Noise Management & Its Control
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Lecture - 16
The Continuity Equation and Gas Law

Hello again. Welcome to Noise Management and its Control. Today is the fourth day for the third week of this course and what we plan to do today is address two other equations which are the continuity equation as well as the gas law and once we have the 3 equations together, then we will again synthesize these and come up with the final form of our 1-D wave equation. So, the first equation we are going to deal with today is the continuity equation.

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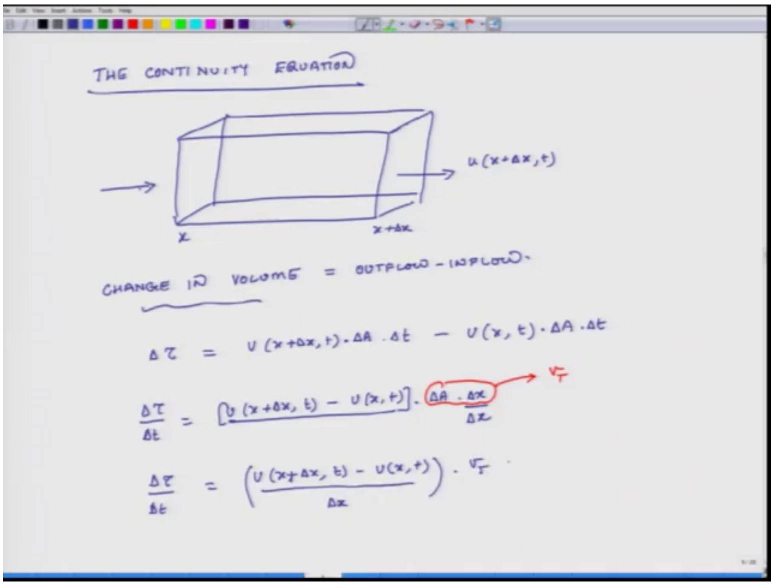

So, as we had discussed earlier the continuity equation essentially is a expression of conservation of mass. So, consider the case that you have a space into which flow is coming in and flow is going out to. So, this is a different things then what we look at earlier.

So, you have a physical region in which air or water or some fluid is moving in and at the other end fluid is moving out. Now if we have more fluid coming in and let us fluid going out this thing will expand and if the opposite happens then it will shrink. So, using that thought process, we can say that change in volume of fluid element volume is equal to outflow minus inflow, because if there is no change in volume, then what does that mean that mass is not being concerned its going somewhere. So, it has to be accounted for that is where why we are talking about mass conservation.

So, let us say that that delta this change in volume we call it delta tau and what is outflow. So, what is outflow I mean if I look at you know this 3 d volume element, then what is outflow? Outflow is; whatever is the velocity here. So, this is location x, this is location x plus delta x, then the velocity is u at location x plus delta x and time t. So, I multiply this U by A. So, that is the; so, it will be U x plus delta x times t times delta a and this is the change outflow happening in one second, it is happening in one second.

So, if the total time in which we are interested is delta t, then that is delta t, this is the outflow and the inflow is U x t times delta a times delta t. So, here you are not freezing time, we are measuring change in the volume over a period of time which is delta t because if delta t was 0, then there would be any change, right. So, we have to measure change in volume over a period of time. So, I can write this as delta tau over delta t equals U x plus delta x comma t minus U xt times delta a and I can multiply and divide by delta x. So, this delta a times delta x is what VT volume of the element. So, this equals delta tau over delta t, U x plus delta x comma t minus U xt delta x times Vt.

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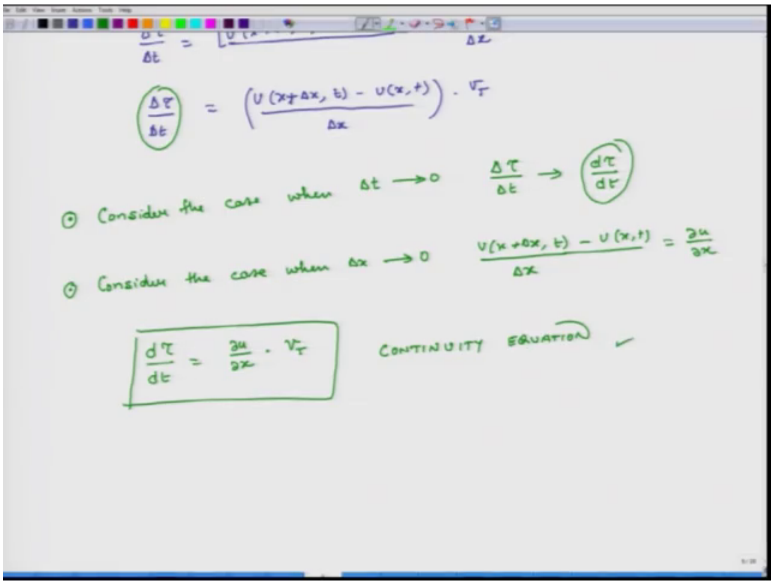

So, now consider the case when delta t is approaching 0, when delta time is time is becoming extremely small then what will happen delta tau over delta t approaches total derivative of change in volume divided by dt. So, this is the total change because this delta t is the total change, it is not just based on position it is the total change in volume and when I divide it by. So, very small amount of time then I get total derivative of tau with respect to t. So, if I am making these observations over very small period of time then this term becomes total derivative of tau with respect to t. So, this is one thing.

Second thing we do is we consider simultaneously at the same you know consider the case, when delta x is very small, then my on the left hand side I get U x plus delta x comma t minus delta x minus U x t and that is equal to partial derivative of u in space. So, my equation becomes d tau over dt equals del u over del x times V t. So, this is the continuity equation this is the continuity equation.

So, now what we have done is we have developed the momentum equation, this is the continuity equation which is based on conservation of mass and the third equation we are going to work on is the gas law. So, the next equation we are going to work on is the gas law. So, that relates to process equation.

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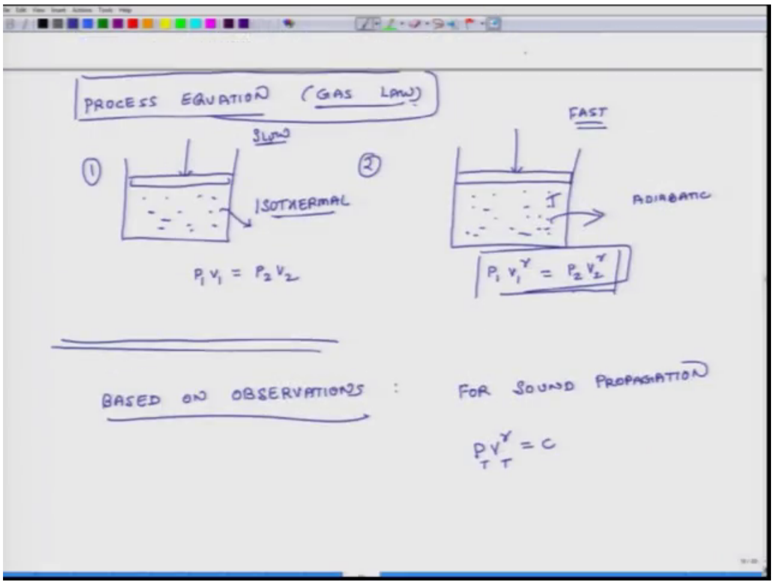

So, what we have done till. So, far is the Newton’s equation and we should remember that the Newton’s law is applicable in all cases it does not matter; what is the situation, it matters in all sorts of situation because it is a universal law matters; it is applicable in all sorts of situations.

The second equation which we developed was the continuity equation and once again its based on law of conservation of mass. So, it is also applicable in all sorts of situations all sorts of situations the third equation we are going to work on is the process equation which is the gas law and this may not be necessarily applicable in all situations.

For instance if I have a cylinder and I press here and I and there is gas here and if I press it extremely slowly, then as I keep on compressing the gas the pressure on the gas will go up as the pressure in the gas goes up its temperature when you press gas it pressure temperature goes up, but because the process of compression is extremely slow the temperature of the gas goes up which means that heat gets a chance to leak to the outside atmosphere, right because out inside the temperature has gone up outside the temperature is same.

So, heat flows from inside to outside and because the process is very slow whatever heat gets generated it gets released to the outside. So, the temperature remains the same on the in the gas. So, in this case the process is an isothermal process it depends on the nature of the process, but. So, this is one situation the second situation could be the same thing the system is the same, but it is very fast very fast. So, here it is extremely slow here, it is extremely fast and what is happening here.

As I am compressing the gas and I am doing that very fast the pressure rises significantly and very quickly and a lot of heat gets generated because the rate of because the change in pressure is very large and so, temperature here becomes high, but it does not there is not sufficient time for this heat extra heat which is generated in the air to leak out to the outside atmosphere even though inside temperature is now much higher than outside temperature.

So, here because the system was very slow it was an isothermal process and here because the system is very fast, it is an adiabatic process because heat is we are is unable to escape to the outside it does not have time, it has it takes time for things to move and heat actually travels very slowly. So, it does not have sufficient time to get out of the system and neutralize the temperature. So, that outside and inside temperature is same.

In the first case heat has a lot of time because I am pressing it very slowly. So, heat has a lot of time to leak to the outside world and inside and outside temperature does not change. So, the system could be the same, but based on the nature of process the gas law would be different. So, here in first case what will be the gas law it will be P 1 V 1 equals P 2 V 2 and in the second case the process will be P 1 V 1 to the power of the gamma is equal to P 2 V 2 to the power of gamma and then there are others processes also there could be; if there could be another process which could be poly tropic in nature or could be isobaric in nature and so on and so forth.

So, now the question is that when sound travels when there is sound propagation happening in air; what did we discussed earlier that essentially there are pressure fluctuations and pressure fluctuations move from one point to other point to third point and so and so forth. So, as the sound is spreading these pressure fluctuations are moving from one point to other point to the third point and now we have to figure out which process should we choose which process should we choose.

If we choose a wrong process, then finally we will end up getting wrong results. So, we have to make the right choice you have to make the right choice it turns out that if you use this adiabatic process P 1 V 1 to the power of gamma then it gives us good results. So, based on observations based on observations and experiments we are going to make a choice. So, these are not my observation based on observation of lot of people in earlier times we are going to make a choice that we are going to say that when pressure and volume are is changing due to propagation of sound those changes in volume and pressure are governed by the adiabatic equation.

So, we can say that for sound propagation we will say that PV to the power of gamma equals constant and it is what P it is total pressure and total volume; so, to give you a historical backdrop when this topic was being discussed initially. So, Newton was one of the first guys who address this problem and he made a wrong choice he assumed that when sound propagates in air he thought that the process is isothermal.

So, he considered PV PT VT as constant and later when a lot of experiments were done it was found that this choice was wrong. So, once that realization was made then there was another person by the name Rayleigh he said- no, the process is not isothermal, but rather adiabatic in nature and once that assumption was made. And then lot of experiments were conducted it was found that the prediction of sound propagation velocity was theoretical prediction was very close to experimentally measured value.

So, based on all that experimental data we are going to make this choice PT VT to the power of gamma equals constant.

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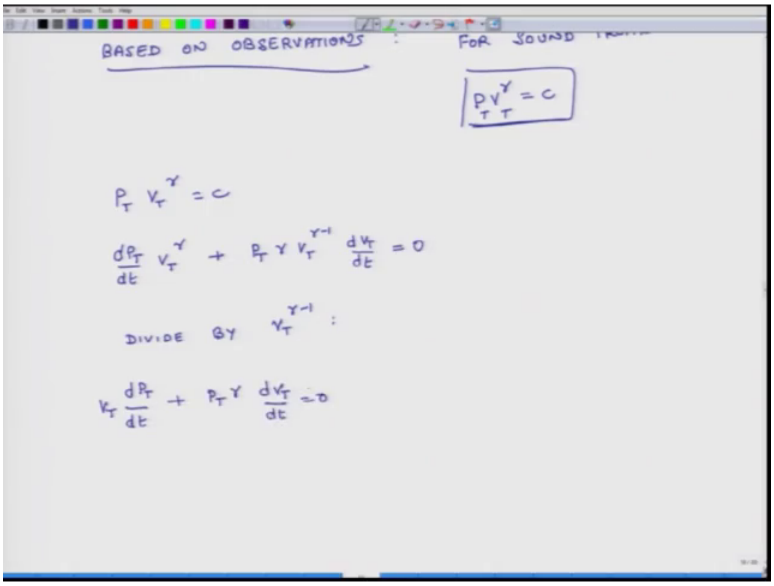

So, that is how we are going to start. So, what we are going to do is rewrite this and now I am going to differentiate it. So, I am going to take the total derivative. So, d PT over d t V T to the power of gamma plus PT gamma VT to the power of gamma minus 1 times d VT over dt equals 0.

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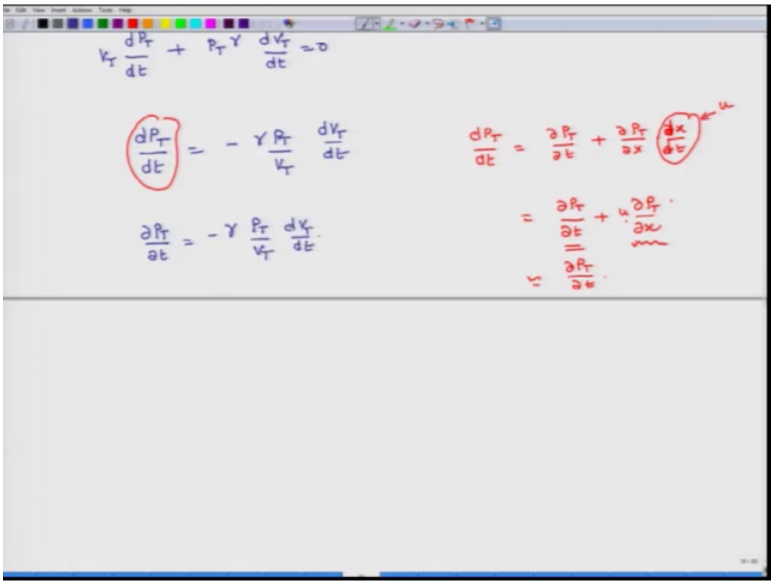

So, I can rearrange this and I can write it as d PT over dt equals minus gamma PT over VT times dv t over dt.

Now this guy is the total derivative of pressure in time. So, I can write it as d PT over d t is equal to partial of this pressure in time plus del PT over del x times del x over over dx over dt and what is this u right. So, this is equal to; so, this relation now we know. So, once again this term is very small compared to this term for the same reason this is dependent on u as a small entity and pt. So, this term is negligible. So, this I can approximate it as partial of pressure with respect to time.

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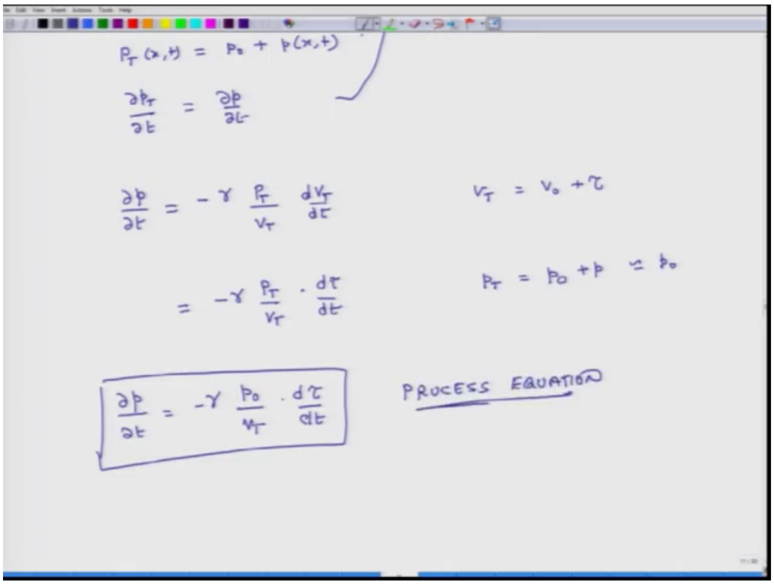

So, I plug this back in and I get partial of pressure with respect to time equals minus gamma PT over VT d VT over dt. So, the other thing we have to remember is PT changes with x and t and this equals P naught plus P of x and t. So, del PT over del t is equal to del P over del t. So, I put this back in my earlier equation and I get partial of incremental pressure with respect to time equals minus gamma PT over VT d VT over dt and we also note that VT is equal to V naught plus tau. So, this I can rewrite it as minus gamma P t over VT times d tau over dt and I make the third assumption that PT is equal to P naught plus P and this is P approximately equal to P naught.

Finally, I am left with is del P over delta t is equal to minus gamma P naught over VT times d tau over dt. So, this is my process equation this is my process equation and this is based on the adiabatic gas law because and we chose adiabatic gas law because we have a lot of experimental data which shows that if we use adiabatic assumption then our experimental results and our theoretical results they agree extremely well.

If we use some process law, then they do not agree. So, that is the basis on which we have used this and then as we were developing this equation we omitted the lawn non-linear term u times del PT over del x because we assumed that it is very small and that assumption is based on our original assumption that all changes in pressure velocity and everything they are extremely small. So, these are the 3 equations the continuity equation the process equation and the momentum equation.

What we will do in our next class is now, we will rewrite these equations and then synthesize them and we will combine them. And finally, we will come up with the final single one dimensional gas wave propagation equation which we will start solving and learn how sound propagates in air.

With that we close our discussion for today, and we will meet once again tomorrow.

Thank you.