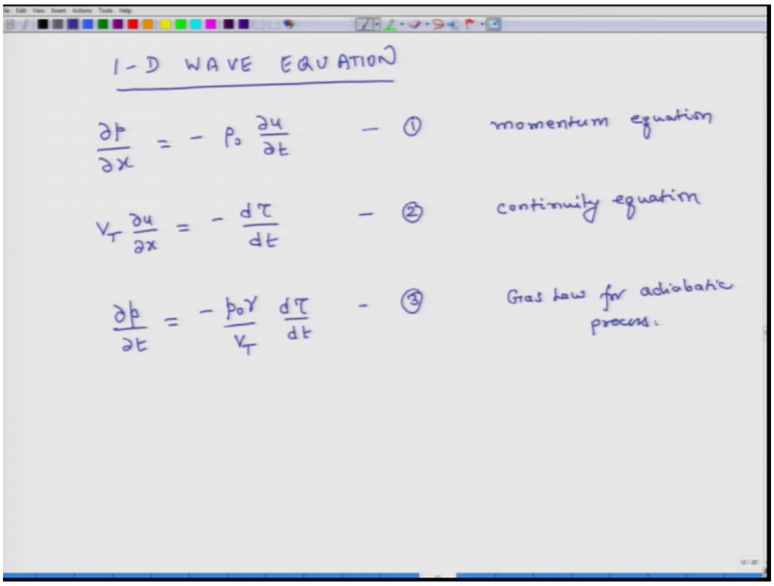
Noise Management & Its Control
Prof. Nachiketa Tiwari
Department of Mechanical Engineering
Indian Institute of Technology, Kanpur

Lecture - 17
One dimensional wave equation

Hello. Welcome to Noise Management and its Control. Today is the fifth day of the third week of this course and what we plan to do today is a develop the; finally, develop the one dimensional wave equation using the 3 equations which we had developed in earlier classes. So, specifically what we have developed were the momentum equation which was based on Newton's second law the continuity equation which was based on conservation of mass and the gas equation which was based on the relationship between pressure and volume in a gas when it follows an adiabatic process.

So, what we will do is we will use these three equations and combine them together to come up with the final 1-D wave equation.

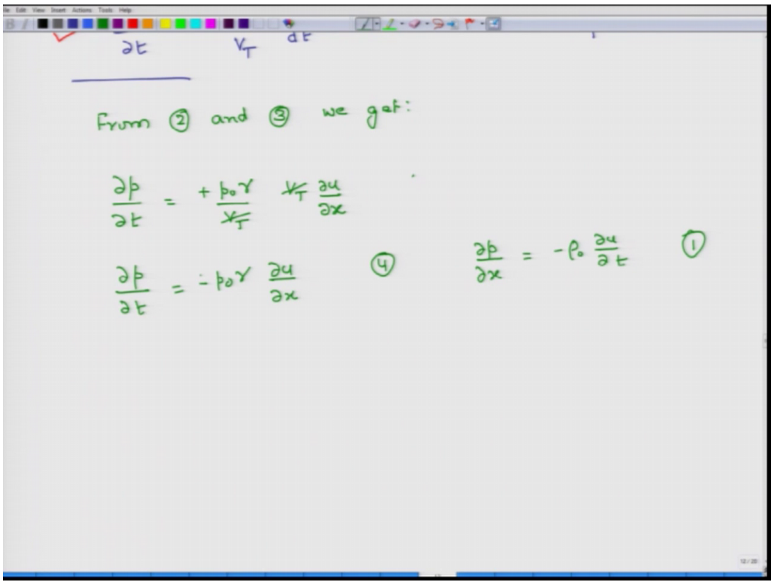
(Refer Slide Time: 01:09)



So, that is what we are going to develop and to recap the three equations which we had developed were partial of pressure with respect to position equals minus rho naught del u over del t. So, let us call this equation one and this was based on. So, this is the momentum equation then the next equation was V T. Del u over del x equals minus total derivative of change in volume with respect to time. So, this is equation 2 and this is the continuity equation and the third equation was partial of pressure with respect to time equals minus p naught gamma over V T beta over dt.

So, that is the gas law for adiabatic process. So, what you see especially in the momentum equation is what it says is that. So, on the left hand side is the partial derivative of pressure with respect to x. So, essentially this is the gradient of pressure. So, what this equation says is that if there is gradient if the pressure changes in a fluid then because of that change of fluid the fluid will experience acceleration which is del u over del t and then of course, it has to be multiplied by rho naught because that is change in momentum that is what it means.

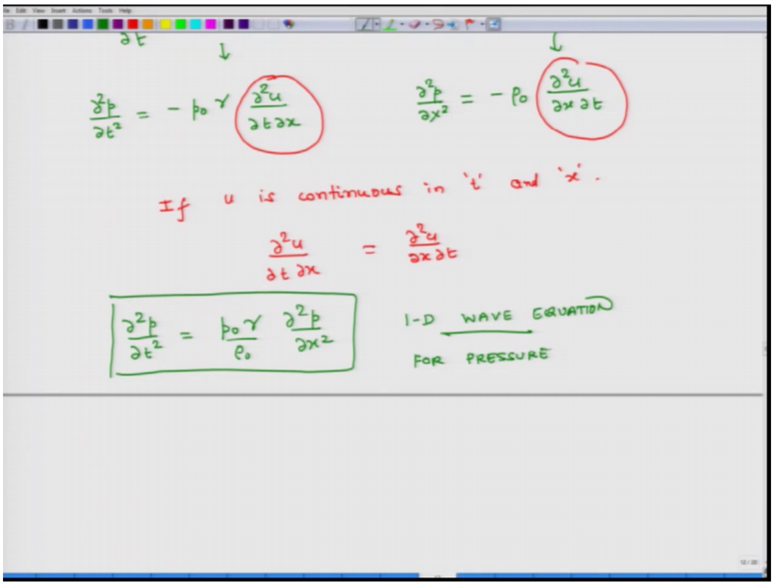
(Refer Slide Time: 03:37)



So, now what we are going to do is we are going to synthesize these 3 equations and come up with the final 1-D equation and the first thing we will do is we will we see that equation 2 and equation 3 on the right side we have tau total derivative of tau with respect to t. So, we will eliminate tau from equations 2 and 3. So, what we get is from 2 and 3. So, del p over del t equals minus p naught gamma over V T and d tau over d t is minus V T del u del x. So, V T del u over del x and this negative sign goes away from here because theta over dt already has a negative sign. So, this becomes positive. So, this is. So, we will just rewrite this, because V T cancels out is equal to p naught gamma del u over del x.

So, this is equation four and of course, for this is the momentum equation. So, this is I am rewriting equation one here del u over del t s del u over del t. So, I think I missed a negative sign here. So, it is here now we see that in equations four and one we look at equation four and one they are in pressure which is p and u. And now our aim is to eliminate let us say u from here. So, what we do is we differentiate equation four with respect to time and differentiate equation one with respect to x.

(Refer Slide Time: 06:06)

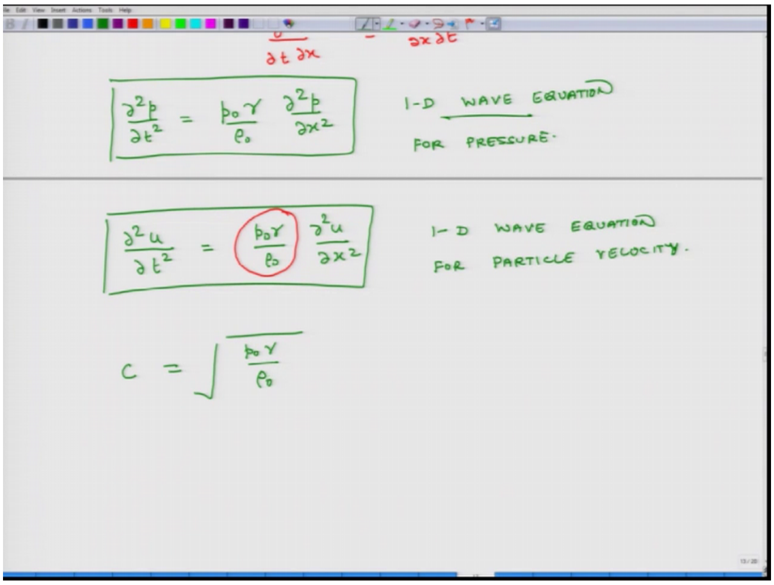


So, when we do that we get second derivative of pressure with respect to time. So, this is a partial derivative is equal to minus p naught gamma del 2 u over del t del x and from here we get del 2 p over del x square equals minus rho naught del 2 u over del x del t now we note that here I have the second derivative of u first with respect to x, then with t and here I have second derivative of u first respect to time and then with respect to x. So, these are across derivatives and if u is continuous in time and x and it is differentiable, then we can say that these cross derivative are same.

So, we make this assumption that u and its first derivatives are continuous in time in x and if that assumption holds then these 2 things can be considered as the same which one. So, so these things can be eliminated. So, once this is eliminated what I have is. So, if that is the case then what I have is del 2 p over del t square is equal to p naught gamma by rho naught times del 2 p over del x square. So, that is my 1-D wave equation for pressure.

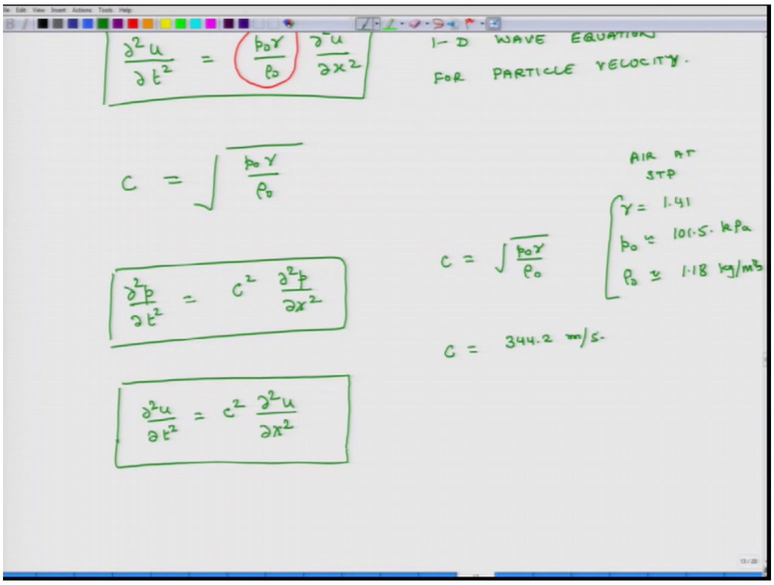
So, in this 1-D wave equation both the sides they involve pressure on the left side has second derivative in time the right side has second derivative in x and similarly. So, what we have done here is that from these 3 equations, we had eliminated tau and u we can do a similar mathematics and eliminate tau and p and using the same approach.

(Refer Slide Time: 09:04)



We will also get a similar equation for velocity. So, that will be del 2 u over del t square is equal to p naught gamma over rho naught del 2 u over del x square and this is 1-D wave equation for particle velocity for particle velocity. So, these are. So, in both these cases the equation is for one dimensional wave propagation. Now let us look at this term p naught gamma over rho naught. So, what we do is we define a new parameter called C and we see that C equals square root of p naught gamma over rho naught. So, if we do that then the equations for pressure and velocity become del 2 p over del t square equals C square del 2 p over del x square.

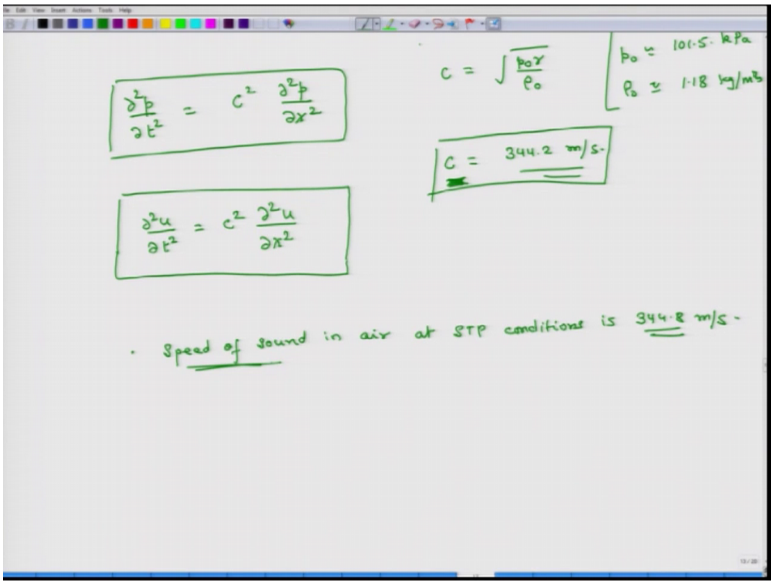
(Refer Slide Time: 10:24)



And similarly for velocity particle velocity our relation is second derivative of u with respect to time equals C square times second derivative u with respect to x. Now we have said that C equals p naught gamma over rho naught and if we plug in the values of p naught and rho naught and gamma. So, gamma equals for an ideal gas, it is 1.41, p naught is approximately equal to 101.5 kilo Pascal, s rho naught is approximately 1.18 kg per square per cubic meters. So, this is for air at STP conditions.

So, if we plug-in these values we found find that C equals 344.2 meters per second and now what we will try to do is we will try to figure out what is this parameter C all about.

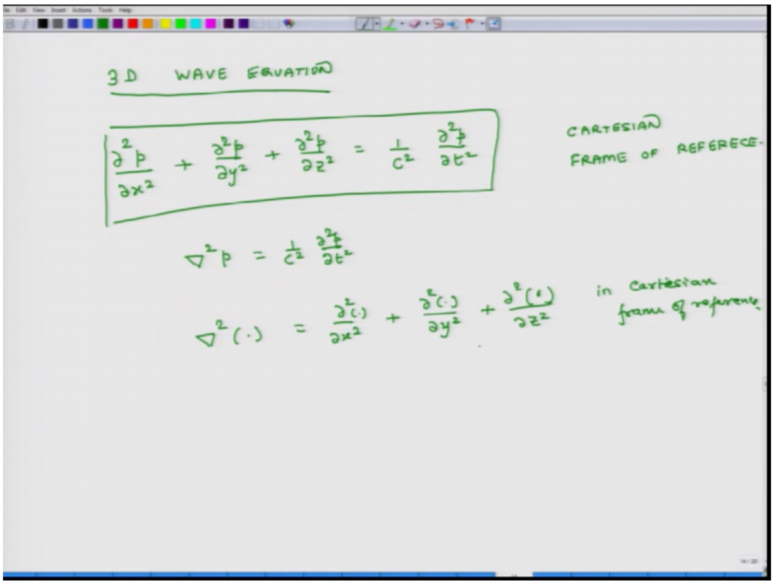
(Refer Slide Time: 12:11)



Now, if you measure the speed of sound in air experimentally you find that speed of sound in air at STP conditions is 344.8 meters per second. So, what is note is that this speed of sound is very close to this value of C. So, is it by accident or what? So, that is what we will later explode that and what we will find this that physically C indeed represents the speed of sound, but that understanding will develop once we actually try to solve this equation. And then we will figure out we will find that this coincidence between the speed of sound and the value theoretically computed value of C is not accidental, but before we do that I wanted to share a couple of other equations.

So, till so far; what we have done is that we have developed 1-D wave equation for pressure and 1-D wave equation for velocity, but in reality sound does not have to be travelling only in one dimension only in very special circumstances, it travels in one dimensions.

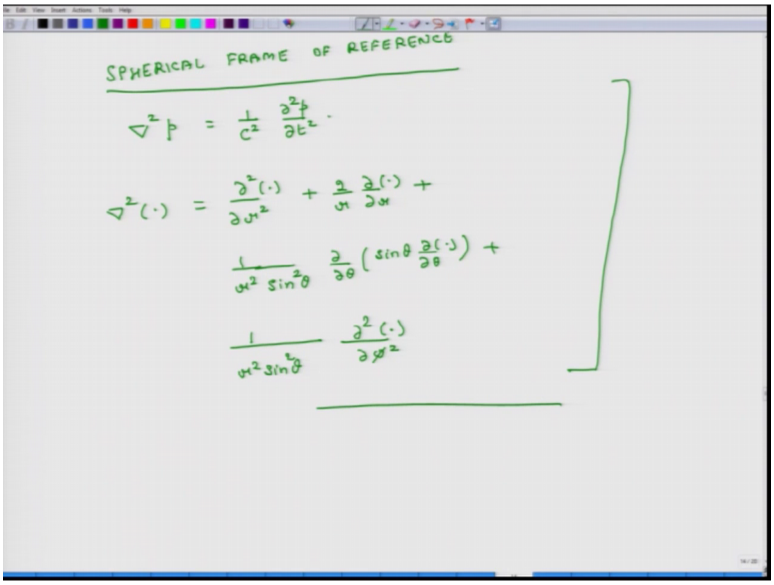
(Refer Slide Time: 13:51)



So, when sound is propagating in all the 3 directions then its propagation is governed by 3 d.

3 d wave equation and what is the nature of this wave equation it is del 2 p over del x square plus del 2 p over del y square plus del 2 p over del z square equals 1 over C square del 2 p over del t square. So, this is for a Cartesian frame of reference I can also express the same equation I can rewrite the same equation as grad square p equals 1 over C square del 2 p over del t square and the definition of this term operator grad square is. So, is this in Cartesian frame of reference Cartesian frame of reference this is the definition of this operator del square if we go if we use a different frame of reference.

(Refer Slide Time: 16:03)



Let us say spherical frame of reference we use the spherical frame of reference the equation in terms of this del square still remains the same. So, it is del square del square p equals 1 over C square del 2 p over del t square it does not change, but the definition of this operator that changes. So, in Cartesian fame the definition of the operator was this entire thing in spherical frame r coordinates are r theta and phi, right.

So, this is no they should not be a sin square here; there should be a sin square here and this is del square by del phi square. So, this is the spherical frame reference. So, this covers our wave equation. What we will do in the next classes we will actually try to solve this equation and then we will develop an interpretation for this parameter C and we will actually find out in our next class that C indeed actually represents the velocity of sound propagation.

So, with this we close for today and we will meet once again tomorrow.

Thanks.