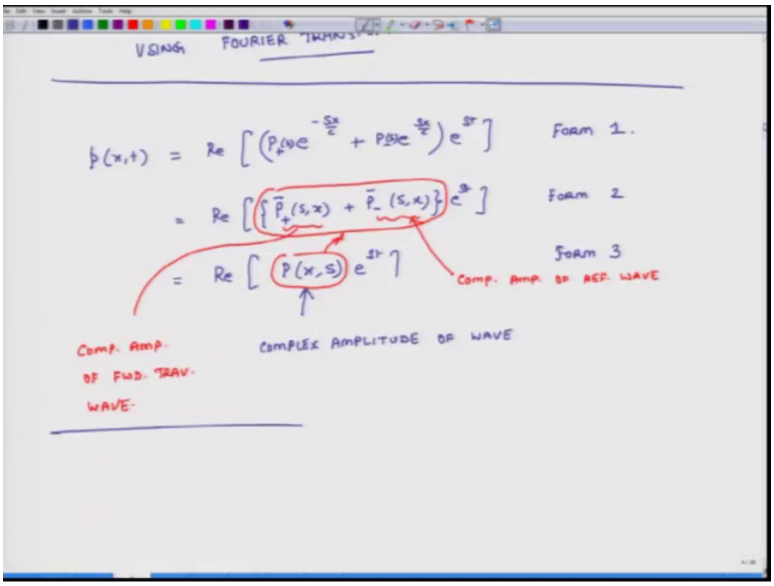

And this thing is known as complex amplitude of wave complex amplitude of the wave this not complex amplitude of the forward travelling wave or the backward travelling wave of the total wave which is there that is represented by p this thing is complex amplitude of reflected wave. And this guy is complex amplitude of forward travelling wave now these curve for all positions.

So, this comes for all positions and all frequencies these are the complex amplitudes. So, the next thing is. So, we can develop similar relations for velocity as well. So, we have developed transmission line equation for pressure. And similarly we can have a transmission line equation for velocity and by analogy I can have a transmission line equation for velocity.

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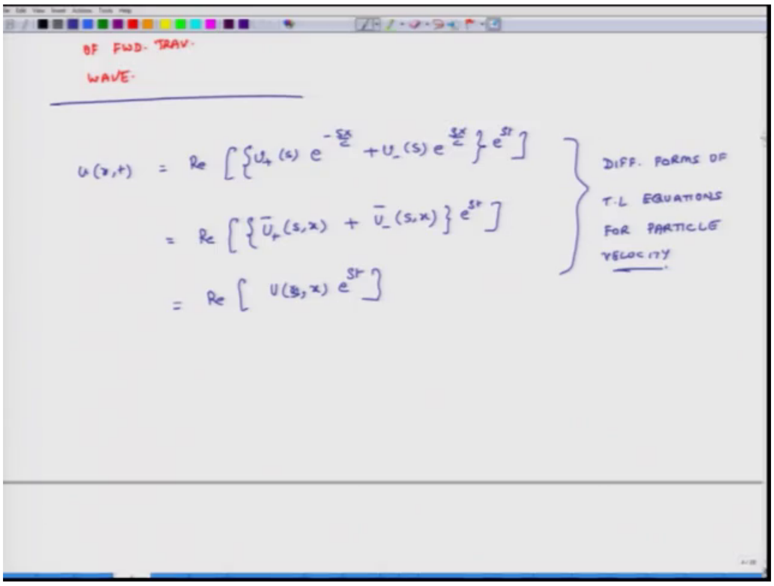

So, u of xt is equal to real of u plus S e to the power of minus S x over c plus u minus S e to the power of S x over c e to the power of S t. So, this is the first form and like we did in the case of pressure I can also have u plus bar S comma x plus u minus bar S comma x e to the power of St, this is mathematically different form from 2 and the third form which is the most condensed one is u S comma x e to the power of S t. So, these are three different. So, these are different forms of transmission line equations for particle velocity these are transmission line equations for particle velocity.

So, what we have done today is we have developed transmission line equations for pressure and we have developed transmission line equations for particle velocity for one dimensional waves moving in a with respect to partition frame of reference. What we will do in the next class is will continue this discussion and specifically we resolve a one single problem. And then later; what we will develop is a relation between equation 1 and equation 2 between transmission line equation for pressure and transmission line equation for velocity will develop relations between these 2 equations also.

So, that concludes our discussion for today. And it has been pleasure talking to you, I look forward to seeing you all tomorrow.

Thank you. Have a great day.