**Noise Management & Its Control**
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**Lecture – 18**
**Solution for one dimensional wave equation**

Hello, welcome to noise management and its control. Today is the last day of the third week of this course and today, we will close our discussion on the one d wave equation and specifically what we are going to discuss today is are 2 things; one is that we will develop some general solutions for the one d wave equation.

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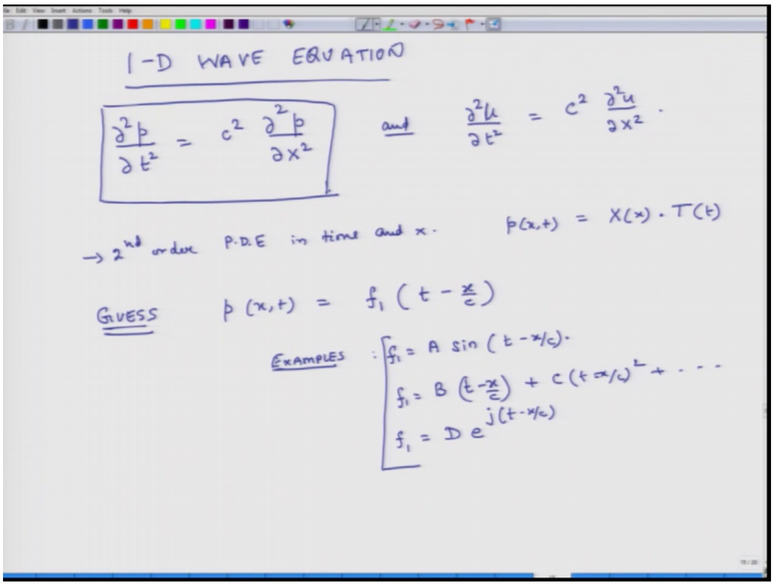


And next what we will do is that we will develop a physical interpretation of the parameter c. So, our one d wave equation; so, where one d wave equation is second derivative partial derivative of pressure with respect to time equals c square del 2 p over del X square. So, this is for pressure and similarly we have del 2 u over del t square equals c square del 2 u over del X square. So, in this class we will primarily focus on the pressure wave equation, but if we wanted to develop solutions for velocity wave equation the theory and the process will be more or less identical. So, we do not have to worry about the velocity wave equation. So, now, what we will do is that we will try to figure out a solution for this pressure wave equation.

So, what we see is that this is a second order partial differential equation. So, it is a second order partial differential equation in time and X, now how do we develop a solution for it. So, there are several approaches you may have learnt in terms of how to solve these types of equations one is through the method of variable separable where you assume that pressure which is a function of X and t is itself made up of 2 functions one is purely dependent on X.

So, capital X is purely dependent on position X and then the other thing is another function t which is purely dependent on t and if we use this and plug into it then we can separate the variables and develop a solution for this other approaches that we can guess some solution we can guess some solution and if our guesses are correct then when we plug that solution back into this pressure wave equation it should satisfy the equation. So, if we make a guess and if you plug it into the first equation the left hand side and right side if they are indeed equal then we say that our guesses are correct. So, that is what we will do. So, we will guess and we will say that lets guess that p of X and t is equal to some function f 1 which depends on t minus X by c.

So, function f 1 does not depend on t alone it does not depend on X alone rather it depends on a combination of t and X in such a way that we have to do t minus X over c. So, examples of such functions what could be put some examples. So, there could be infinite search function some examples one example would be a sin t minus X by c another example could be B t minus X by c plus. So, this is another example plus capital c times t minus X by c whole square and I can extend this series another example could be an expense exponentially thing d e to the power of j t minus X by c.

So, the point is that as long as t and X by c come in this form t minus X by c in this form it will be a function of t minus X over c. So, we are saying that any function which is a function which depends on t minus X over c is a potential solution for the pressure wave equation. So, this is our guess and now what we will try to see is whether our guesses correct or not.

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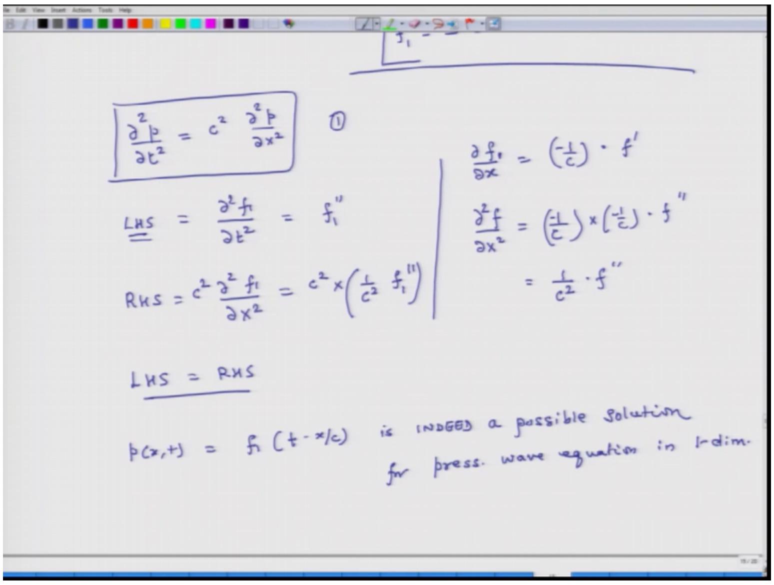


So, we will rewrite the pressure wave equation. So, our pressure wave equation is del 2 p over del 2 p square is equal to c square del 2 p over del X square. So, let us number this as equation one and the lhs of this equation left hand side equals del 2 f 1 divided by del t square and the right hand side of this equation is equal to del 2 f 1 over del X square and that equals. So, so we will actually calculate the right side so far. So, what is del f over del X is equal to minus 1 by c times del f over del t, we can say that or I can call it f prime.

So, this is equal to f 1 prime prime similarly the second derivative del 2 f over del X square is equal to minus 1 over c times minus 1 over c times f time prime. So, this is equal to 1 over c square times f prime prime. So, what is a right side it is c square times second derivative of f 1 with respect to X and that equals c square into 1 by c square f prime prime.

So, what we see is that left hand side is equal to the right hand side. So, what we have guessed is indeed a possible solution it may not be the only solution, but it is indeed a possible solution for our one dimensional wave equation. So, we can conclude that p of X and t is equal to f 1 t minus X over c is indeed a possible solution for pressure wave equation in one dimensional in one dimension.

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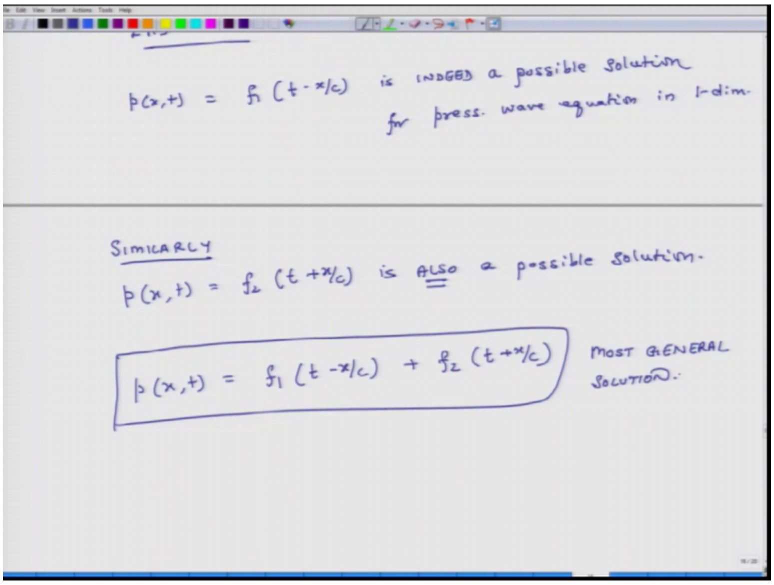


Similarly, we can also say that p of X t equals f 2 t plus X over c is also a possible solution. So, these are the most general solutions we can think off f 1 which depends on t minus X over c and f 2 which depends on t plus X over c these are the most general solutions. So, in an overall sense we can say that the most general solution for the pressure wave equation equals f 1 t minus X over c plus f 2 t plus X over c. So, this is the most general solution.

We will look at a simple case and we will initially consider only this part of the solution.

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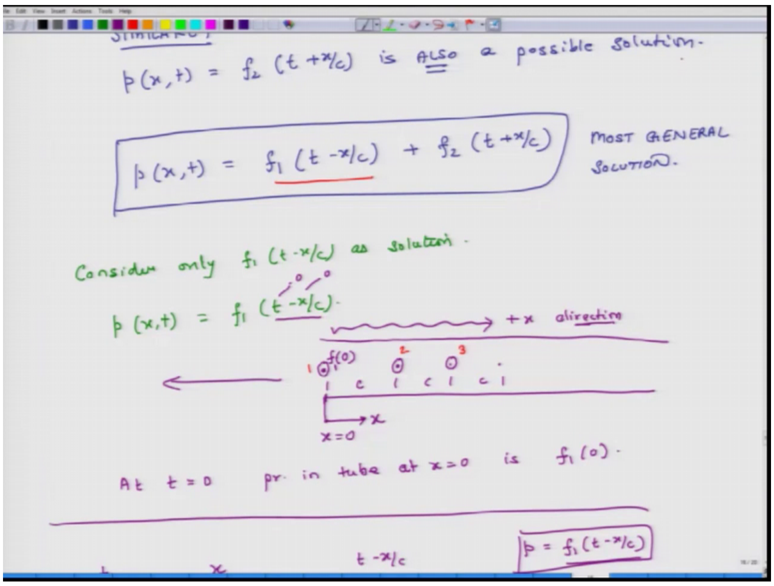

So, initially we will consider only and we will see what it means only f 1 t minus X over c as solution then p of X t equals f 1 t minus X over c and at this point of time what we will do is. So, this is a solution for a wave which is traveling in one dimension it is f 1 represents a wave which is traveling in one dimension f 2 also represents a wave which is travelling in a single dimension X dimension, but we will see what these 2 different forms mean f 1 and f 2. So, consider a simple tube and let us say this is my X axis and X as a starting from here. So, the value of X is X is equal to 0 at this origin and the wave is travelling in this direction and we say and when we are observing the tube we note that at time t is equal to 0 at time t is equal to 0 the pressure at the origin is let us say f 1 of 0.

At time t is equal to 0 pressure is f 1 of 0 why because in this when t is equal to 0 in the brackets this number is 0 and at origin X is also 0. So, in the bracket whatever is the only thing which is left is 0, right. So, what is the pressure at origin at time t is equal to 0 f 1 of 0. So, at time t is equal to 0 pressure in tube at X is equal to 0 is f 1 of 0.

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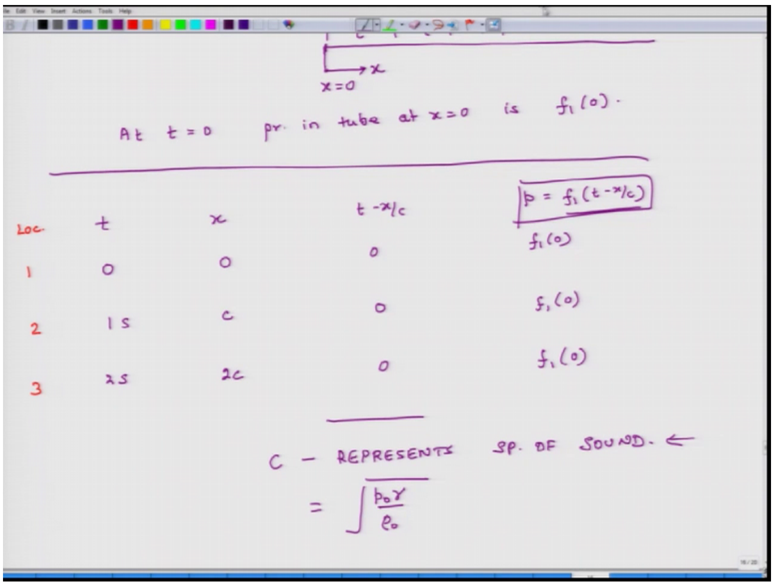


We will make a table. So, we will in this table we will have time we will have X we will have t minus X over c and we will have p equals f 1 t minus X over c. So, we are said that at time t is equal to 0 at location X is equal to 0 t minus X over c is 0 and what is the pressure f 1 of 0 now consider time one second one seconds after one second we want to see where what is the pressure what is the pressure and we want to find the pressure at X is equal to. So, we want to find the pressure here. So, at time t is equal to one second we want to find the pressure here and this distance is c meters. So, what is the value of X in this case c, right.

It is c meters and what is the value of t minus X over c 0 and what will be the pressure it will still be p minus X over c which is at location 2 right this is location one is f 1 0 consider time 2 seconds at 2 seconds we are interested in finding the pressure at location 2 location number 3. So, I will call this location one this location 2 location 3 I will write it here location 1, 2, 3. So, at location 3 what is the coordinate or the position it is 2 c what is the value of t minus X over c 0. So, what will be the pressure f 1 of t minus X over c and this is 0. So, what this tells us is that if there was some sound present at time t is equal to 0 at origin and if its value was f 1 0 the same sound travels to location 2 which is at a distance c away from location one after one second and the same sound travels to location 3 at time 2 seconds right and that location 3 is 2 c distance away from the origin which means that the sound this sound signal which was initially at origin location one travels to location 2 after one second and what is its speed the distance is c.

The time is one. So, its speed is c meters per second and it travels to location 3 after 2 seconds and again the speed is c meters per second. So, from this table we conclude that c represents speed of sound this is one thing t c represents speed of sound and it is for this reason and mathematically we can calculate it as p naught gamma over row naught and we have to take this square root of it. So, c presents the speed of sound and its value can be calculated like this and if we calculated we find that this number is very close to the experimentally observed value of speed of sound and it and because it represent the speed of sound it indeed comes close to the theoretical value or the actual measured value of the sound speed. So, that is one the second observation we would like to note is that f 1 t minus X over c this function.

It represents a sound wave which is propagating in the positive X direction right because it is t minus X over c it represents a sound wave which is travelling in the positive X direction, but there is no reason for sound not to travel in this direction sound from here it can travel in positive X and as well as negative X direction. So, the neg.

The wave which is travelling in the negative X direction be represented as t plus X over c f 2 t plus X over c and if we make the same table like we made for f 1, we will find that for f 2 also the speed of sound is same which is c, but the direction of sound propagation is negative X direction it is negative X direction.

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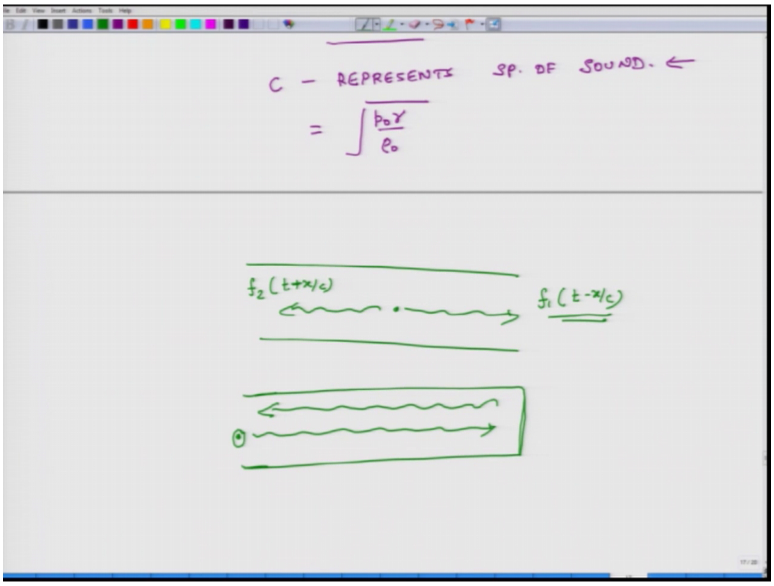


So, in reality when sound travels in the positive X direction the motion of the move the propagation of sound is represented by f 1 p minus X over c and in the negative X direction the propagation of sound waves is represented by f 2 t minus X over c t plus X over c now in reality these things these functions f 1 and f 2 they are important, if suppose I have a closed tube and suppose I start generating sound at the open end of the tube what happens sound travels in the positive X direction that I can represent by f 1 and then it hits this wall and then it gets reflected and the reflected sound I can represent it as by f 2.

So, this is the physical interpretation of f 1 and f 2 and the physical interpretation of c is that it indeed represents the speed of sound propagation. So, that concludes the discussion for today and starting next week we will start solving this one dimensional wave equation for a lot of practical applications and then after that we start looking at noise how it propagates how it can be measured; how it can be managed and controlled. So, with that we close the discussion for today and we look forward to seeing all of you on Monday of the next week.

Thank you very much, bye.