Noise Management & Its Control
Prof. Nachiketa Tiwari
Department of Mechanical Engineering
Indian Institute of Technology, Kanpur

Lecture - 10
Complex Numbers

Hello. Welcome to Noise Measurement and its Control. Today is the fourth day of the second week of this course. And in the second half of this week what we plan to do is, explain to you a couple of very important concepts. Initially we will start by discussing in brief about complex numbers, and some of you may have forgotten how different operations in the world of complex numbers work. So, we will have a very quick review of complex numbers. Then we will explain you; what is a linear function, what is a linear system, and then we will close the week by discussing about transfer functions.

So, with this overall plan, we are going to start our discussions for today. And we shall start with talking about: we shall start with discussion on Complex Numbers.

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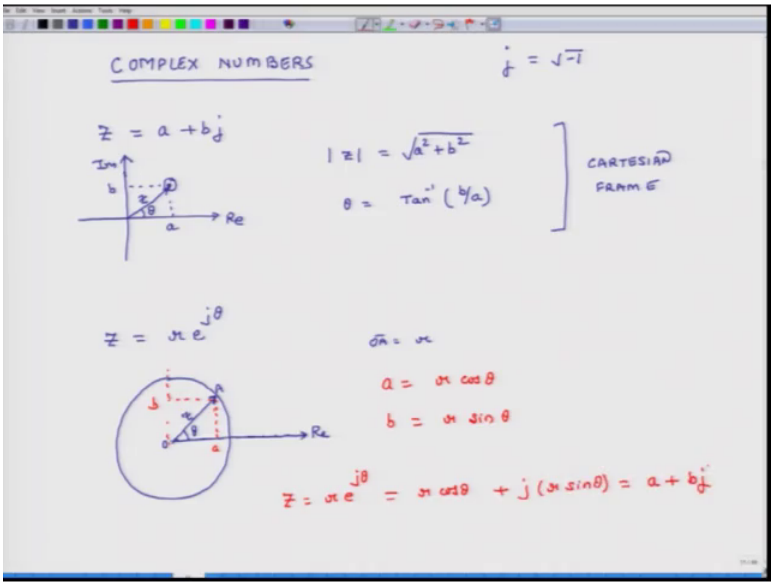

So, complex numbers we can express them in two formats. So, one is z, if it z is a complex number it has a real part, and it has an imaginary part; so a plus b j. Now in this discussion, in this course most of times we will not use this I, as the square root of negative 1, but rather we will define j as square root of negative 1, because a lot of times when we want to designate I, it will be used in the sense of current. So, instead of i square root of negative 1 is defined as j. So, z is equal to a plus b j this is one format.

So, if I have to plot this, this is my real axis, this is my imaginary axis. So, this is a, this is b, and this is z. So, that is y. Now the magnitude of z is a square plus b square, and this theta is tan inverse of b over a, tan inverse b over a. I can also express z in a polar format. So, this is in a Cartesian format, but in polar format, I can express z as a constant r a real numbers r, actually I will use smaller case r e j theta.

So, in this depiction this is a, this is my real axis, and this is z, this is theta, this is z. So, o a o a is the magnitude of this z r, and theta is the angular position angular coordinate. So, this is a polar. In a polar frame of reference, this is how I can explain a complex number. I can depict a complex number and I can move from Cartesian to polar, polar to Cartesian using some relations. So, if I want to map in Cartesian system, then this is a and this is b. So, a is equal to r cosine theta b is equal to r sin theta ok.

So, I can also express the same thing. So, I can express z equals r e j theta equals r cosine theta plus j times r sin theta, and I can also call this a plus b j. So, next we will look at some identities, important identities.

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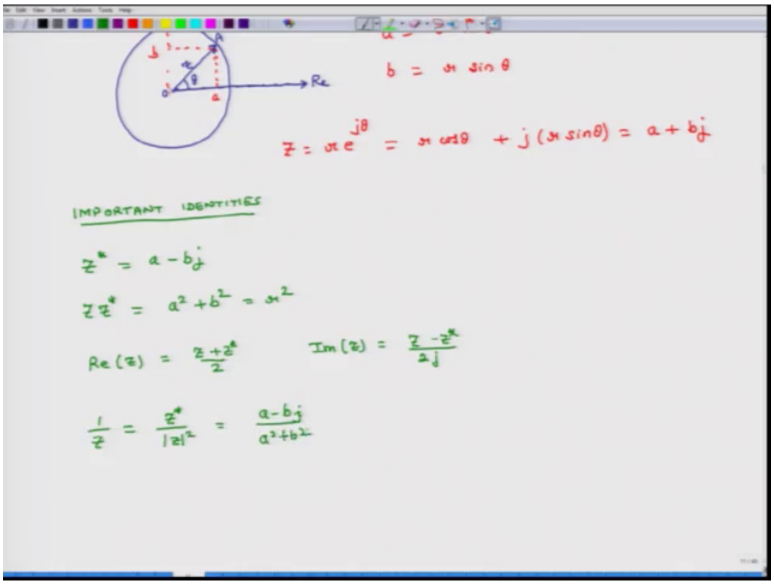

So, the first thing is that we define that z star is complex conjugate of z, and it is defined as a minus b j, and if that is the case then z z star is a square plus b square and that is equal to r square. Then we can also say that real of z is equal to z plus its complex conjugate divided by 2, and imaginary of z is equal to z minus z star divided by 2 j.

Another identity is that 1 over z equals z star by mod of z square, and this is equal to a minus b j divided by a square plus b square, and then there are some numbers.

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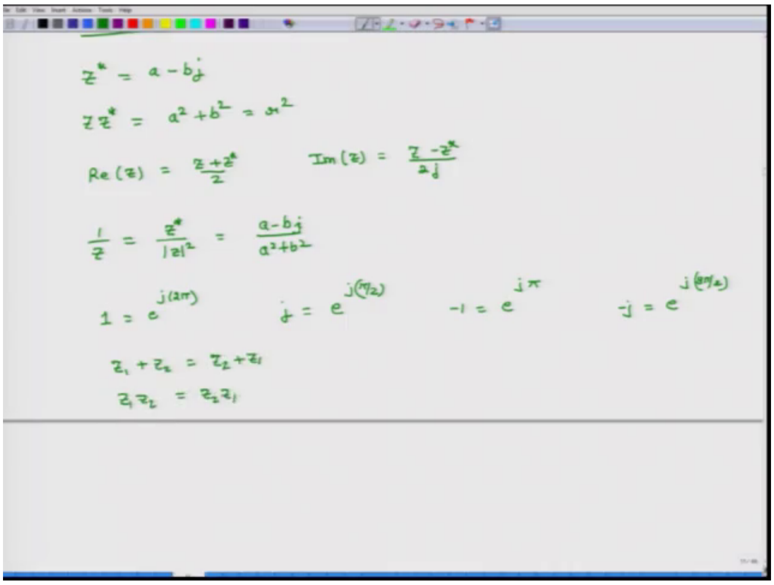

So, 1, I can express 1 as e to the power of j times 2 pi, because if I use the Euler's formula then e to the power of j times 2 pi is cosine of 2 pi plus j times sin of 2 pi, it comes to be 1 j equals e to the power of j pi by 2 minus 1 equals e to the power of j times pi and minus j equals e to the power of j times 3 pi by 2, and then lastly some two other identities, which I will write down. So, when we add up z and two complex numbers it is commutative.

So, z 1 plus z 2 equals z 2 plus z 1, and z 1 z 2 equals z 2 z 1. So, now, we will do a couple of examples. So, that we become a little more comfortable.

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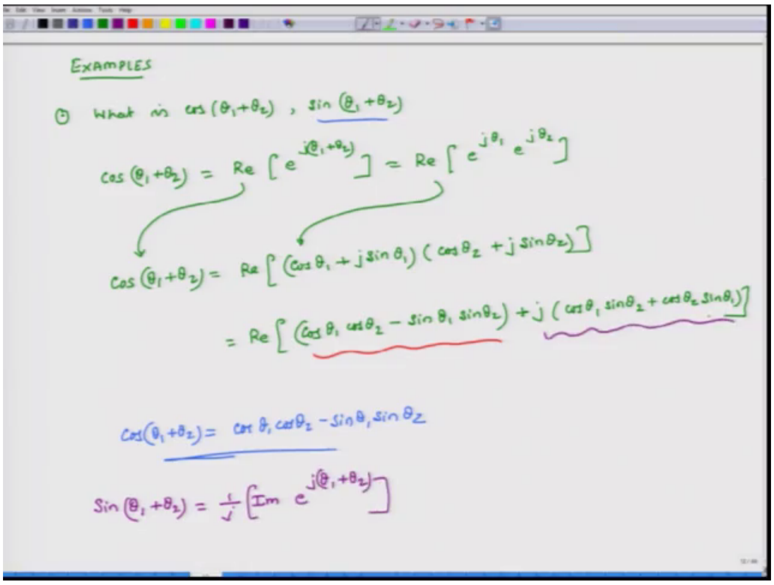

Examples: so what is cosine of theta 1 plus theta 2, and sin of theta 1 plus theta 2. Suppose I want to develop expressions for cosine of some sub two angles, and sine of sums of two angles: one way to figure out this relation is that cosign of theta 1 plus theta 2 is what real of e to the power of j theta real of e to the power of theta 1 plus theta 2.

So, this is equal to real of e to the power of j theta 1 e to the power of j theta 2. So, from this I get cosine of theta 1 plus theta 2 is equal to. From this side I get real of cosine theta 1 plus j sin theta 2 j sin theta 1 cosine theta 2 plus j sin theta 2, and this I get cosine theta 1, cosine theta 2 minus sin theta 1 sin theta 2 plus j cosine theta 1 sin theta 2 plus cosine theta 2 sin theta 1. So, we note that the real of left side is just this thing. So, I can write cosine theta 1 plus theta 2 is equal to cosine theta 1 cosine theta 2 minus sin theta 1 sin theta 2, this is one example.

Likewise, I can also develop expression for sin theta 1 sin theta 2. So, I will not do the entire exercise, but I will just start, and then you can do the rest in your homes. So, sin theta 1 plus sin theta 2 is equal to 1 over j imaginary of e to the power of j theta 1 plus theta 2, and actually when you do the math, this is the term you end up with, and when you divide it by j, we get this, you do two more examples.

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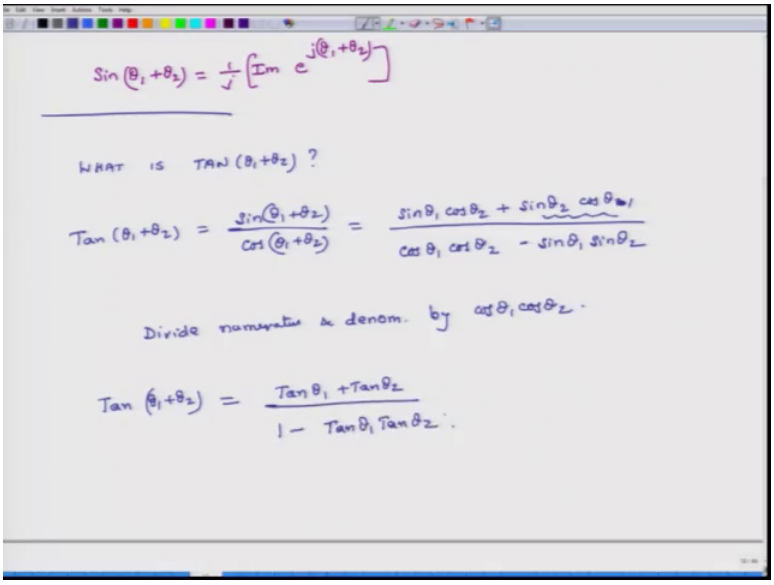

So, what is tan of theta 1 plus theta 2. So, tan of theta 1 0 plus theta 2 is equal to sin of theta 1 plus theta 2 divided by cosine of theta 1 plus theta 2, and we just now learnt what are the values of sin theta 1 plus theta 2 and cosine theta plus theta 2.

So, we will just plug in those numbers. So, sin theta 1 cosine theta 2 plus sin 2 cosine theta 1. Oops divided by cosine theta 1 cosine theta 2 minus sin theta 1 sin theta 2, and what we do is, we divide the numerator numerator and denominator by cosine theta 1 cosine theta 2. So, what we get tan theta 1 plus tan theta 2. So, if I divide sin theta 1 cosine theta 2 by cos theta 1 cos theta 2. I get tan theta 1, when I do the operation of this guy, I get tan theta 2 and then as denominator I am left with 1 minus tan theta 1 tan theta 2.

One last question we will do is.

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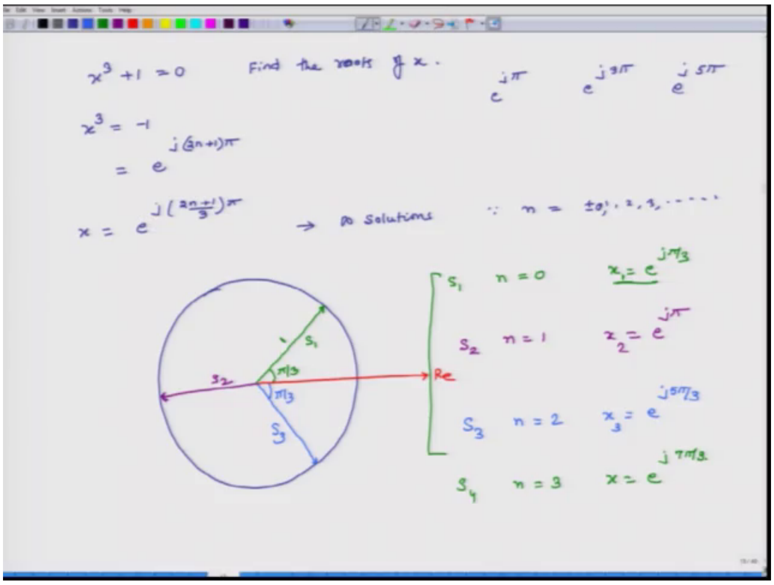

If y equals minus 1, or I will say that y cube equals minus 1, actually 1 reframe it as x cubed plus 1 equals 0. So, suppose this is an equation, find the roots of x. So, x equals minus 1, and I can express minus 1 as e to the power of what j what times how much. So, e to the power of j pi is a is equal to minus 1 e to the power of j times 3 pi is also equal to minus 1 e to the power of j times 5 pi is also equal to minus 1 and so on and so forth. So, I can say that it is 2 n plus 1 pi. So, x equals e to the power of j 2 n plus 1, three times pi. Now we will see what. So, how many solutions we have.

So, these are n finite solutions, because n is ranging from plus minus 1 to 3 and so on and so forth and infinite solutions. So, we will plug these solutions. So, for n is equal to 0. So, this is, actually this could be 0 also. So, n is equal to 0 x is equal to e to the power of j times pi over 3 pi over 3 is 60 degrees. So, and what is the amplitude the modulus of x 1, it is 1. So, modulus is 1, this is pi over 3 radians; n is equal to 0, n is equal to 1. in that case x equals e to the power of j pi.

So, the second solution is this guy right. So, this is solution one, this is solution two, and S 3 actually this is not visible easily. So, S 3 n is equal to 2, when n equals 2, then it becomes 4 plus 1 5 5 by 3 x is equal to e to the power of j 5 pi over 3, which is minus 60 degrees S 4 n is equal to 3, if n is equal to 3, then what x is equal to e to the power of j 7 pi by 3, which is same as the first solution.

So, these first three solutions repeat themselves. So, they are infinite solutions, but they are repeating the three unique solutions are S 1 S 2 S 3, these are the three roots. So, the first root is x 1 is equal to e to the power of j pi over 3. Second one is, second root is x 2 equals e to the power of j pi, and the third root is x 3 equals e to the power of j times 5 pi over 3. So, these are three unique solutions for this problem.

So, this covers our discussion on complex numbers. This was a quick preview of complex numbers. And what we will do in the next class is, we will start discussing about transfer functions, complex time signals, linear systems and so on and so forth. So, with that we close for today and have a nice day.

Thanks.