Noise Management & Its Control
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Lecture - 15
The momentum equation

Hello, welcome to noise management and its control. Today is the third lecture or the third day of the third week of this course. And what we plan to do today is a continuation of the discussion which we were having yesterday and specifically what we will do is we will develop the momentum equation in today's class.

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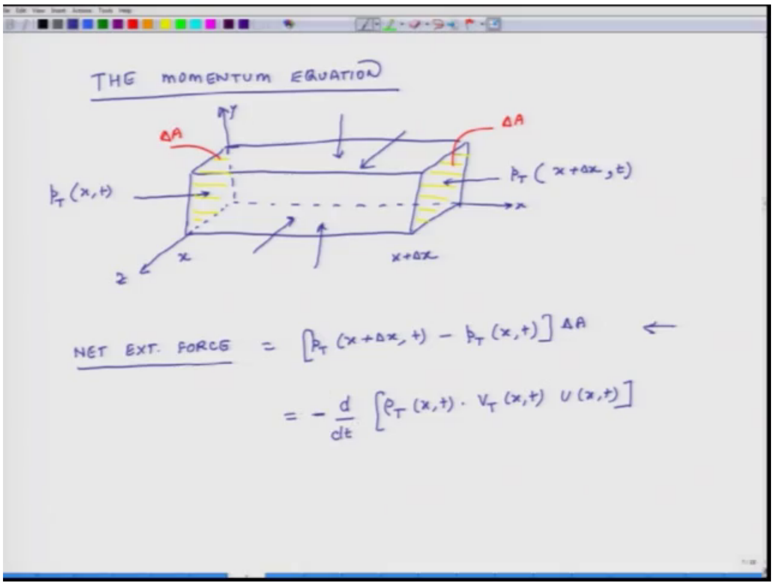

So, our focus today is going to be the momentum equation. So, essentially what this equation is going in what we are going to do in this equation is that we will look at the whole fluid volume in which sound is traveling, and we will take a small portion of that fluid volume and we will develop a free body diagram of that fluid volume. So, we will identify all the forces which are acting on that small piece of fluid. And whatever is the extra force that will be equated to the rate of change of momentum of that mass and that is basically same as the Newton's second law.

So, let us make a small piece of fluid element. So, let us say the coordinate of the first position, first starting point of this fluid element is x, and this fluid element is delta x long. So, this is x plus delta x, there is pressure acting on this phase. And what is the pressure we are interested in finding out the total pressure, so it is P T, and p T is a function of position and time. So, what we are looking at is we are taking a picture of this fluid element at time t equals t, at time t equals t, we are taking a snapshot at time t; and at exactly at that time we are seeing we are going to measure all the forces which are acting on this body. So, the pressure is acting on in this direction.

Similarly, in this direction there will be another pressure it will also be P T, but the coordinate of at this location is not x, but it is x plus delta x. And the time is still the same because we are taking a snapshot at time t equals t. There are also forces on this surface on the top surface and on the bottom surface, they are also pressures, but the change of pressure in the, so this is my x direction, this is my y direction, this is my z direction, the change of pressure in the y direction is 0. So, the total net force acting in y direction is 0. Similarly, there are forces acting on this phase and this phase, but again because the flow is one-dimensional, the change of change in forces on these two opposite surfaces in the z direction and y direction, they are 0. So, we are not really worried about how forces and pressures are changing in y and z direction, because they balance out each other. So, there is no motion in the y direction and z directions. So, the only changes which exist in the system are in the x direction, at x is equal to x the pressure is p T at location x and at time t; and at x is equal to x plus delta x it is a slightly different pressure.

The other thing we say is let us say that this area - this cross sectional area is delta A delta A. So, net external force on this fluid element is equal to what p T x plus delta x comma t minus p T x, t and times delta A that is the next external force. And what is the direction in which these forces acting it is acting in this direction, because pressure acts inwards, pressure tries to compress something. So, this is the next external force, this should be equal to, so what does Newton's second law says that whatever is the net external force on a small fluid element that is equal to rate of change of momentum of the system.

So, what is momentum of the system, it is the mass of this fluid element. So, mass is what, density at x, t times volume, so this is the mass of the system. And if I multiply this by velocity, what did I call. So, it is velocity is u. So, this is momentum of the system and it is equal to the rate of change of momentum, so d over d t and because the pressure acts inwards I have a negative sign here. Now, remember this is a mass particle and we said that the mass particle is constant. So, assume that think about it is a small balloon, neither the mass is leaving nor the mass is coming in.

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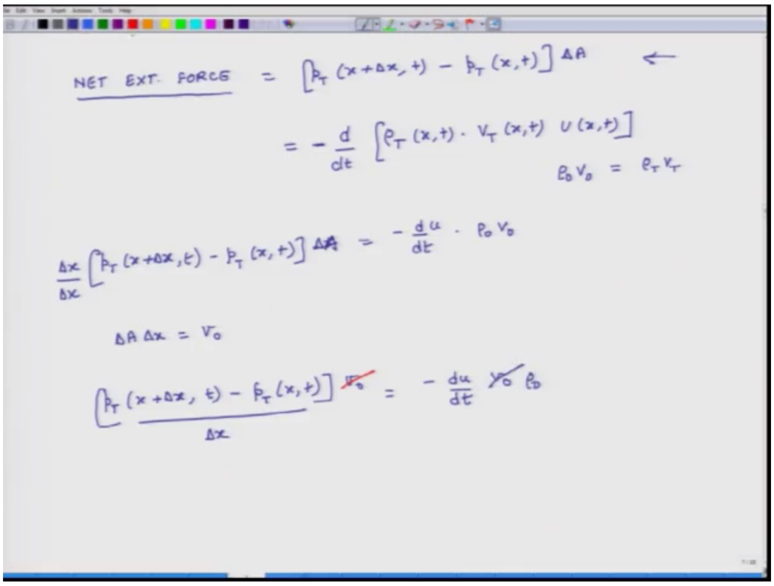

So, the left side of the system is p T x plus delta x minus p T x t times delta A that is the force is equal to minus d over dt and because it is a constant mass particle I can say that rho naught v naught is same as rho t v t. Because the mass of the particle is not changing it is just becoming fatter or thinner, no mass is exiting or leaving the thing. So, this is there. So, all I have to do is du times rho naught v naught. The next thing what I do is I multiply this and divided by delta x. So, we know that delta A times delta x is equal to v naught. So, this is this length of the fluid element is delta x cross sectional area is delta A. So, the overall volume is v naught. So, I can rewrite this as p T x plus delta x times t comma t minus p T x t v naught divided by delta x is equal to minus d u over dt times v naught. And there has to be a rho naught also I am sorry. So, v naught gets cancelled from both sides. So, what I am left with this goes away this goes away and I am left with rho naught.

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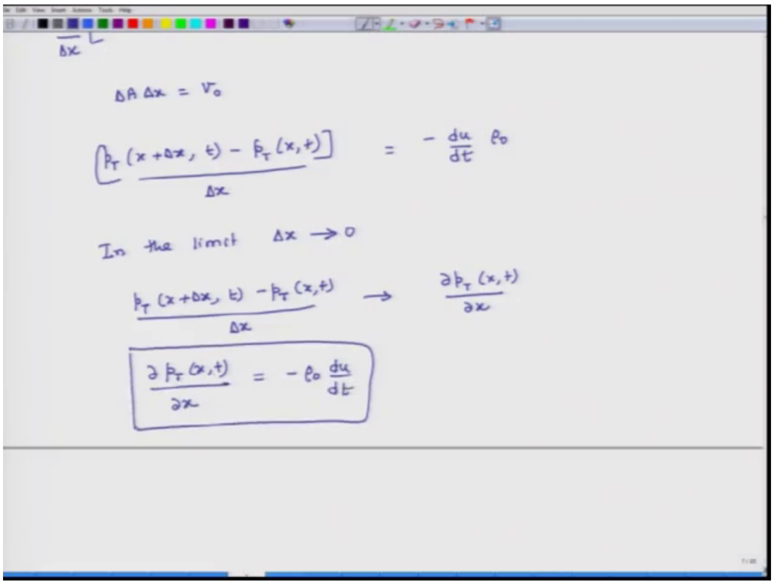

So, now we move one step further. If delta x approaches 0 in the limit, so it will not in the limit, delta x approaching 0 p T x plus delta x comma t minus p T x, t divided by delta x. It approaches what; it is a total derivative or partial derivative. It will be the partial derivative because when we have a partial derivative only x because only x is changing time is not changing in total derivative both x and time would change. So, here only x is changing. So, it is a partial derivative. So, I can write it as minus not partial derivative of p T with respect to x is equal to minus rho naught du over dt, and we will simplify this a little bit further.

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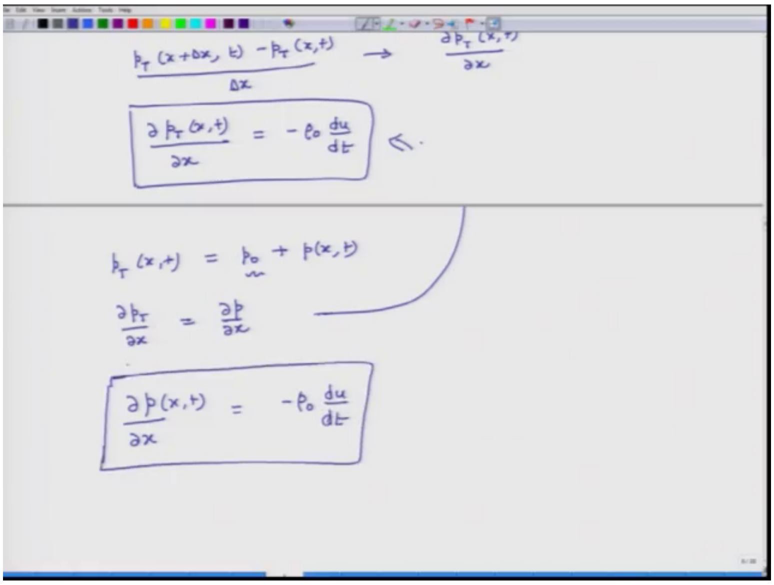

So, we know that p T which is the function of x and t is equal to what p naught plus p of x and t. So, I can say that partial of pt with respect to x is same as partial of p with respect to x because p naught is a constant. So, if I plug this back into this guy, what I get is partial of t and t is a function of x and time is equal to minus rho naught du over dt.

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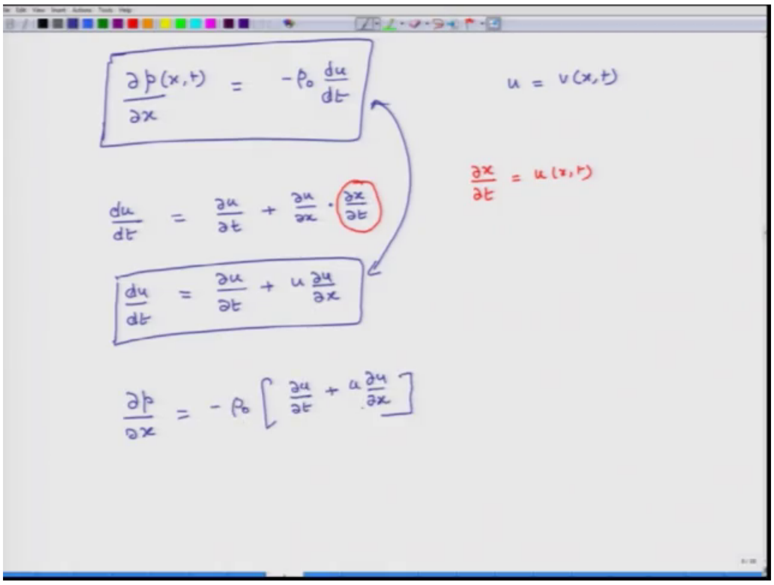

Next thing we know d u over dt. So, we know that du depends on what x and time. So, this is the total derivative of u and that I can rewrite it as del u over del t plus del u over del x times del x over del t. Now, think about this term del x over del t. What does x represent, x represents the position of this fluid particle in Cartesian space in the Cartesian frames work system, it is the location of this fluid particle which x represents. So, if x changes then the location of fluid particle is changing. So, it is the position. So, what that means is that del x over del t is what is actually the velocity of the fluid particle. So, with that understanding I can write it as del u over del t plus u del u del over del x.

So, what I do is I combine these two guys and I get del p over del x equals minus rho naught del u over del t plus u del u over del x. And now we will make one final simplification. So, what we had said was that we had said that these velocities, pressures changes, so these are all small change small numbers p u, rho naught may not be small, but u is a small entity p is a small entity and so on and so forth. And also we had said that, so u represents what velocity of the particle right, v represents velocity of the particle. So, this is a small number and this is also its partial derivative is also a small number.

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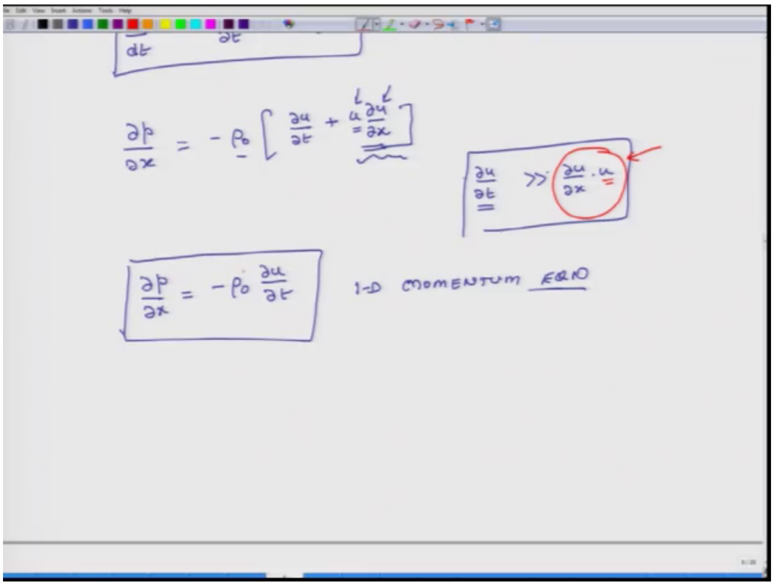

So, what that means, is that del u over del t is very large compared to del u over del x times u because these are products of two small numbers and this is a small number. So, if that is the case then I can omit this term and I can write del p over del x is equal to minus rho naught del u over del t. So, this is my final form for 1-D momentum equation. And in this 1-D momentum equation, we have neglected this term, because we are saying that the whole system is having very small fluctuations in pressure density and velocity. If u was large, if u was large for instance when there are explosions and things like that then u could be very large; in that case we cannot omit this term and then the system becomes non-linear because I have u multiplied by its partial derivative in x, so it becomes a non-linear system. But here the 1-D momentum equation is a linear equation it depends on the first power of p it also depends on first power of u, so it is a linear equation right. So, that is important to understand.

So, this is one equation which we have developed tomorrow we will develop another equation which is the continuity equation. So, with that we close our discussion, and we will meet once again tomorrow, and we will discuss the continuity equation as well maybe even the gas law tomorrow.

Thank you.