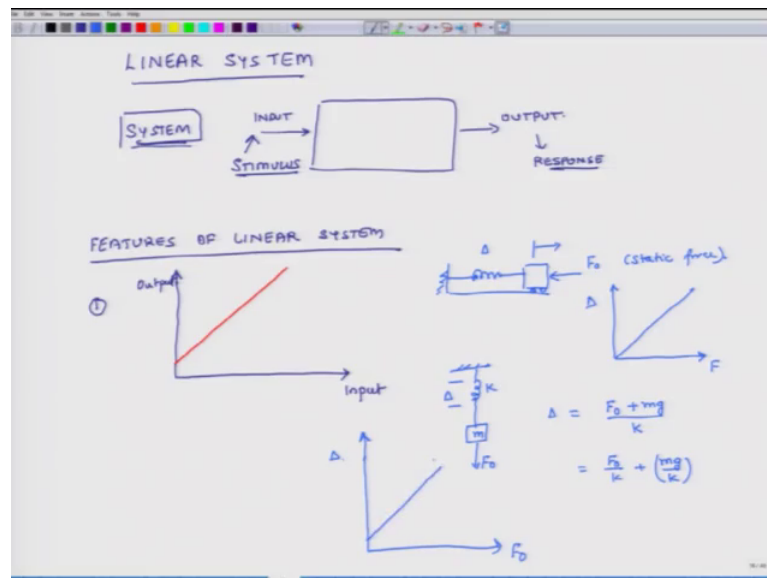


Noise Management & Its Control
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Lecture – 12
Linear Systems

Hello. Today is the last day of this week and what we plan to do today is introduce a new topic it is a very important concept and it is about linear systems. Because whatever we will be discussing in this course will be about linear systems and it is important to understand from a physical standpoint as to what is a linear system.

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So, we will discuss about linear systems. So, any system for example, a television, we will just represent the system as a box it has some input and it has some output. It can have multiple inputs it can have multiple outputs also. But for purposes of simplicity we will just consider a single input and a single output. So, a television is a system, a human body could be a system right.

So, what could be an input to a human body? I press or pinch, this could be an input I am exciting and as a consequence my muscles move and things respond in all sorts of ways, so that could be an output. So, input relates to the stimulus how am I exciting or stimulating the system that is what input is all about and output is about response how is the system responding or behaving in presence of that stimulus in presence of that

stimulus. So, this is any system this is not a linear system or anything like that it could be any system it could be a linear system or it could be a non-linear system.

Now, for linear systems you will not go into the mathematical definition of linear systems, but from a practical standpoint we will identify some of the important features of linear systems. So, features of linear systems features of leader system. First feature is that if I make a graph of input versus output the line has to be a straight one. The line need not go through the origin, but it has to be a straight line.

Example, so we will see an example consider a spring and the mass and I am applying a static force F_{naught} this is static force. So, in this case the stimulus is the force and what is the response of the system? Response of the system is displacement in the spring. So, I will call it delta deflection in the spring. So, if I plot in this case stimulus versus response it will be a straight line. So, it is a linear system it is a system which is linear between delta and F_{naught} maybe on other parameters it may not be, but between delta and F_{naught} it is a linear system if I excite it using F_{naught} the response curve is a straight line.

Another example would be the same thing, but there is a mass, there is a stiffness and I am also applying a force. So, in this case the deflection in the spring delta will be what it will be $F_{\text{naught}} + mg$ divided by K . So, this will be equal to $F_{\text{naught}} / K + mg / K$ now if I plot the response versus stimulus F_{naught} is the stimulus. So, if I keep on increasing the force deflection starts keeps on growing, but at zero force also there is some deflection right. So, this curve will not go through the origin, but even then it is a linear system. So, this is the first thing that if I plot the relationship between input and output it is a straight line.

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The image shows a whiteboard with handwritten notes. At the top, there is a free-body diagram of a mass m with a downward arrow labeled F_0 and a displacement Δ indicated. To the right, the following equations are written:

$$\Delta = \frac{F_0 + mg}{k}$$
$$= \frac{F_0}{k} + \left(\frac{mg}{k}\right)$$

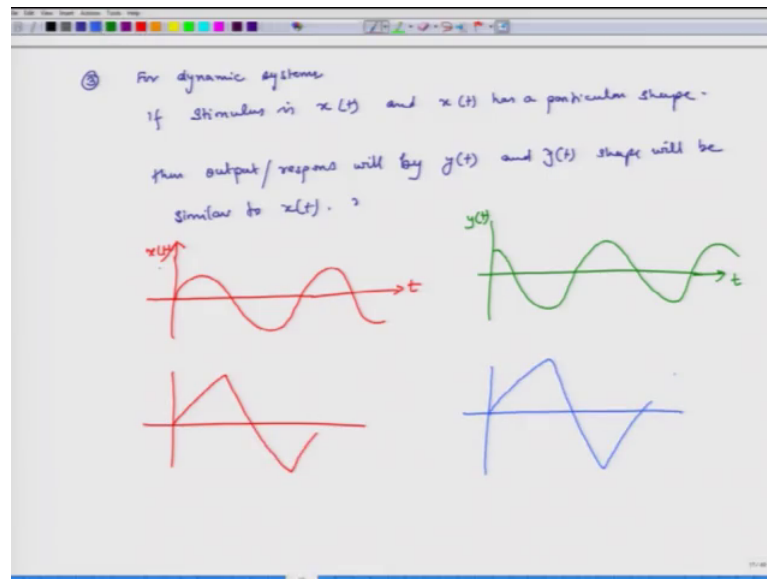
Below the equations is a graph with displacement Δ on the vertical axis and force F_0 on the horizontal axis. A straight line starts from the origin and goes up and to the right. Below the graph, the text reads: "③ If the stimulus is small, then response will be small and as well." followed by "Ex." and two diagrams of a ball on a surface. The first diagram shows a ball at the bottom of a concave-up curve, representing a stable equilibrium. The second diagram shows a ball at the top of a concave-down curve, representing an unstable equilibrium.

A direct consequence of that thing of a linear system is that if the stimulus is small then response will be small and will be small as well. What does that mean? That if I disturb the system by a very small amount the system will respond also by changes which will be extremely small in magnitude. I mean they may not be the same you know there may be some amplification, but the change in the response will be also extremely small it is not that if I disturb it by an extremely small amount there will be extremely large response responses those systems are not necessarily linear they are not linear systems for instance an example. So, example if I disturb this ball by a small amount I if I apply a very little force on this ball what will happen the ball will shift by a small amount right, it will be a linear system.

But if I disturb the ball in this case by a small amount what will happen to the ball even a very very little displacement it will cause very large you know changes if I apply extremely small force it will be sufficient to for it to fall down which is an extremely large deflection. So, this can be thought as a linear system, but not necessarily this one.

Third, these two observation one and observation two they are good for static systems as well as dynamic systems static systems as well as dynamic systems.

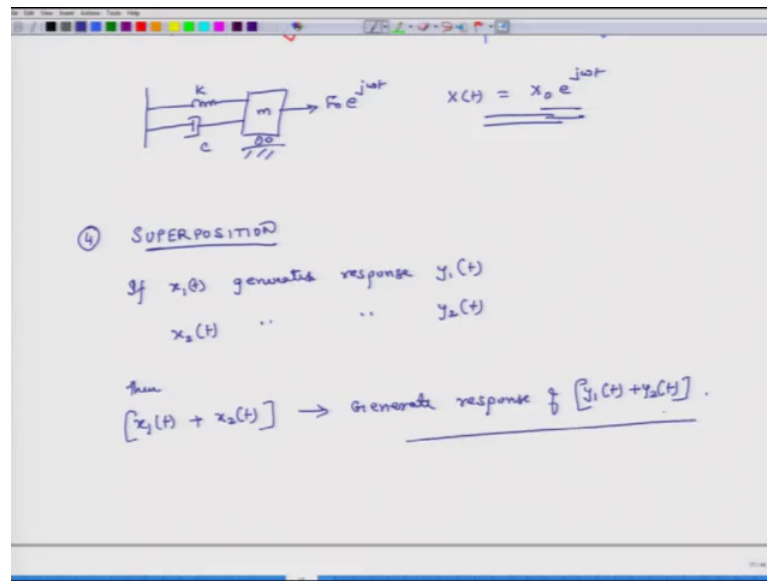
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Now, I am going to tell you a special feature of dynamic systems, for dynamic systems if stimulus is $x(t)$ right. So, I can excite it let us say by a sinusoidal function. So, if $x(t)$ has a particular shape then output or response will be $y(t)$ and $y(t)$ shape will and shape I am using the term loosely, but shape will be similar to $x(t)$ will be similar to $x(t)$ what does do I mean by this. So, the question point is that if my response, if my stimulus is like this, so this is time and this is $x(t)$.

Then this is time and my response it may be sinusoidal, maybe with a different phase but it will be like this. So, if this is a sinusoidal thing this will also be sinusoidal amplitudes may be different phase may be different, but frequency will be the same, shape will be the same. If response is if displacement you know stimulus is this then it has to be something similar. So, it is for this reason that a lot of times when we solve this type of a problem and we say that it is excited by a force which is the stimulus $e^{j\omega t}$ and if we are interested in finding out displacement then we say that $x(t)$ is what? Is $x(t)$ not $e^{j\omega t}$ because it has a similar behavior. This thing will not hold true if the system was non-linear, it will hold true only if the system is going to be linear in x and y ; x and F . So, this is the third thing.

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And the last one, the last feature of the system is superposition. What do I mean by that? That if $x_1(t)$ generates response $y_1(t)$ and $x_2(t)$ generates response $y_2(t)$ then $x_1(t)$ plus $x_2(t)$ together they will generate $y_1(t)$ plus $y_2(t)$. So, this is also a very important feature of a linear system. Again, this four thing also does not work if the system is non-linear, it will work only for linear systems. So, what does that mean? If a string gets stressed by 2 centimeters when I apply 5 kg force and the same spring gets stressed by 5 centimeters when I apply how much.

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Then when I add up both those forces the total displacement will also be I can add up and I do not have to separately calculate the displacement.

So, this concludes our discussion for today. I wanted to discuss another part today on transfer functions, but I think I will leave it for the next week and I hope what I have covered is something you have grasped, if you have any questions or problems please let us know. Please shoot your questions to the tutors and with that I wish you the best of weekend and we will meet once again on Monday.

Thank you very much bye.