# Noise Management & its Control

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## Lecture - 14
## Introduction to one dimensional wave equation

Hello. Welcome to Noise Management and its Control. This is the second day of the third week of this course. And what we plan to do today as well as the remaining part of this week is development of 1-D wave equation.

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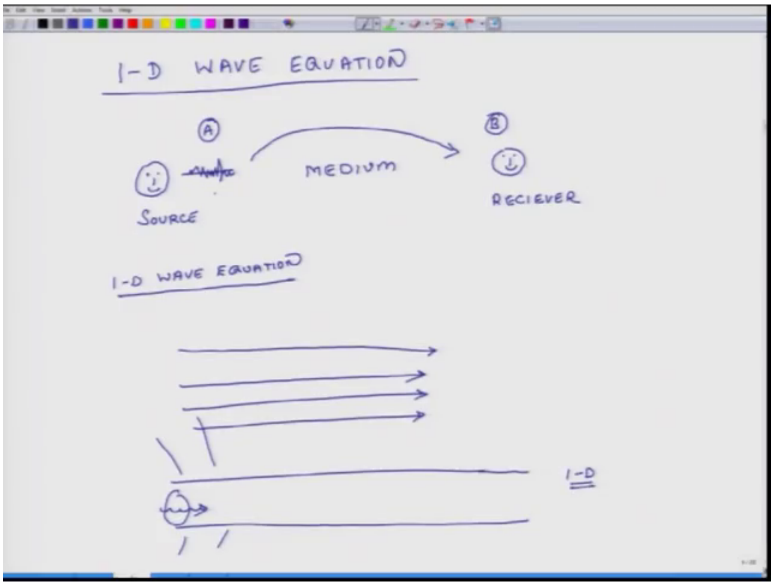


So, that is what we planned to do today. And specifically today, we will actually introduce the concept of 1-D wave equation 1-D wave equation and to give you some idea what is that we are talking about is. So, this is what we plan to do. So, suppose there is a source and there is a receiver and when the source generates sound and here we have the medium. So, when the source generate sound in this case, it could be a person when he or she talks there are pressure fluctuations pressure fluctuations and these pressure fluctuations somehow get transmitted through the medium and these pressure fluctuations are received after sometime by the receiver. So, what we are interested in figuring out is what.

How does sound or these pressure fluctuations get transmitted from point a; which is the source to point b which is the receiver, this is what we are interested in figuring out and this we can do when we talk about the 1-D wave equation. So, why is it one dimensional because here all we are interested in finding out is that if sound is travelling only in one direction now in reality this may not always be true for instance if there is a bird which chirping in air then the sound from the bird it gets propagated in all the directions. So, that may not be a one dimensional wave propagation.

But there could be cases where sound propagates only in a particular single direction for instance if I have a pipe long pipe and if by put my mouth here and here I generate some sound then the sound will just travel through the length of the pipe, it will not go outside the pipe and it will not you know propagate in other directions it will just propagate in along the length of the pipe. So, this kind of situations where sound propagates only in one dimension only in one dimension and this is a Cartesian frame of reference. So, it is travelling; let us say only in the x direction, then how do we understand the nature of propagation of sound when sound is just traveling in one single direction. So, what that is what we plan to understand for starters because if we get a feel of that.

Then we can extend that understanding to two and three dimensions also. So, that is what we plan to do in today's lecture as well as the remaining lectures.

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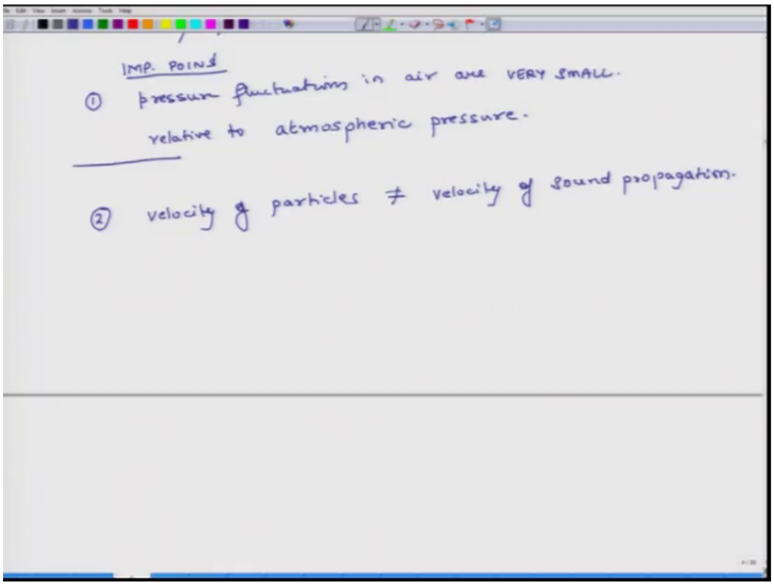


So, as we are doing this we have to remember some important things. So, one thing is. So, we will assume something assumptions. So, first thing what we will assume is that when we speak or sound is being produced in air that we have discussed earlier that when we say that sound is being produced essentially; what is being what is happening is that there are very small pressure fluctuations in the air. So, we assume that pressure fluctuations in air are very small; very small compared to what related to atmospheric pressure the second thing; we this is not an assumption.

So, this is this is an important assumption the second thing is I will not call this as a list of assumption the first one is an assumption, but. So, I will just call it important points the second thing we have to remember is when sound is propagating we had seen in the first lecture itself, it propagates because particles vibrate and they transmit energy to adjacent particles. And in this way the energy gets transmitted from point a to point b, right. So, they have to be particles for sound to propagate and the velocity of particles you know velocity of particles is not the same as velocity of sound propagation this is important to understand.

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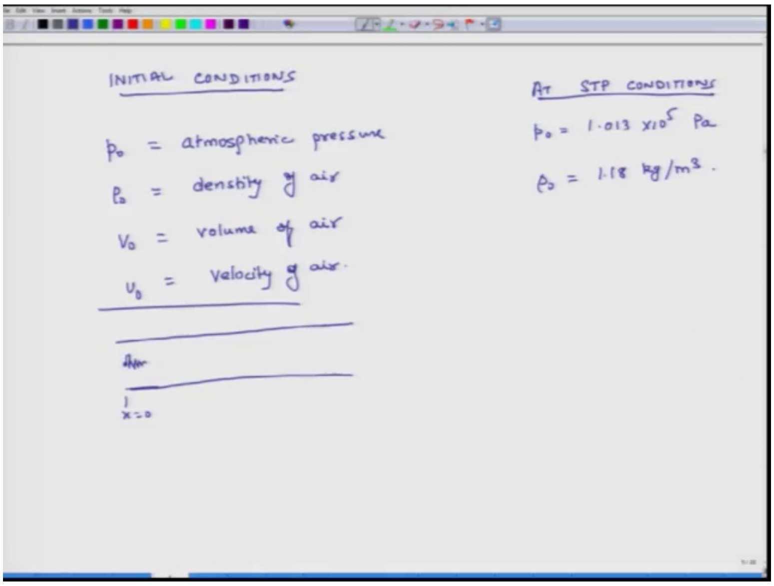


So, with this these two important points we will get back and we will start formulating the 1-D wave equation. So, suppose there is a medium and the initial conditions are what are the initial conditions when there is no sound in this medium then this is my pressure P naught is atmospheric pressure rho naught is the density of air V naught is volume of air and u naught is velocity of air.

So, when there is no sound it could be that there is a bunch of air which is just blowing fast at a constant speed its moving, and the pressure of that air is P naught rho naught is density of air V naught is volume of that sample of air and at standard temperature pressure conditions. We will note that P naught is 1.013 into 10 to the power of 5 Pascals and rho naught which is the density is 1.18 kilograms per cubic meters. So, this is the initial condition. And now consider the fact that in such an air there could be a small sources which generates sound.

So, when it is generating sound let us consider the source as the origin that is the coordinate of that location is x is equal to 0. So, so it would be say duct initially in the duck there is no sound and this is x is equal to 0 and then the source it generates some sound at x is equal to 0. So, when this small disturbance is created the pressure in the entire tube after sometime it will be different. So, we will write down the equations for that pt is the pressure in air at position x and at time t is equal to P naught plus small disturbance which is lowercase p.

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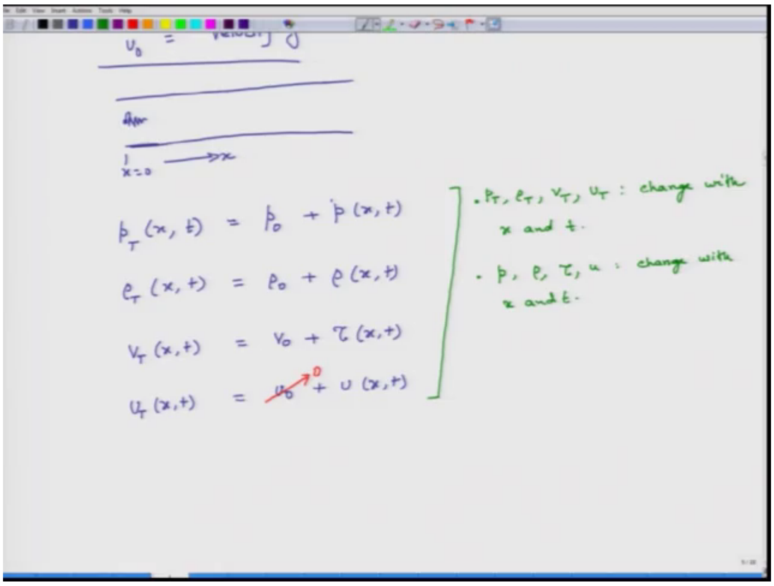


So, this I will rewrite it as V naught. So, at the total pressure we just distinguish between total and time, I will can make it capital T. So, total pressure at any point. So, if this is my x coordinate at any point in air is P naught plus small p x t where p small p is the disturbance caused due to this initial sound which has been generated. Similarly total density and this density will vary from point to point and time to time will be the initial density which was constant in the hole tube.

Initially plus some disturbance in the densities which I called rho and that disturbance will vary with x and t similarly volume of the air is V T and this is V naught plus now here I will use a slightly different terminal I will call it tau tau x t and velocity U T; it is t equals u naught plus u x t. Now we can assume that initially the air was not having any velocity it was just sitting. So, if that is the case then this thing goes to 0.

So, u t is same as lowercase u x t. So, we will make some comments at this p t rho t V t u t they changed with x and time. So, their functions of x and time similarly p rho tau and u they also change with x and time, but P naught rho naught V naught they are constant they do not change the x and time, because those are the initial conditions for the system.

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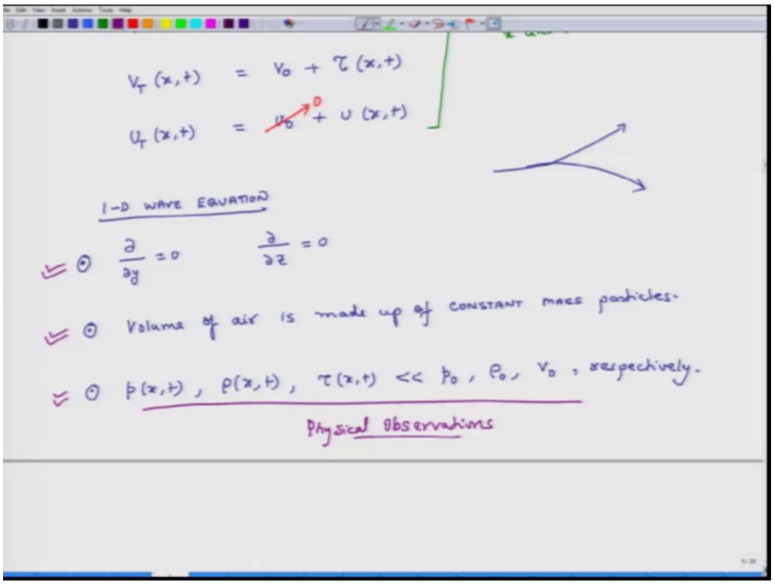


Now, remember that the equation which we are developing is 1-D wave equation which means that there is no velocity or pressure change in y direction and z direction right nothing is changing in y direction and the z directions. So, because it is a one die one dimensional wave equation later when we start doing mathematics we should recognize that derivative partial derivative of any entity p rho V u and so on and so forth in the y direction is 0 and partial derivative of any entity in the z direction is also 0.

So, this is one assumption we are making because its one dimensional wave equation the second assumption we are going to make is that volume of air now this is the important assumption is made up of constant mass particles constant mass particles. So, what does that mean that they were in very large number of particles in the system and particles maybe moving from one point to other second point to third point and so on and so forth, but as they are moving from one point to other their mass us not changing.

So, as the mass is not changing what is happening either they are expanding or contracting, but the mass is remaining same and also there is no splitting of air if it mass not changing, then it is not that our flow is doing this because if it does this then some particles will have to split. So, it is one dimensional no branching of flow. So, mass is not changing. So, the volume can change volume of each particle could change, but its mass is vary not changing right thing about like a small balloon very small balloon the air contained in it has fixed mass, but if the pressure goes in goes up then the balloon becomes larger and the pressure becomes smaller pressure balloon becomes smaller, but the mass of all these little balloons in the system it is not changing.

The energy of each of these little mass particles that could also change balloon could become hotter when it gets compressed the pressure increases and it becomes hotter when it expands it becomes cooler. But once again the mass of each of these little particles it is not changing, this is an important assumption and the third assumption is p rho and tau are very small compared to P naught rho naught and V naught respectively.

So, the first assumption what is the basis of the first assumption because we are assuming that the system is one dimensional what is the basis of second assumption I mean; this is the mathematical format we are using to analyze the problem and the third assumption what is the basis this is based on physical observations because most of the times when sound is generated. And when you see, and you measure the amount of fluctuations in pressure density and velocity you find that these fluctuations are extremely small compared to the pressure density and volume of the air when everything was at rest. So, changes in pressure density and volume they are extremely small and this is based on actual measurements and physical observation. So, that is the basis of our equation. So, now, that we have set up the problem.

What we will do is we will develop mathematical relationships between P rho V and U mathematical relationships and then based on those relationships will try to understand how is pressure changing from point a to point b and how do we; what kind of mathematical relationships do we use.

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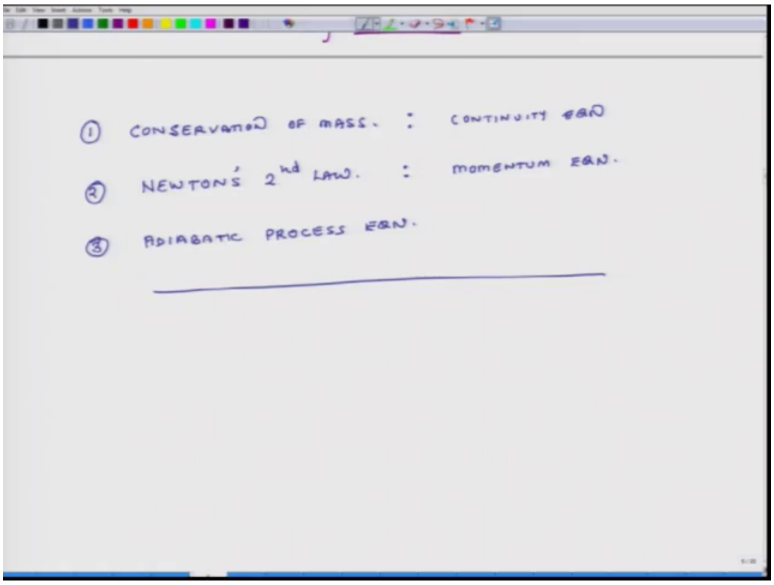


So, we use broadly three mathematical relations to connect p rho and u to connect p rho and u what are those three mathematical relations. So, the first relation is conservation of mass. So, we will write any equation that as sound is propagating or as air is changing its volume and things like that the mass has to remain conserved right the mass cannot change. So, we will develop an equation from there the second equation. We will use to connect all these variable is Newton’s second law.

So, I will go back to the first equation which we will develop using conservation of mass we will call that equation the continuity equation the second equation which we will use based on Newton’s second law which says force equals mass times acceleration basically that is the equation, we will use that will give us an equation which is known as momentum equation and then the third equation which we will use is the process equation and we know that when air gets compressed and it expands it can be in also it can exhibit all types of behavior that behavior could be adiabatic in nature it could be isothermal it could be isomeric and so on and so forth.

But again, based on physical observations we have found that when sound propagates in air the fluctuations in pressure in air are extremely fast. So, that process can be approximated as an adiabatic process. So, the third equation which we will use is adiabatic process equation. So, this is also known as; now I will just call it process equation.

So, this is how we are going to develop the one dimensional wave equation. We will connect this parameters p rho V and u using conservation of mass Newton's second law and adiabatic process. And as a consequence we will arrive at the final one dimensional wave equation.

So, with that we conclude our discussion and we will meet once again tomorrow. And we will continue this discussion on one dimensional wave equation tomorrow as well.

Thank you.