Noise Management & Its Control

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Lecture – 11
Complex Time Function

Hello, welcome to Noise Management and its Control. Today is the 5th day of the second week of this course and what we plan to do today is introduced a new concept known as complex time function.

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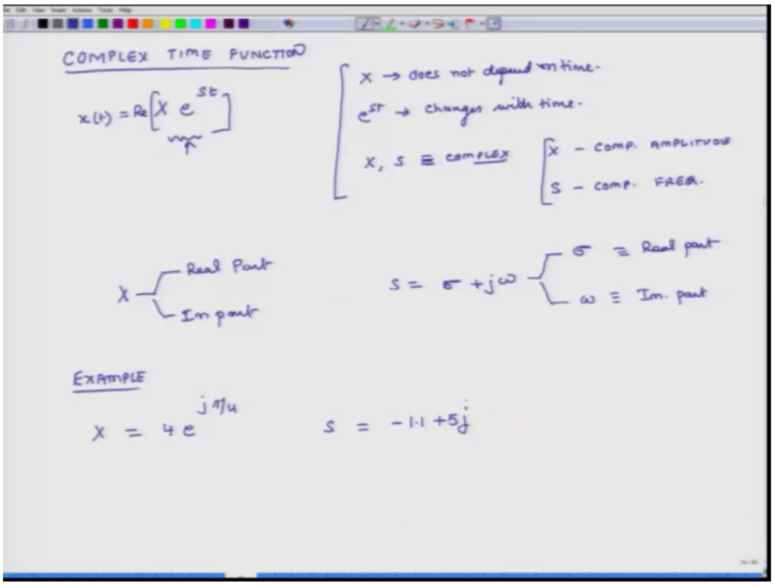

So, yesterday we learnt about complex numbers and today we will use that knowledge to understand something known as complex time function.

So, what is a complex time function? It is a function which is complex in characters. So, it is has a real part and an imaginary part and it changes with time changes with time. So, this is how we define it. So, it will have an amplitude capital X and it will have a time dependent part e to the power of st. So, X does not depend on time, but it can be complex and e to the power of st changes with time. Now X and S are complex X and S are complex, X complex amplitude and S is also complex it is called complex frequency is called complex frequency. So, X is complex amplitude and S is complex frequency. So, X will have a real and an imaginary part and same thing is true of X also.

Now, the X could represent anything it could represent pressure it could represent displacement velocity acceleration whatever so, but it will be a complex entity. So, whatever is the physically real thing is I will define it as x t is equal to real of complex time function. So, if this represents complex time function for acceleration then X will be the actual acceleration as a function of time. If the thing in brackets represents complex time function for pressure than small x as a function of time will be the pressure actual pressure the real pressure; physically measuring pressure. So, this is complex and its real component is the physically measurable pressure which we will measure at a point of time. So, that is there. So, this is complex time functions.

Now, X as a real part and an imaginary part and same thing is true for S and S we typically write it as sigma plus j omega where sigma is the real part of complex frequency and omega is the imaginary part of complex frequency. So, we will do an example. So, that we understand this better actually we will do two examples.

So, the first example is X equals, so X is complex, it has an amplitude 4 e to the power of j pi over 4 and S let us say it is minus 1.1 plus 5 j. So, what is the overall signal?

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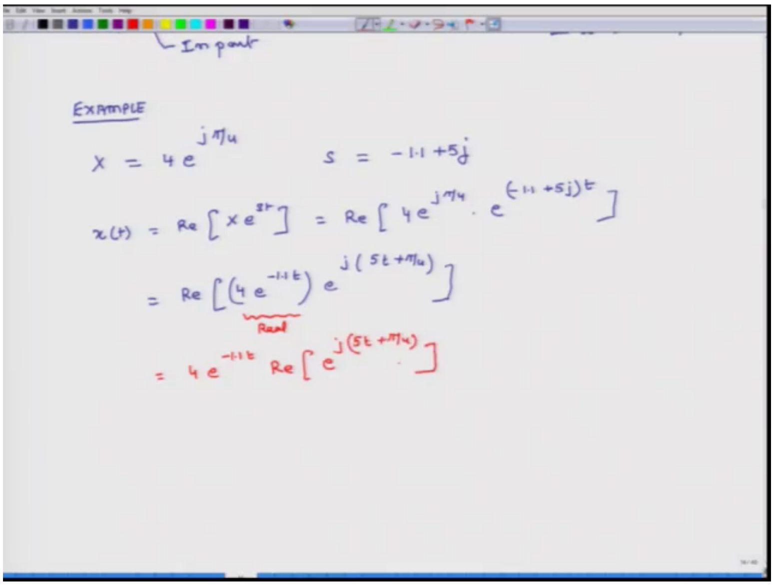

The actual signal let say X represents pressure then small x also represents the actual pressure capital X represents complex pressures then small x represents actual pressure. So, actual pressure is equal to real of X times e to the power of st based on the relation we wrote earlier and this is equal to real of 4 e j pi over 4 times e to the power of minus 1.1 plus 5 j t .

So, that is equal to real of. So, what I am going to do is I am going to rearrange all this. So, 4 e to the power of minus 1.1 t times e j 5 t plus pi by 4. Now, this is entirely real. So, I can take it out right away. So, I get 4 e to the power of minus 1.1 t times real of e to the power of j 5 t plus pi by 4. So, x t is 4 e to the power of minus 1.1 t cosine 5 t plus pi by 4.

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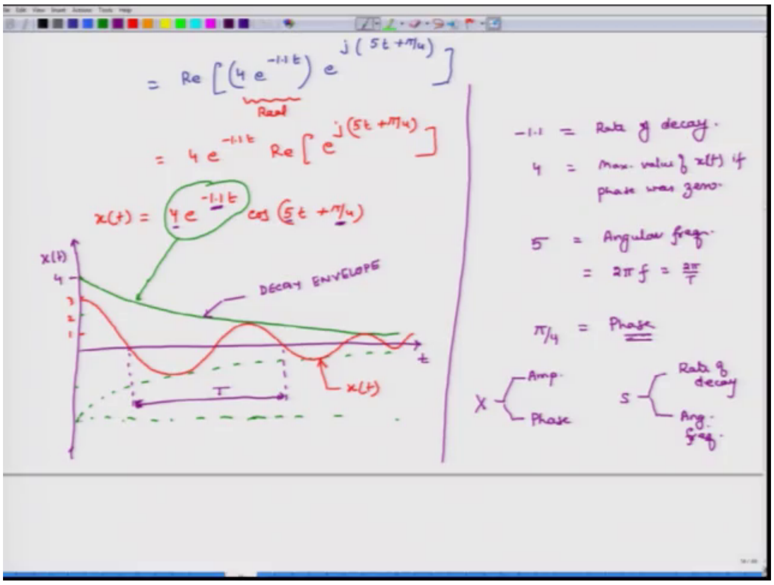

So, we will plot this. So, on the X axis I am going to plot time and on the y axis I am going to plot x of t, but before I plot the entire function I will just plot this function. First I will plot just this function. So, at time t is equal to 0 the function in green it has a value of 4 right. So, let say this is 2 4 and as time increases this value which is in green it becomes smaller. So, this curve will looks something like this the one in green and what I will do is I will take its mirror image like this I am just taking its mirror image, but the solid line represents 4 e to the power of minus 1.1 times t.

So, now, I am going to plot the entire function. At time t is equal to 0 the thing in the red the thing in green is having a value of 4, but at t is equal to 0, 5 t is 0. So, what is cosine pi by 4? At time t is equal to 0 is cosine of pi by 4, cosine pi by 4 is 1 over root 2. So, this entire function is 1 over root 2 times 4, 1 over root 2 is 0.707, so that is 2.8. This is 1 2 3.

So, it is slightly less than 3, 2.8 and this thing will have a cosine shape. So, it will look something like this like this. So, this is x t.

Now, let us try to understand the meaning of all the terms which we have seen till so far. So, this curve is called decay envelope it tells how fast this red curve is going to decay and the speed of this decay is determined by which parameter.

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It depends on one of this number right, now depends on 1.1. So, minus 1.1 is rate of decay. The next thing is 4, what is 4? 4 is the maximum possible value of this function maximum possible value, if this 5 pi by 4 was not there then the value of this function would have been 4 at t is equal to 0. So, this is the maximum value of x t if phase was 0. Next thing is 5. So, we have taken care of this guy we have taken care of this guy, let us look at 5, what is 5?

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Ah?

Student: Angular Frequency

It is the angular frequency. So, it is the angular frequency and this is equal to 2 pi times frequency of the system. So, if I know angular frequency 5 then from this I can calculate the frequency of the system which is 5 by 2 pi and yeah or I can also call it 2 pi over T where T is the time period. And what is the time period? It is 2 pi by 5, but physically this is the time period on the graph, that is a time period on the graph right. It is the distance between 2 peaks or 2 (Refer Time: 12:33) or 2 places where it is cutting the X axis in the same direction so that is the time period. And then the last thing is pi over 4 which is this, this is the phase ok.

So, X, remember X is the complex amplitude it has it gives us two information amplitude and phase. So, X tells us about amplitude and it tells us about the phase of the signal and S tells us about rate of decay and it also tells us about angular frequency, tells us about rate of decay and angular frequency. So, this is important to understand.

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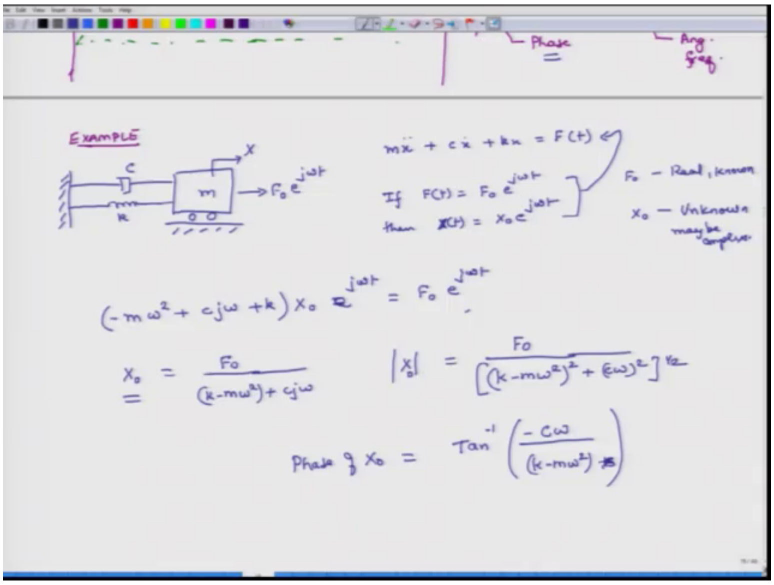

Let us look at a physical example, another example. Suppose I have spring dash part and mass system there is no friction suppose and applying a force F naught e j omega t then I will not into the mathematics maybe we will talk about it later. But the equation which governs the behavior of this systems is mx double dot plus cx dot plus kx is equal to F t and if F t is F naught e j omega t, what we will do is we will develop an expression. So, if F t equals F naught e j omega t then we can write that X t little X t is X naught e j omega t then if I make this substitutions what I get is m omega square plus c j omega plus k X naught e j omega t equals F naught e j omega t. So, here F naught S real and S naught if we do not know F naught is real and it is known because I am applying a known force which is vibrating and I want to figure out what is the value of X naught.

So, I do not know X naught and X naught can be real or complex. So, X naught unknown and may be complex. So, this is the equation I get. So, I plug both these things into this equation and I get this relation and then this e j omega t gets cancelled from both the sides so I get X naught is equal to F naught divided by k minus m omega square plus c j omega. So, X naught is a complex number, it is a complex amplitude of this motion, it is a complex amplitude of this motion and so this X as an amplitude and it as a phase X as an amplitude and it as a phase right that is what we are seeing here. X is a complex number and what is the amplitude of, amplitude of X naught is what? F naught by k minus omega square the c omega square. And X naught because it complex it also has a phase, phase of X naught is equal to tan inverse omega minus yeah this. So, in this case

X naught is the complex amplitude of the complex time signal its amplitude is this thing its phase is this.

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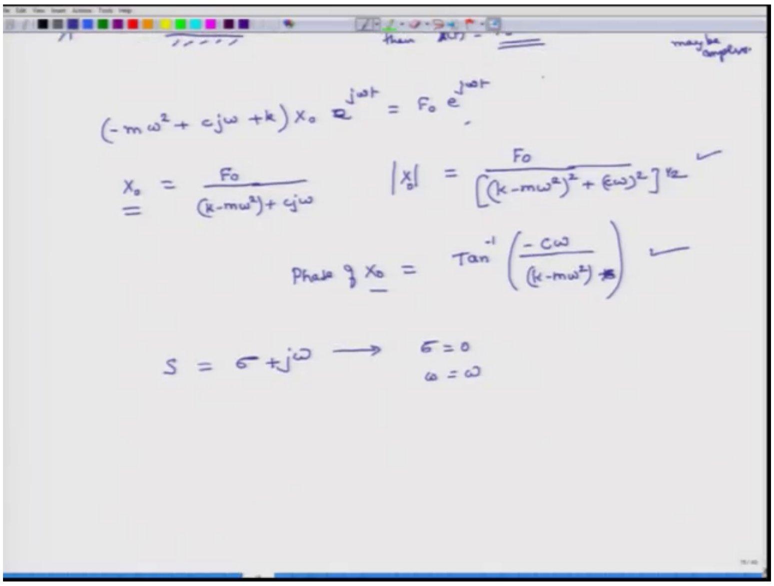

And what is the complex frequency? S is equal to sigma plus j omega, in this case what is the value of sigma now because the complex time signal is X naught e j omega t. So, sigma is equal 0 and omega equals omega because there is no sigma here, understood. So, that is what complex time signals are all about.

So, I think this concludes our discussion for today and tomorrow we will continue this discussion little bit more and we will also start talking about linear systems. With that have a great day and we will meet once again tomorrow.

Thank you.